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Adaptive Granular-Ball Concept-Cognitive Learning for Efficient and Robust Fuzzy Knowledge Representation in Classification Tasks

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***Abstract*—In many real-world scenarios, data unreliability and information-processing uncertainty seriously affect the credibility and reliability of artificial intelligence models. Probing a valuable cognitive learning paradigm is crucial for supporting and implementing the proposed artificial intelligence hypothesis. Concept-cognitive learning (CCL), as an emerging cognitive learning technology, investigates the knowledge-learning process using limited data through the lens of concepts. However, the existing CCL research has one prevalent disadvantage: instability and complex cognition. Thus, this work focuses on constructing an effective and interpretable cognitive learning paradigm for acquiring knowledge. Inspired by a multi-granularity knowledge representation, this paper proposes an adaptive and robust fuzzy knowledge representation paradigm, namely granular-ball concept-cognitive learning(GB-CCL). Unlike existing CCL systems, GB-CCL is a non-parameter cognitive learning system that achieves more accurate, faster adaptive capabilities for concept learning. The concept acquired through cognitive learning is structured conceptual knowledge, which is naturally interpretable and robust. Finally, extensive experiments verify that GB-CCL is an effective cognitive learning method with significantly better performance than state-of-the-art models in classification and robust learning.**

***Index Terms*—Concept-cognitive learning, granular computing, granular-ball computing, fuzzy concept, knowledge discovery.**

I. Introduction

DESPITE the storm of artificial intelligence that has swept the world and deeply affected the development of all walks of life, there are many tough and common problems when different artificial intelligence methods encounter complex scenarios, such as credibility, reliability, and interpretability. Simulating human cognitive processes is fundamental in artificial intelligence and cognitive computing [1], [2]. In exploring how to effectively cognitive and learning a thing, granular computing(GrC) may be viewed as a novel, interesting, and interpretable theory and technology. One emphasizes observing, analyzing, and understanding things at multiple granularities, perspectives, and levels [3]–[5]. Undoubtedly, granular computing has become an active research direction in artificial intelligence and cognitive computing.

Granular-ball computing(GBC) and concept-cognitive learning are two influential GrC studies on information processing and cognitive learning, with different emphases. According to Chen [6], "human cognition has the characteristics of large-scale priority." Then, Xia et al. [7] proposed an efficient, robust, and interpretable multi-granularity computing paradigm, named granular-ball computing. One can depict the distribution characteristics of the sample in the data space and thereby improve the classification performance of rough sets (RSs). Beyond the traditional RSs, the granular ball (GB) has the advantage of being fast and accurate in data analysis. In some sense, the GBC model can simultaneously express both the classical and the neighborhood RSs, enabling it not only to handle continuous data but also to use equivalence classes for knowledge representation. Moreover, extended GBC models have been investigated in many ways to address different problem needs. For instance, papers [8] utilize the accurate classification advantage of GB to propose the GB-KNN for enhancing the classification performance of KNN. GRRS is presented in the paper [9] to improve neighborhood RSs by taking advantage of the adaptive learning of GB. Building upon this, Xia et al. [10] further proposed an effective granular-ball generation approach for general classification tasks. Cao et al. [11] investigated GB-based open and continual feature selection in dynamic environments. Li et al. [12] introduced a granular-ball regeneration clustering method grounded in the principle of justifiable granularity.

Concept-cognitive learning is the science of cognition and learning things via concepts [13]–[15]. As we all know, the concept is the basic unit of thought underlying human intelligence and communication, and the cognitive and learning concepts are important issues for studying granular computing and artificial intelligence [16], [17]. Currently, concept-cognitive learning, an emerging artificial intelligence method for concept learning and knowledge discovery, has received much attention. For instance, Li et al. [18] analyze the concept-learning mechanism from a cognitive psychology principle. Shi et al. [19] establish an incremental CCL model for classi-

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fication tasks. Niu et al. [20] put forward a parallel computing method for concept-cognitive learning. Shi et al. [21] proposed a concurrent CCL method. Zhang et al. [22] studied CCL based on attribute topology. Deng et al. [23] offered a multi-conceptual information acquisition for hierarchical classification. Lin et al. [24] analyzed a concept reduction from a global relevance and redundancy viewpoint. With the in-depth study of CCL in knowledge modeling and method application, some researchers are concerned that it can be combined with machine learning to broaden the research horizon of this theory further. In recent papers, Mi et al. proposed a fuzzy-based CCL model [25] and a semi-supervised CCL method [26]. Guo and Xu successively put forward a series of CCL models for complex data analysis, such as two-way CCL [27], memory-based CCL [28], and fuzzy-granular CCL [29], etc. In addition, Wang et al. [30] proposed a multiview CCL framework that integrates high-order information fusion of fuzzy attributes. Liu et al. [31] further explored a stochastic CCL model for missing multi-label learning. In some sense, CCL aims to explore accurate and effective mechanisms for knowledge discovery through cognitive and learning concepts.

Following the no-free-lunch theorem, we find that although both are popular intelligence information-processing approaches, they have unavoidable deficiencies: GBC sacrifices precision for speed, and CCL sacrifices speed for precision. In this sense, a fruitful marriage of the two may be a good choice for knowledge discovery, as it can help eliminate mutual deficiencies. From an extender view of CCL and GB, we built a granular-ball concept cognitive learning by introducing the granular-ball, three-way concept analysis to the cognitive concept process for high-efficiency exploring knowledge from complex data. The main idea of this work is three-fold: 1) the granular-ball in a fuzzy decision formal context, 2) the justifiable granular-ball generation for concept learning, and 3) the label prediction based on concept recognition. By doing so, GB-CCL achieves highly accurate classification and significantly outperforms its peers. Furthermore, we summarize the main contributions as follows:

- It proposes a novel and adaptive granular-ball concept-cognitive learning framework for fuzzy knowledge representation in classification tasks, namely GB-CCL. Extensive experiments demonstrate the robustness and effectiveness of the proposed GB-CCL method.
- It introduces a fuzzy three-way concept learning mechanism based on granular balls in a fuzzy decision formal context to improve the robustness of concept learning, which leverages granular-ball computing and three-way concept analysis to enhance the robustness of the GB-CCL.
- It designs a justifiable granular-ball generation model for concept learning to acquire structured conceptual knowledge from complex data. Beyond the traditional CCL, GB-CCL enables fast and adaptive acquisition of structured conceptual knowledge.
- It builds an effective label-prediction method based on the fuzzy three-way concept center in the concept-cognitive process, thereby enhancing the concept recognition performance of fuzzy data.

The remainder is organized as follows: Section II revisits several necessary notions for GB-CCL. A granular-ball concept learning mechanism is introduced in Section III. The proposed GB-CCL method is shown in Section IV. In Section V, the experimental analysis is reported, and then this paper is concluded in Section VI for further investigation.

II. Preliminaries

Considering that the proposed GB-CCL is based on the ideas of concept-cognitive learning and granular-ball computing, this section briefly reviews some basic knowledge with regard to them. A more detailed description can be found in the corresponding references [10], [27], [32]. As is well known, the standard CCL methods cannot tackle continuous data directly, and the fuzzy-based CCL is a straightforward approach to discovering the knowledge embedded in continuous data. Nevertheless, in the real scenario, many tasks are described with numerical(or fuzzy) data, especially in classification problems. Hence, throughout this paper, we will consider a fuzzy formal context to build the model.

A quintuple $(G, M, \widetilde{I}, D, J)$ is a regular fuzzy decision formal context, where $(G, M, \widetilde{I})$ and (G, D, J) are, respectively, a conditional formal context and a decision formal context.

- $G = \{x_1, x_2, \dots, x_n\}$ is a nonempty set of objects.
- $M = \{a_1, a_2, \dots, a_m\}$ is a nonempty set of attributes.
- $\widetilde{I} : G \times M \to [0, 1]$ is a fuzzy relation on $G \times M$, where $\widetilde{I}(x, a)$ denotes the fuzzy membership degree of x with respect to a.
- $D = \{d_1, d_2, \dots, d_l\}$ is a decision label set, which induces a decision partition $G/D = \{D_1, D_2, \dots, D_l\}$, where $D_i = \{x \in G \mid J(x, d_i) = 1\}$, $i = 1, \dots, l,$.
- $J : G \times D \to \{0, 1\}$ is a binary relation between objects and decision labels.

A. Concept-cognitive learning system

Let 2^G and 2^D be two power sets of G and D, Γ^M is the set of all fuzzy sets on M. Suppose that $\widetilde{\mathcal{F}} : 2^G \to \Gamma^M$ and $\mathcal{P} : \Gamma^M \to 2^G$ are two set-valued mappings. For simplicity, they are rewritten as $\widetilde{\mathcal{F}}$ and $\mathcal{P}$, respectively. For any $X_1, X_2 \subseteq G$ and $\widetilde{B} \in \Gamma^M$, the pair $\widetilde{\mathcal{F}}$ and $\mathcal{P}$ is called cognitive operators, if the following statements hold:

1) $X_1 \subseteq X_2 \Rightarrow \widetilde{\mathcal{F}}(X_2) \subseteq \widetilde{\mathcal{F}}(X_1)$;
2) $\widetilde{\mathcal{F}}(X_1 \cup X_2) \subseteq \widetilde{\mathcal{F}}(X_1) \cap \widetilde{\mathcal{F}}(X_2)$;
3) $\mathcal{P}(\widetilde{B}) = \{x \in G | \widetilde{B} \subseteq \widetilde{\mathcal{F}}(x)\}$.

Definition 1. *Let $FC = (G, M, \widetilde{I}, D, J)$ be a fuzzy decision formal context, for any $X \subseteq G$ and $\widetilde{B} \in \Gamma^M$, the cognitive operators $\widetilde{\mathcal{F}}$ and $\mathcal{P}$ can be defined as follows:*

$$\widetilde{\mathcal{F}}(X)(a) = \bigwedge_{x \in X} \widetilde{I}(x, a), a \in M, \tag{1}$$

$$\mathcal{P}(\widetilde{B}) = \{x \in G | \widetilde{B}(a) \leq \widetilde{I}(x, a), \forall a \in B\}. \tag{2}$$

Generally speaking, $\widetilde{\mathcal{F}}$ and $\mathcal{P}$ are called a pair of positive cognitive operators. A pair $(X, \widetilde{B})$ is called a fuzzy concept if $\widetilde{\mathcal{F}}(X) = \widetilde{B}$ and $\mathcal{P}(\widetilde{B}) = X$, where X and $\widetilde{B}$ are referred to as the extent and intent of the fuzzy concept $(X, \widetilde{B})$. Similarly, the negative cognitive operators can be defined as follows.

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Definition 2. *Given a fuzzy decision formal context* $FC = (G, M, \widetilde{I}, D, J)$*, for any* $X \subseteq G$ *and* $\widetilde{B} \in \Gamma^M$*, the negative cognitive operators* $\widetilde{\mathcal{F}}^- : 2^G \rightarrow \Gamma^M$ *and* $\mathcal{P}^- : \Gamma^M \rightarrow 2^G$ *are defined as:*

$$\widetilde{\mathcal{F}}^-(X)(a) = \bigwedge_{x \in X} \widetilde{I}^-(x, a), a \in M, \tag{3}$$

$$\mathcal{P}^-(\widetilde{B}) = \{x \in G | \widetilde{B}(a) \leq \widetilde{I}^-(x, a), \forall a \in B\}. \tag{4}$$

where $\widetilde{I}^- = \{< (x, a), 1 - \widetilde{I}(x, a) > | (x, a) \in G \times M\}$ *denotes the complement of* $\widetilde{I}(x, a)$*, representing the non-membership degree of object* x *with respect to* a*.*

Definitions 1 and 2 show that the positive and negative cognitive operators describe the cognitive process between objects and attributes. Meanwhile, some studies [28], [32] combine these two into a unique cognitive operator for simultaneously expressing positive and negative information.

Definition 3. *Given* $FC = (G, M, \widetilde{I}, D, J)$*, for any* $X \subseteq G$ *and* $\widetilde{B_1}, \widetilde{B_2} \in \Gamma^M \times \Gamma^M$*, the fuzzy three-way concept cognitive operator* $\widetilde{\mathcal{F}}^\nabla : 2^G \rightarrow \Gamma^M \times \Gamma^M$ *and* $\mathcal{P}^\nabla : \Gamma^M \times \Gamma^M \rightarrow 2^G$ *are defined as:*

$$\widetilde{\mathcal{F}}^\nabla(X) = (\widetilde{\mathcal{F}}(X), \widetilde{\mathcal{F}}^-(X)), \tag{5}$$

$$\mathcal{P}^\nabla(\widetilde{B_1}, \widetilde{B_2}) = \mathcal{P}(\widetilde{B_1}) \cap \mathcal{P}^-(\widetilde{B_2}). \tag{6}$$

Then, $(X, (\widetilde{B}_1, \widetilde{B}_2))$ is called a fuzzy three-way concept if $\widetilde{\mathcal{F}}^\nabla(X) = (\widetilde{B}_1, \widetilde{B}_2), \mathcal{P}^\nabla(\widetilde{B}_1, \widetilde{B}_2) = X$. More details on fuzzy concept or fuzzy three-way concept can be found in references [25], [32], [33].

B. Granular-ball computing

Granular-ball computing, an important extension of neighborhood rough sets, provides a new multi-granularity computing framework for characterizing data distribution using a pair of radius and center. Thus, it is necessary to introduce the notion of the neighborhood granular in this subsection.

Given $FC = (G, M, \widetilde{I}, D, J)$, for any $x \in G$ and $\widetilde{B} \in \Gamma^M$, the neighborhood granular of x with respect to attribute subset $B \subseteq M$ is defined as:

$$\delta_{\widetilde{B}}(x) = \{x' \in G | dis(x, x') \leq \delta\}, \tag{7}$$

where $dis(x, x') = \sqrt{\sum_{a \in \widetilde{B}} ||\widetilde{I}(x, a) - \widetilde{I}(x', a)||^2}$ and $|| \cdot ||$ denotes the 2-norm, $\delta > 0$ is a distance threshold.

Although the neighborhood relationship can be defined by a radius that captures the similarity between any two samples, selecting a radius is also a significant challenge, particularly in terms of flexibility and generalization. A granular ball, however, can overcome this challenge by defining a center and a radius.

Compared with the neighborhood granular, the significant advantage of GBC is that each granule can be generated adaptively based on the data distribution, and the radius does not need to be set in advance. The radius equals the average distance from all instances in the granular ball to its center, and the radius can also be set as the maximum distance. Hence, a granular ball with a center and radius can be used as input to the learning strategy or as precise measures to represent the sample space in a fuzzy decision formal context, thereby achieving multi-granularity cognitive computing characteristics and accurately describing the instance space of data. Besides, granular-ball generation is another important issue of GBC. As analyzed in paper [10], GB generation can be incorporated into GBC to enhance the classification performance. A detailed granular-ball generation process can be found in Fig. 1.

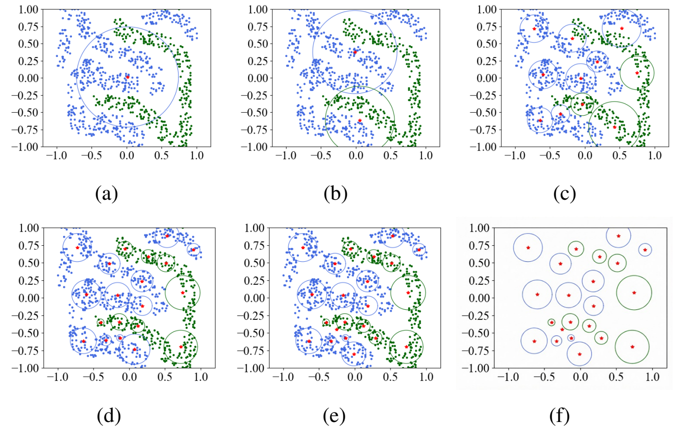

Fig. 1. Granular ball generation process in [10].

III. Granular-ball Concept Learning

This section presents new notions and the related theoretical analysis of the granular-ball concept in fuzzy decision formal contexts. Motivated by GBC and CCL, we offer a granular-ball concept learning model that integrates the merits of granular-ball, fuzzy three-way concept, and concept learning.

A. Granular-ball in fuzzy decision formal context

Generally speaking, fuzzy concept learning is more dependent on initial information, such as a granular concept learning based on object-oriented information was offered in [25], a progressive fuzzy concept according to neighborhood information granular was proposed in [32], and a fuzzy concept learning via cosine similarity granular was studied in [28]. Meanwhile, to enhance the effectiveness of GB-CCL, this subsection primarily defines a novel information granule, i.e., the granular-ball. Building on this thought, we present a granular ball concept learning for the subsequent concept-cognitive process, which can be described as follows.

Definition 4. *Let* $GBs = \{GB_1, GB_2, \ldots, GB_u\}$ *be a set of granular-balls. For any* $GB_i^{D_k}$ *associated with* D_k*, its center* c_i*, radius* r_i*, and purity* $Pur(GB_i^{D_k})$ *are defined as follows:*

$$GB_i^{D_k} = \{x \in G | \sqrt{||\widetilde{\mathcal{F}}(x) - c_i||^2} \leq r_i\}, \tag{8}$$

$$c_i = \frac{1}{|GB_i^{D_k}|} \sum_{x \in GB_i^{D_k}} \widetilde{\mathcal{F}}(x), \tag{9}$$

$$r_i = \frac{1}{|GB_i^{D_k}|} \sum_{x \in GB_i^{D_k}} \sqrt{||\widetilde{\mathcal{F}}(x) - c_i||^2}, \tag{10}$$

$$Pur(GB_i^{D_k}) = \frac{\max_{1 \leq j \leq k} |GB_i^{D_k} \cap D_j|}{|GB_i^{D_k}|}, \tag{11}$$

where $|\cdot|$ denote the cardinality of a set.

Definition 5. *Let $\mathcal{S}^{D_k} = \{GB_1^{D_k}, GB_2^{D_k}, \ldots, GB_s^{D_k}\}$ be a set of granular balls associated with D_k, where $s = |\mathcal{S}^{D_k}|$. We say that $\mathcal{S}^{D_k}$ is a* granular-ball subspace *of D_k if the following properties hold:*

1) $\forall GB_i^{D_k} \in \mathcal{S}^{D_k}$, $GB_i^{D_k} \neq \emptyset$.
2) $\forall GB_i^{D_k} \in \mathcal{S}^{D_k}$, $GB_i^{D_k} \subseteq D_k$.
3) $\bigcup_{i=1}^{s} GB_i^{D_k} = D_k$.
4) $min\ |\mathcal{S}^{D_k}|$, s.t. items 1) and 2) hold.

As indicated in definitions 4 and 5, we only consider the basic form of GB for a fuzzy decision formal context, and a more concrete agenda for GB generation is presented in Section IV. Furthermore, as discussed above, we can learn concepts using the granular-ball and cognitive operators.

B. Fuzzy three-way concept based on GB

As analyzed in [27], [28], [32], the effectiveness of the three-way concept lies in its emphasis on simultaneously characterizing information from both positive and negative perspectives, so three-way concept analysis can be incorporated into a CCL model to improve its accuracy in describing cognitive processes, especially in a fuzzy environment. Following this principle, we incorporate the granular-ball into the concept learning with the fuzzy three-way concept to model a novel fuzzy concept learning mechanism.

Theorem 1. *Given $FC = (G, M, \widetilde{I}, D, J)$ and $\mathcal{S}^{D_k} = \{GB_1^{D_k}, GB_2^{D_k}, \ldots, GB_s^{D_k}\}, s = 1, 2, \ldots, |D_k|$ with $G/D = \{D_1, D_2, \ldots, D_l\}$ on FC. For any $GB_j^{D_k} \subseteq D_k$, $(\mathcal{P}\widetilde{\mathcal{F}}(GB_1^{D_k}) \cap \mathcal{P}^{-}\widetilde{\mathcal{F}}^{-}(GB_1^{D_k}), (\widetilde{\mathcal{F}}(GB_1^{D_k}), \widetilde{\mathcal{F}}^{-}(GB_1^{D_k})))$ is a fuzzy three-way concept.*

Proof: To prove this theorem, one can divide it into two steps:

1) prove $\mathcal{P}^{\nabla}(\widetilde{\mathcal{F}}(GB_i^{D_k}), \widetilde{\mathcal{F}}^{-}(GB_i^{D_k})) = \mathcal{P}\widetilde{\mathcal{F}}(GB_i^{D_k}) \cap \mathcal{P}^{-}\widetilde{\mathcal{F}}^{-}(GB_i^{D_k})$ is valid;

2) prove $\widetilde{\mathcal{F}}^{\nabla}(\mathcal{P}\widetilde{\mathcal{F}}(GB_i^{D_k}) \cap \mathcal{P}^{-}\widetilde{\mathcal{F}}^{-}(GB_i^{D_k})) = (\widetilde{\mathcal{F}}(GB_i^{D_k}), \widetilde{\mathcal{F}}^{-}(GB_i^{D_k}))$ is valid.

- For 1), according to definition 3, we have
$$\mathcal{P}^{\nabla}(\widetilde{\mathcal{F}}(GB_i^{D_k}), \widetilde{\mathcal{F}}^{-}(GB_i^{D_k})) = \mathcal{P}\widetilde{\mathcal{F}}(GB_i^{D_k}) \cap \mathcal{P}^{-}\widetilde{\mathcal{F}}^{-}(GB_i^{D_k}).$$
- For 2), according to definition 2, we further obtain
$$\begin{aligned}&\mathcal{P}\widetilde{\mathcal{F}}(GB_i^{D_k}) \cap \mathcal{P}^{-}\widetilde{\mathcal{F}}^{-}(GB_i^{D_k})\\ &= \{y \in D_k | \widetilde{I}(y,a) \geq \textstyle\bigwedge_{o \in GB_i^{D_k}} \widetilde{I}(o,a),\ a \in M\}\\ &\cap \{y \in D_k | \widetilde{I}^{-}(y,a) \geq \textstyle\bigwedge_{o \in GB_i^{D_k}} \widetilde{I}^{-}(o,a), a \in M\}\\ &= \{y \in D_k | \widetilde{I}(y,a) \geq \textstyle\bigwedge_{o \in GB_i^{D_k}} \widetilde{I}(o,a), a \in M\}\\ &\cap \{y \in D_k | 1 - \widetilde{I}^{-}(y,a) \leq 1 - \textstyle\bigwedge_{o \in v} \widetilde{I}^{-}(o,a), a \in M\}\\ &= \{y \in D_k | \widetilde{I}(y,a) \geq \textstyle\bigwedge_{o \in GB_i^{D_k}} \widetilde{I}(o,a), a \in M\}\\ &\cap \{y \in D_k | \widetilde{I}(y,a) \leq \textstyle\bigvee_{o \in GB_i^{D_k}} \widetilde{I}(o,a), a \in M\}\\ &= \{y \in D_k | \textstyle\bigwedge_{o \in GB_i^{D_k}} \widetilde{I}(o,a) \leq \widetilde{I}(y,a) \leq \textstyle\bigvee_{o \in GB_i^{D_k}} \widetilde{I}(o,a), a \in M\}.\end{aligned}$$
Thus, for any $a \in M$, we have
$\widetilde{\mathcal{F}}(\mathcal{P}\widetilde{\mathcal{F}}(GB_i^{D_k}) \cap \mathcal{P}^{-}\widetilde{\mathcal{F}}^{-}(GB_i^{D_k})(a) = \bigwedge_{o \in GB_i^{D_k}} \widetilde{I}(o,a) = \widetilde{\mathcal{F}}(GB_i^{D_k})(a)$.
Meanwhile, for any $a \in M$, we obtain
$\mathcal{P}\widetilde{\mathcal{F}}(GB_i^{D_k}) \cap \mathcal{P}^{-}\widetilde{\mathcal{F}}^{-}(GB_i^{D_k}) = \{y \in G_i | \bigwedge_{o \in GB_i^{D_k}} \widetilde{I}^{-}(o,a)\} \leq \widetilde{I}^{-}(y,a) \leq \bigvee_{o \in GB_i^{D_k}} \widetilde{I}^{-}(o,a), a \in M\}, \widetilde{\mathcal{F}}^{-}(\mathcal{P}\widetilde{\mathcal{F}}(GB_i^{D_k}) \cap \mathcal{P}^{-}\widetilde{\mathcal{F}}^{-}(GB_i^{D_k}))(a) = \bigwedge_{o \in GB_i^{D_k}} \widetilde{I}^{-}(o,a) = \widetilde{\mathcal{F}}^{-}(GB_i^{D_k})(a)$.
Hence,
$$\widetilde{\mathcal{F}}^{\nabla}(\mathcal{P}\widetilde{\mathcal{F}}(GB_i^{D_k}) \cap \mathcal{P}^{-}\widetilde{\mathcal{F}}^{-}(GB_i^{D_k})) = (\widetilde{\mathcal{F}}(GB_i^{D_k}), \widetilde{\mathcal{F}}^{-}(GB_i^{D_k})).$$

By combining 1) and 2), this theorem is proven.

Based on Theorem 1, we can obtain structured conceptual knowledge by defining a new fuzzy concept, the granular ball, from both positive and negative perspectives. In this way, the fuzzy three-way concepts learned have the advantage of being more robust and precise than the way in [13], [25], [28], [32]. And then, we can build the corresponding fuzzy three-way concept space as follows.

Definition 6. *Let $\mathcal{S}^{D_k} = \{GB_1^{D_k}, GB_2^{D_k}, \ldots, GB_s^{D_k}\}(s = 1, 2, \ldots, |D_k|)$ be a granular-ball subspace on FC. The fuzzy three-way concept subspace $\widetilde{\mathcal{G}}^{D_k}$ based on granular-balls is defined as:*
$$\begin{aligned}\widetilde{\mathcal{G}}^{D_k} &= \{(\mathcal{P}\widetilde{\mathcal{F}}(GB_i^{D_k}) \cap \mathcal{P}^{-}\widetilde{\mathcal{F}}^{-}(GB_i^{D_k}), (\widetilde{\mathcal{F}}(GB_i^{D_k}),\\ &\quad \widetilde{\mathcal{F}}^{-}(GB_i^{D_k})))| GB_i^{D_k} \in \mathcal{S}^{D_k}\}.\end{aligned} \tag{12}$$

Theorem 2. *Let $\mathcal{S}^{D_k}$ be a granular-ball subspace of D_k, and $\widetilde{\mathcal{G}}^{D_k}$ be the corresponding fuzzy three-way concept subspace constructed from $\mathcal{S}^{D_k}$. Then the following property holds:*
$$1 \leq |\widetilde{\mathcal{G}}^{D_k}| \leq |\mathcal{S}^{D_k}|. \tag{13}$$

Proof: The proof can be divided into four cases:

1) If $|\mathcal{S}^{D_k}| = 1$, i.e., D_k contains only one granular-ball, then the fuzzy three-way concept subspace generated from it has exactly one concept. Thus, $|\widetilde{\mathcal{G}}^{D_k}| = 1$.
2) If $|\mathcal{S}^{D_k}| > 1$, i.e., D_k contains multiple granular-balls and for each $x \in D_k$, there exists at least one $GB_i^{D_k} \in \mathcal{S}^{D_k}$ such that $x \in GB_i^{D_k}$, then at least two concepts will be generated. Hence, $|\widetilde{\mathcal{G}}^{D_k}| \geq 1$.
3) If exist $GB_i^{D_k}, GB_j^{D_k} \in \mathcal{S}^{D_k}$ such that $(\mathcal{P}\widetilde{\mathcal{F}}(GB_i^{D_k}) \cap \mathcal{P}^{-}\widetilde{\mathcal{F}}^{-}(GB_i^{D_k}), (\widetilde{\mathcal{F}}(GB_j^{D_k}), \widetilde{\mathcal{F}}^{-}(GB_j^{D_k}))) = (\mathcal{P}\widetilde{\mathcal{F}}(GB_j^{D_k}) \cap \mathcal{P}^{-}\widetilde{\mathcal{F}}^{-}(GB_j^{D_k}), (\widetilde{\mathcal{F}}(GB_j^{D_k}), \widetilde{\mathcal{F}}^{-}(GB_j^{D_k})))$ then two different granular-balls generate the same fuzzy three-way concept, and thus $|\widetilde{\mathcal{G}}^{D_k}| < |\mathcal{S}^{D_k}|$.
4) If for all $GB_i^{D_k}, GB_j^{D_k} \in \mathcal{S}^{D_k}$ with $i \neq j$, their corresponding concepts are distinct, then each granular-ball generates a unique concept, giving $|\widetilde{\mathcal{G}}^{D_k}| = |\mathcal{S}^{D_k}|$.

By combining 1), 2), 3), and 4) this theorem is proven.

By theorem 1 and definition 6, we obtain the fuzzy three-way concept based on GB via two pair of cognitive operators(i.e., $(\widetilde{\mathcal{F}}, \mathcal{P})$ and $(\widetilde{\mathcal{F}}^{-}, \mathcal{P}^{-})$), and then $\widetilde{\mathcal{G}} = \{\widetilde{\mathcal{G}}^{D_1}, \widetilde{\mathcal{G}}^{D_2}, \ldots, \widetilde{\mathcal{G}}^{D_l}\}$ is established, in which the GB is the initial information granular for fuzzy three-way concept learning. Unlike [27], [28], [32], using GB as information cues can characterize data distributions faster and has the advantage of no parameters.

IV. The Proposed GB-CCL Method

Based on the above discussion, this section first presents the proposed GB-CCL, which uses granular balls for initial concept learning. Then it discusses a new label-prediction method within the concept-cognitive process. Furthermore, we also summarize the overall procedure for GB-CCL.

A. Justifiable granular-ball for concept learning

Note that definitions 4 and 5 only give the general form of the GB and GB subspace and do not discuss how to generate a justifiable GB and its GB space in a fuzzy decision formal context. Thus, this subsection is necessary to propose an initial granular-ball generation for concept learning in a fuzzy decision formal context.

Given a fuzzy decision formal context $FC = (G, M, \widetilde{I}, D, J)$, where $G/D = \{D_1, D_2, \ldots, D_l\}$, the justifiable granular-ball generation method consists of four parts: initialization, update of granular divisions, granular-ball generation, and concept learning.

- **Initialization:** Randomly select l objects in different decision subspaces as the initial granular center set $\hat{C}_0 = \{c_1, c_2, \ldots, c_l\}(c_i \in D_i)$. For each center c_i, define the initial granular division as

$$div_0(c_i) = \{x \in G \mid c_i = \arg\min_{c_j \in \hat{C}_0} \|x - c_j\|\}. \quad (14)$$

Each $div_0(c_i)$ is called an initial granular diversion of G.

- **Update granular division:** At the w-th granular division, for any $div_{w-1}(c_i), i = 1, 2, \ldots, l$ in the last granular division process. If $div_{w-1}(c_i)$ is *pure* (all objects have the same decision label), stop dividing: $Div_w^i = div_{w-1}(c_i)$. Otherwise, $div_{w-1}(c_i)$ is impure with h different classes. Keep the original center c_i and randomly select $h-1$ objects from other classes as new centers $\hat{C}_w^i = \{c_1, \ldots, c_h\}$. Update the granular division as

$$div_w(c_j) = \{x \in div_{w-1}(c_i) | c_j = \arg\min_{c_k \in \hat{C}_w^i} \|x - c_k\|\}, \quad j = 1, 2, \ldots, h. \quad (15)$$

Repeat until all granular divisions are pure or satisfy a given purity threshold.

- **Granular-ball generation:** Let the final granular centers and corresponding granular division are $\hat{C} = \{c_1, c_2, \ldots, c_f\}$ and $Div = \{div(c_1), div(c_2), \ldots, div(c_f)\}$, respectively. The decision label of each granular division is given by $DivLab = \{D(c_1), D(c_2), \ldots, D(c_f)\}$. Then, the granular balls induced by Div are generated based on the granular centers and radius, which could be described as follows:

$$r_i = \frac{1}{|div(c_i)|}\sqrt{\sum_{x \in div(c_i)} ||x - c_i||^2}, \quad (16)$$

$$GB_i^{D(c_i)} = \{x \in D(c_i) | dis(x, c_i) \le r_i\}. \quad (17)$$

Then, the quadruple $(c_i, r_i, GB_i^{D(c_i)}, D(c_i))$ is called a justifiable granular ball, and the justifiable granular-ball space is $\mathcal{B} = \{(c_i, r_i, GB_i^{D(c_i)}, D(c_i)), i = 1, 2, \ldots, f\}$.

- **Concept learning:** The granular ball concept learning could be characterized based on the above justifiable granular-ball generation. For any $(c_i, r_i, GB_i^{D(c_i)}, D(c_i)) \in \mathcal{B}$, the fuzzy three-way concept on this granular ball is defined as follows:

$$\widetilde{\mathcal{F}}^{\nabla}(GB_i^{D(c_i)}) = (\widetilde{\mathcal{F}}(GB_i^{D(c_i)}), \widetilde{\mathcal{F}}^{-}(GB_i^{D(c_i)})), \quad (18)$$

$$\mathcal{P}^{\nabla}(\widetilde{\mathcal{F}}(GB_i^{D(c_i)}), \widetilde{\mathcal{F}}^{-}(GB_i^{D(c_i)})) = \mathcal{P}(\widetilde{\mathcal{F}}(GB_i^{D(c_i)})) \cap \mathcal{P}^{-}(\widetilde{\mathcal{F}}^{-}(GB_i^{D(c_i)})). \quad (19)$$

Then, its fuzzy three-way concept subspace $\widetilde{\mathcal{G}}^{D_k}$ can be constructed as follows.

$$\widetilde{\mathcal{G}}^{D_k} = \{(\mathcal{P}\widetilde{\mathcal{F}}(GB_i^{D(c_i)}) \cap \mathcal{P}^{-}\widetilde{\mathcal{F}}^{-}(GB_i^{D(c_i)}), (\widetilde{\mathcal{F}}(GB_i^{D(c_i)})\widetilde{\mathcal{F}}^{-}(GB_i^{D(c_i)})))| GB_i^{D(c_i)} \in D_k\}. \quad (20)$$

Finally, the complete fuzzy three-way concept space is $\widetilde{\mathcal{G}} = \{\widetilde{\mathcal{G}}^{D_1}, \widetilde{\mathcal{G}}^{D_2}, \ldots, \widetilde{\mathcal{G}}^{D_k}\}$.

Remark: Compared with existing granular-ball methods, this justifiable granular-ball generation approach effectively removes boundary and noisy samples, and leverages the resulting granular-ball clusters as adaptive information granules for fuzzy three-way concept learning.

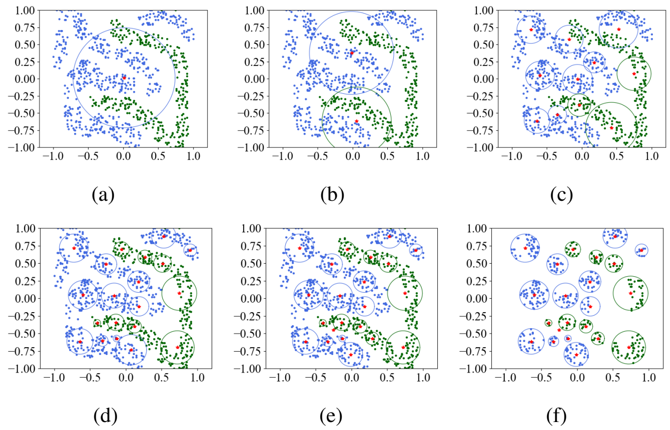

Fig. 2. Justifiable granular ball generation process. (a)-(f) depict the splitting process at different steps. Compared with the existing approaches that utilize class centers as representatives for subsequent learning in [10], the proposed method removes boundary and noisy samples and leverages the resulting granular-ball clusters as adaptive information granules for concept learning.

After the above four steps, we obtain the justifiable granular-ball generation method for concept learning. Note that $\mathcal{B} = \{(c_i, r_i, GB_i^{D(c_i)}, D(c_i)), i = 1, 2, \ldots, f\}$ and $\widetilde{\mathcal{G}} = \{\widetilde{\mathcal{G}}^{D_1}, \widetilde{\mathcal{G}}^{D_2}, \ldots, \widetilde{\mathcal{G}}^{D_l}\}$ are necessary elements in label prediction at the concept-cognitive processes. A diagram of the GB generation process is shown in Fig.2.

The time complexity of the granular-ball generation phase (Lines 2–18) mainly depends on the number of decision classes l, the refinement iterations t, the number of objects n, and the feature dimensionality m. Since each refinement involves distance or similarity computations in the m-dimensional space and scans the involved objects at most once, this phase requires $O((l-1)t\,nm)$ time in the worst case. The justifiable granular-ball representation further introduces an additional cost of $O(km)$ by computing the center and radius of the k granular balls. In the fuzzy three-way concept learning stage (Lines 19–24), each object is processed only within its corresponding granular ball, leading to an intent learning cost of $O(nm)$. The context learning incurs $O(knm)$ due to comparisons between k granular balls and n objects. By retaining the dominant terms, the overall worst-case time complexity of Algorithm 1 is $O\big(((l-1)t + k)nm\big)$.

B. Label prediction in concept-cognitive process

In concept-cognitive learning, we strive to develop an effective conceptual model with satisfactory classification per-

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Algorithm 1 Justifiable granular-ball generation for concept learning

Input: A fuzzy decision formal context $FC = (G, M, \widetilde{I}, D, J)$.
Output: Granular-ball space $\mathcal{B}$, fuzzy three-way concept space $\widetilde{\mathcal{G}}$.

Initialize $\mathcal{B} = \emptyset$, $\widetilde{\mathcal{G}} = \emptyset$;
for each $D_k \in G/D$ **do**
  Initialize $\widetilde{\mathcal{G}}^{D_k} = \emptyset$, $Div_0 = \emptyset$;
  Randomly select one object from each decision class in G/D as the initial granular center set
  $\hat{C}_0 = \{c_1, c_2, \ldots, c_{|G/D|}\}, \quad c_i \in D_i$
  for all $c_i \in \hat{C}_0$ **do**
    Compute $div_0(c_i)$ according to Eq.(14);
    if $div_0(c_i) \subseteq D_k$ **then**
      $Div_0 \leftarrow Div_0 \cup \{div_0(c_i)\}$;
    else
      Update $div_0(c_i)$ according to Eq.(15);
    end if
  end for
  for all $div(c_i) \in Div_0$ **do**
    Compute the granular-ball radius r_i according to Eq.(16);
    Generate the granular ball $GB_i^{D(c_i)}$ according to Eq.(17);
    $\mathcal{B} \leftarrow \mathcal{B} \cup \{(c_i, r_i, GB_i^{D(c_i)}, D(c_i))\}$;
  end for
end for
for all $GB_i^{D(c_i)} \in \mathcal{B}$ with $D(c_i) = D_k$ **do**
  Construct a fuzzy three-way concept $(X, (\widetilde{A}_1, \widetilde{A}_2))$ according to Eqs.(18) and (19);
  Construct a fuzzy three-way concept subspace $\widetilde{\mathcal{G}}^{D_k}$ according to Eq.(20);
  $\widetilde{\mathcal{G}}^{D_k} \leftarrow \widetilde{\mathcal{G}}^{D_k} \cup \{(X, (\widetilde{A}_1, \widetilde{A}_2))\}$;
end for
$\widetilde{\mathcal{G}} \leftarrow \widetilde{\mathcal{G}} \cup \{\widetilde{\mathcal{G}}^{D_k}\}$;
return $\mathcal{B}$ and $\widetilde{\mathcal{G}}$.

formance, yet the actual situation is often less than ideal. Hence, pseudo-concepts [25] and progressive concepts [13], [32] are a great way to recognize fuzzy concept ontologies. Effective concept recognition requires concepts with high representation. Consequently, a label-prediction strategy is proposed in this subsection to identify objects effectively.

Definition 7. *Let $\mathcal{B} = \{(c_i, r_i, GB_i^{D(c_i)}, D(c_i)), i = 1, 2, \ldots, f\}$ be a granular-ball subspace, and $(\mathcal{P}\widetilde{\mathcal{F}}(GB_i^{D(c_i)}) \cap \mathcal{P}^{-}\widetilde{\mathcal{F}}^{-}(GB_i^{D(c_i)}), (\widetilde{\mathcal{F}}(GB_i^{D(c_i)}), \widetilde{\mathcal{F}}^{-}(GB_i^{D(c_i)})))$ be the fuzzy three-way concept induced by $GB_i^{D(c_i)}$. Then, its positive and negative centers are defined as follows:*

$$C^{+}_{GB_i^{D(c_i)}} = \frac{\sum_{x \in \mathcal{P}^{\nabla}(\widetilde{\mathcal{F}}(GB_i^{D(c_i)}), \widetilde{\mathcal{F}}^{-}(GB_i^{D(c_i)}))} \widetilde{\mathcal{F}}(x)}{|\mathcal{P}^{\nabla}(\widetilde{\mathcal{F}}(GB_i^{D(c_i)}), \widetilde{\mathcal{F}}^{-}(GB_i^{D(c_i)}))|}, \quad (21)$$

$$C^{-}_{GB_i^{D(c_i)}} = \frac{\sum_{x \in \mathcal{P}^{\nabla}(\widetilde{\mathcal{F}}(GB_i^{D(c_i)}), \widetilde{\mathcal{F}}^{-}(GB_i^{D(c_i)}))} \widetilde{\mathcal{F}}^{-}(x)}{|\mathcal{P}^{\nabla}(\widetilde{\mathcal{F}}(GB_i^{D(c_i)}), \widetilde{\mathcal{F}}^{-}(GB_i^{D(c_i)}))|}, \quad (22)$$

where $C^{+}_{GB_i^{D(c_i)}}$ and $C^{-}_{GB_i^{D(c_i)}}$ denote the positive and negative centers of the fuzzy three-way concept, respectively.

Definition 8. *Let o be a newly input object with fuzzy membership degree $\widetilde{\mathcal{F}}^{+}(o)$ and non-membership degree $\widetilde{\mathcal{F}}^{-}(o)$. For any fuzzy three-way concept induced by $GB_i^{D(c_i)}$, the fuzzy three-way concept recognition degree is defined as:*

$$\mathcal{R}_{o,GB_i^{D(c_i)}} = \sqrt{\|\widetilde{\mathcal{F}}^{+}(o) - C^{+}_{GB_i^{D(c_i)}}\|^2 + \|\widetilde{\mathcal{F}}^{-}(o) - C^{-}_{GB_i^{D(c_i)}}\|^2},$$

where $\|\cdot\|$ denotes the 2-norm. This measure quantifies the distance of object o from the positive and negative centers of the fuzzy three-way concept.

Algorithm 2 Label prediction in concept-cognitive process

Input: Fuzzy three-way concept space $\widetilde{\mathcal{G}} = \{\widetilde{\mathcal{G}}^{D_1}, \widetilde{\mathcal{G}}^{D_2}, \ldots, \widetilde{\mathcal{G}}^{D_l}\}$, newly input object set ΔG
Output: $\mathcal{R}^{*}_{o,D_k}$ and predicted label D_k^{*}

1: **for all** $o \in \Delta G$ **do**
2:   Obtain $(\widetilde{\mathcal{F}}^{+}(o), \widetilde{\mathcal{F}}^{-}(o))$;
3:   **for all** $\widetilde{\mathcal{G}}^{D_k} \in \widetilde{\mathcal{G}}$ **do**
4:     Initialize $\mathcal{R}^{*}_{o,D_k} \leftarrow +\infty$
5:     **for all** $GB_i^{D(c_i)} \in \widetilde{\mathcal{G}}^{D_k}$ **do**
6:       Compute positive center $C^{+}_{GB_i^{D(c_i)}}$ and negative center $C^{-}_{GB_i^{D(c_i)}}$ according to Eq.(21) and Eq.(22);
7:       Compute concept recognition degree:
$$\mathcal{R}_{o,GB_i^{D(c_i)}} = \sqrt{\|\widetilde{\mathcal{F}}^{+}(o) - C^{+}_{GB_i^{D(c_i)}}\|^2 + \|\widetilde{\mathcal{F}}^{-}(o) - C^{-}_{GB_i^{D(c_i)}}\|^2}$$
8:       **if** $\mathcal{R}_{o,GB_i^{D(c_i)}} < \mathcal{R}^{*}_{o,D_k}$ **then**
9:         $\mathcal{R}^{*}_{o,D_k} \leftarrow \mathcal{R}_{o,GB_i^{D(c_i)}}$;
10:       **end if**
11:     **end for**
12:   **end for**
13:   Predicted label: $D_k^{*} \leftarrow \arg\min_{D_k} \mathcal{R}^{*}_{o,D_k}$;
14: **end for**
15: **return** $\mathcal{R}^{*}_{o,D_k}$ and D_k^{*}.

Intrinsically, the recognition value $\mathcal{R}_{o,D(c_i)}$ of a fuzzy three-way concept reflects the similarity between the two concepts. In other words, the greater the value of $\mathcal{R}$, the smaller the relationship between the two concepts. Thus, for any newly input object o characterized from positive and negative information, we can calculate the concept recognition between $(o, (\widetilde{\mathcal{F}}^{+}(o), \widetilde{\mathcal{F}}^{-}(o)))$ and any concept in the fuzzy three-way concept space according to definition 8. Likewise, we can also calculate a global minimum concept recognition by pairwise comparison and then predict its label, denoted by ${D_k}^{*} = \arg\min_{D_k} \mathcal{R}_{o,D(c_i)}$. The detailed procedure is presented in Algorithm 2. In the concept center computation at Line 6, each object contributes to only one corresponding concept center, resulting in a time complexity of $O(nm)$. In addition, the concept recognition process (Lines 7–8) compares the newly added objects in ΔG with the existing k concepts under m features, leading to a complexity of $O(k|\Delta G|m)$. Therefore, the overall worst-case time complexity of Algorithm 2 is $O\big((n + k|\Delta G|)m\big)$. By combining Algorithms 1 and 2, the total worst-case time complexity of the proposed method is $O\big((((l-1)t + k)n + k|\Delta G|)m\big)$.

C. Overall procedure

As illustrated in Fig.3, the overall procedure of GB-CCL consists of three stages: 1) justifiable granular-ball generation, 2) granular-ball concept learning, and 3) label prediction in the concept-cognitive process. From the discussions mentioned above, a justifiable granular-ball space $\mathcal{B} = \{(c_i, r_i, GB_i^{D(c_i)}, D(c_i)), i = 1, 2, \ldots, f\}$ generation is a vital step in GB-CCL. In this process, we characterize a GB

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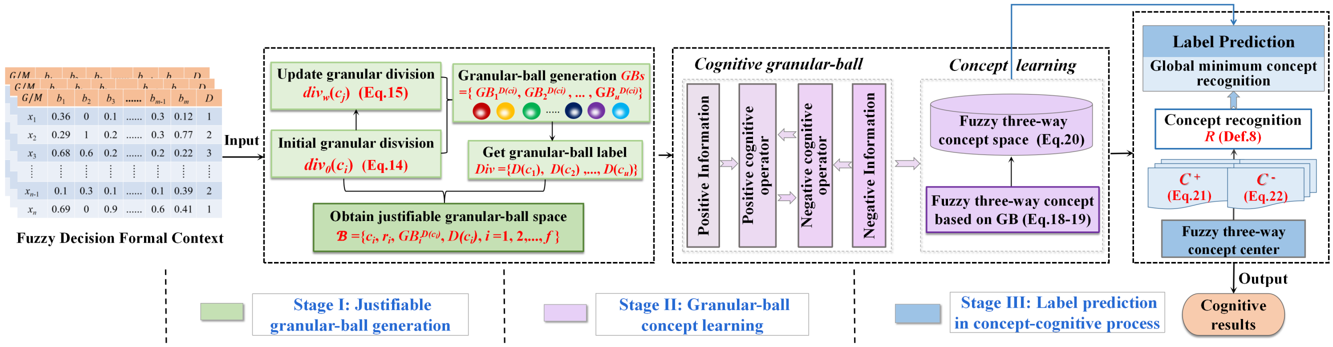


Fig. 3. The overall procedure of the proposed GB-CCL.

adaptive learning method based on decision division G/D, where information granulation is achieved by splitting granular divisions from large to small, resulting in a generated GB that exhibits fast and accurate characterization of data distribution.

Furthermore, we make full use of the justifiable GB learned in the first stage to carry out concept learning, in which a pair of positive and negative cognitive operators(i.e., $(\widetilde{\mathcal{F}}^{+}, \mathcal{P}^{+}), (\widetilde{\mathcal{F}}^{-}, \mathcal{P}^{-})$) are used to distribute knowledge acquisition from two perspectives of positive information and negative information so as to obtain structured conceptual knowledge, i.e., acquire fuzzy three-way concept based on GB and the corresponding fuzzy three-way concept space(i.e., $\widetilde{\mathcal{G}}^{D_k} = \{(\mathcal{P}\widetilde{\mathcal{F}}(GB_i^{D_k}) \cap \mathcal{P}^{-}\widetilde{\mathcal{F}}^{-}(GB_i^{D_k}), (\widetilde{\mathcal{F}}(GB_i^{D_k}), \widetilde{\mathcal{F}}^{-}(GB_i^{D_k})))|GB_i^{D_k} \in \mathcal{S}^{D_k}\}$) for concept recognition in the subsequent stage.

Finally, we design a concept recognition mechanism based on a concept center(i.e., $C^{+}_{GB_i^{D(c_i)}}$ and $C^{-}_{GB_i^{D(c_i)}}$) for label prediction in the concept-cognitive process. In this stage, we can describe its positive and negative information(i.e., $(\widetilde{\mathcal{F}}^{+}(o), \widetilde{\mathcal{F}}^{-}(o))$) and then identify it with the concept in the fuzzy three-way concept space $\widetilde{\mathcal{G}} = \{\widetilde{\mathcal{G}}^{D_1}, \widetilde{\mathcal{G}}^{D_2}, \ldots, \widetilde{\mathcal{G}}^{D_k}\}$, where the label of the concept with the smallest recognition degree is our prediction label for the new object.

V. Experiment Analysis

In this section, we design various numerical comparative experiments compared with other approaches on the public datasets. In the subsequent experiments, we aim to answer the following research questions(Rqs): **Rq1:** Does the proposed GB-CCL method outperform related state-of-the-art peers? **Rq2:** What are the differences between GB-CCL, GB, and other CCL mechanisms? **Rq3:** How does the new viewpoint influence the robustness of GB-CCL?

A. General settings

1) Datasets: To evaluate the performance of our GB-CCL model, we chose 15 public datasets from the UCI Machine Learning Repository [1] to make comparisons, and detailed information about them is shown in Table I. Meanwhile, all the experimental datasets are fuzzified into the interval $[0, 1]$, and they are first fuzzified according to the following equation.

$$\widetilde{I}(x, a) = \frac{f(x, a) - min(f(a))}{max(f(a)) - min(f(a))},$$

where the $f(x, a)$ denotes the value of object x under feature a, the $max(f(a))$ and $min(f(a))$ are the maximum and minimum values of all objects on feature a.

TABLE I
Basic description of 15 selected datasets

No.	Dataset Full Name	Dataset Abbre.	Sample	Feature	Class	Area
D1	Cardiotocography	Card	2126	21	3	Life Science
D2	Dermatlogy	Derma	366	35	6	Life Science
D3	Dry Bean	DryB	13611	17	6	Computer Science
D4	FrogsMFCCs	Frogs	7195	23	60	Other field
D5	Magic	Magic	19020	11	2	Physical Science
D6	Mice Protein Expression	MiPE	1077	69	8	Life Science
D7	Movement	Move	360	91	12	Other field
D8	Parkinsons	Park	195	23	2	Life Science
D9	Seeds	Seeds	210	8	3	Life Science
D10	Segmentation	Segm	2310	19	7	Other field
D11	Wine Quality White	WiQW	4898	12	11	Business
D12	Colon	Colo	62	2000	2	Life Science
D13	Leukemia	Leuk	72	7130	2	Life Science
D14	Lung	Lung	203	3312	5	Life Science
D15	Yale	Yale	165	1025	15	Computer Science

2) Baselines: To enhance the convincing experimental comparison, we compare GB-CCL with 11 representative state-of-the-art peers. These baselines include the multi-layer ensemble evolving fuzzy inference method(MEEFI) [34], fuzzy edited nearest neighbor classifier(FENN) [35], semi-supervised optimal margin distribution machine(ssODM) [36], random forest model(RF) [37], weighted fuzzy concept-cognitive learning(WFCCL) [38], memory-based CCL for fuzzy context(M-FCCL) [28], robust fuzzy-based concept-cognitive learning(R-FCCL) [39], granular-ball K-nearest neighbor classifier(GB-KNN) [8], concept-cognitive learning with collaborative multiscale(CM-CCL) [40], and clustering method with granular-ball(GBCT) [41]. In addition, five ablation experiments are carried out to illustrate the effectiveness of the similarity measures, granular ball generation, concept learning, and prediction mechanisms.

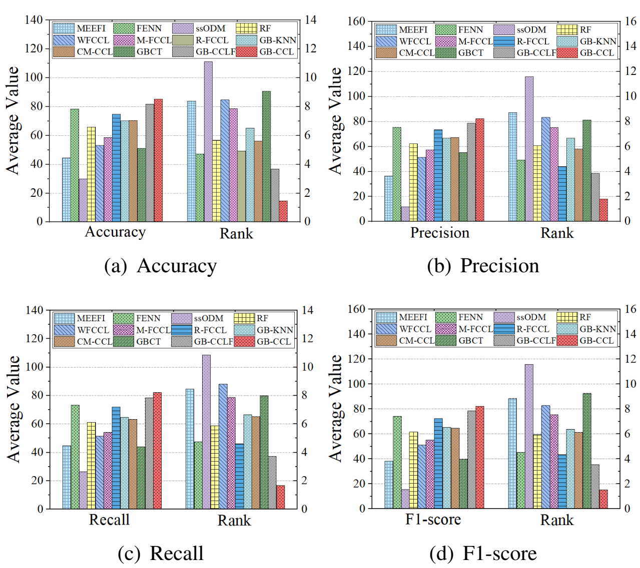
3) Experimental designs: As introduced above, extensive experiments were conducted to evaluate the performance of

[1]http://archive.ics.uci.edu/ml/datasets.php

the proposed GB-CCL model against competing methods. Specifically, to reduce the randomness of the evaluation results, 10-fold cross-validation is employed for each dataset and the compared method for a fair comparison. All the parameter settings are consistent with their corresponding references. Meanwhile, five evaluation measures, including Accuracy("↑"), Precision("↑"), Recall("↑"), F1-score("↑"), and Concept learing time("↓") are selected to make comparison from different perspectives, where "↑" and "↓" indicate "larger the better" and "smaller the better". Moreover, to further illustrate the robustness of GB-CCL in a noise environment, each data is randomly added noise from 2% to 18% with a step of 2% for classification evaluation. All algorithms are implemented on a public computer with OS: Microsoft WIN10; Processor: Intel(R) Core(TM) i7-6800K CPU @ 3.4GHz12; Memory: 62.7GB; Programming language: MATLAB2020a.

B. Performance comparison(Rq.1)

To demonstrate the superiority of GB-CCL in the classification task, we compare it with eleven representative methods in this subsection. The detailed results are recorded in Tables II-V, where Ave denotes the average performance, and all the excellent results are bold. On D14, CM-CCL did not finish within the allowed runtime and is denoted as "$--$". Meanwhile, for a clear comparison, average values and ranks of these compared methods are shown in Fig.4. From these comparison results, we have three main observations as follows:



(a) Accuracy (b) Precision (c) Recall (d) F1-score

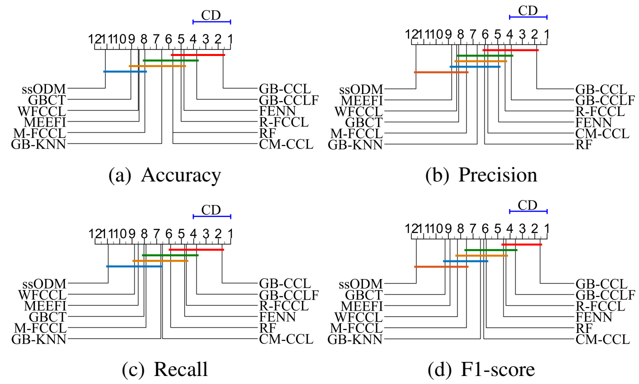
Fig. 4. Average results of GB-CCL and other eleven representative methods.

• The classification results of accuracy, precision, recall, and F1-score for the proposed GB-CCL method are obviously superior to other methods(i.e., MEEFI, FENN, ssODM, WFCCL, CM-CCL, GBCT, GB-CCLF) on the tested data. Concretely, the ranks of GB-CCL versus the selected comparison methods also show that our method has an obvious advantage in classification problems.

• Note that the proposed GB-CCL method achieves the maximum average values and minimum rank of four experimental indexes on 15 datasets, illustrating its excellent performance in classification tasks.

• Particularly, the ssODM is invalid for classification on the WiQW dataset and has poor performance in the Frogs dataset.

Moreover, to examine whether the classification performance differences among the compared methods are statistically significant, Friedman's test [42] is employed at a significance level of $P = 0.05$. The null hypothesis states that all methods perform equally, which is rejected when the p-value is smaller than the predefined threshold. Specifically, the p-values for accuracy, precision, recall, and F1-score are 4.61×10^{-14}, 9.31×10^{-14}, 6.64×10^{-13}, and 3.40×10^{-16}, which is far lower than 0.05, indicating there has a significant difference among compared methods.



(a) Accuracy (b) Precision (c) Recall (d) F1-score

Fig. 5. Nemenyi test on GB-CCL and eleven representative methods.

Based on the above results, Nemenyi's post hoc test [43] is further conducted to assess the pairwise differences between GB-CCL and each baseline. The results are illustrated in Fig. 5. At the significance level of $P = 0.05$, the critical difference(CD) value is 4.3025 with $N_d = 15$ datasets and $N_m = 12$ methods. As shown in the figure, GB-CCL achieves the highest overall rank and is significantly better than the other 7, 6, 7, and 8 methods in terms of accuracy, precision, recall, and F1-score.

C. The differences between the GB and other CCL(Rq.2)

In this subsection, we provide a further analysis by comparing our method with GBC models and other CCL approaches in classification performance and concept learning time.

1) Comparison of classification performance: The classification results of comparing seven concept-cognitive learning and granular-ball computing methods are summarized in Tables II-V. Meanwhile, the classification accuracy of ablation experiments is also shown in Table VI. From these results, we have three main observations as follows:

• Our proposed GB-CCL method achieves higher accuracy/precision/recall/F1-score than two granular-ball computing methods(GB-KNN and GBCT) and five concept-cognitive learning models(WFCCL, M-FCCL, R-FCCL, CM-CCL, and GB-CCLF) in most cases. Concretely, the ranks of GB-CCL versus the selected comparison methods also show that our method is obviously superior to others.

• Compared with the granular-ball concept-cognitive learning without fuzzy concept, i.e., GB-CCLF, our method

TABLE II
CLASSIFICATION ACCURACY(%) COMPARISON WITH OTHER ELEVEN POPULAR METHODS ON 15 DATASETS

No.s	MEEFI	FENN	ssODM	RF	WFCCL	M-FCCL	R-FCCL	GB-KNN	CM-CCL	GBCT	GB-CCLF	GB-CCL
D1	80.76±1.91	80.57±2.70	46.90±3.39	76.71±4.27	59.08±3.52	41.77±4.11	76.43±1.41	67.59±2.77	77.28±2.86	52.35±0.33	78.60±2.35	**81.52**±2.34
D2	94.55±2.85	84.68±5.70	30.59±4.70	89.34±5.54	91.53±3.91	91.33±3.35	93.29±4.60	71.01±9.69	89.10±6.87	55.59±0.40	95.10±3.58	**95.65**±3.18
D3	13.79±1.21	71.93±0.96	14.89±0.60	88.41±0.82	67.18±4.89	48.08±1.49	**91.60**±0.73	86.72±1.58	65.46±3.25	38.59±1.87	85.45±0.95	86.72±1.01
D4	15.37±6.44	86.39±1.52	0.72±0.32	0.85±0.27	54.84±3.21	44.74±2.50	81.03±1.94	64.17±2.77	73.91±2.22	11.01±0.00	86.27±1.02	**86.39**±0.96
D5	35.16±0.82	80.80±0.76	64.84±0.82	**82.78**±0.75	0.07±0.00	73.20±1.35	81.64±0.81	72.94±1.64	74.20±0.68	63.02±3.19	76.39±0.76	75.80±0.96
D6	32.60±9.84	93.78±2.36	13.92±2.36	76.60±4.03	33.53±6.70	40.02±3.95	97.49±1.33	66.68±7.85	90.81±1.40	19.50±4.44	97.68±1.97	**97.96**±1.56
D7	11.11±4.54	69.17±8.12	6.67±2.99	6.11±3.41	39.44±8.16	48.33±9.82	48.33±12.23	70.83±8.41	60.83±6.73	16.94±9.82	81.94±3.53	**83.89**±4.86
D8	57.13±18.28	84.05±5.74	75.32±10.19	83.08±6.40	53.34±16.9	44.16±12.38	44.16±12.38	85.08±11.64	85.11±8.46	84.61±0.11	91.37±7.89	**93.26**±6.57
D9	45.71±16.37	89.05±3.21	33.33±14.02	92.38±5.59	86.19±7.92	90.95±9.38	91.90±6.75	83.81±11.71	65.71±12.46	75.24±17.51	83.33±6.04	**93.81**±6.37
D10	32.51±5.45	93.46±1.90	14.29±2.62	93.59±1.32	67.88±5.03	64.11±4.15	93.51±2.17	80.56±4.58	88.83±2.25	53.42±9.31	92.60±2.50	**93.98**±2.84
D11	51.53±2.47	48.61±1.87	0.00±0.00	57.43±2.38	41.10±2.49	36.65±3.28	54.65±2.52	60.76±1.01	50.41±3.30	34.16±4.59	53.16±2.75	**61.74**±1.84
D12	67.38±11.61	67.14±22.64	35.71±12.94	69.05±14.89	41.67±12.26	67.86±21.77	64.29±10.29	57.86±11.93	91.61±7.36	69.44±7.86	64.52±24.37	66.19±25.09
D13	79.46±22.51	84.46±10.58	34.64±18.75	81.79±13.30	50.36±22.87	71.25±18.22	85.89±15.07	75.00±17.58	83.81±11.16	59.68±6.84	91.61±9.89	**91.61**±9.89
D14	18.24±11.23	93.62±6.48	68.43±13.64	81.79±12.27	69.31±18.04	68.07±11.06	88.50±9.73	79.48±12.47	–	87.83±4.78	93.64±6.98	**93.64**±6.98
D15	30.88±10.02	49.34±13.62	6.76±6.89	9.85±7.92	40.51±7.62	49.01±9.88	29.56±15.31	32.02±9.95	59.34±13.57	44.22±5.62	56.76±11.23	**56.76**±11.23
Ave	44.41±8.37	78.47±5.88	29.8±6.28	65.98±5.54	53.07±8.23	58.64±7.78	74.82±6.48	70.30±7.71	75.46±5.90	51.04±5.11	81.89±5.72	**83.93**±5.71
Rank	8.40	4.73	11.13	5.67	8.47	7.87	4.93	6.53	5.60	9.07	3.67	1.47

TABLE III
CLASSIFICATION PRECISION(%) COMPARISON WITH OTHER ELEVEN POPULAR METHODS ON 15 DATASETS

No.s	MEEFI	FENN	ssODM	RF	WFCCL	M-FCCL	R-FCCL	GB-KNN	CM-CCL	GBCT	GB-CCLF	GB-CCL
D1	**84.08**±5.55	77.69±5.70	29.5±3.05	72.86±5.02	52.58±4.54	46.6±16.80	80.14±5.15	66.33±5.31	80.06±12.64	50.00±70.71	75.42±3.17	80.68±2.58
D2	93.74±4.00	81.26±5.85	5.33±1.08	88.82±6.07	91.44±5.09	92.26±3.37	93.63±4.77	64.71±8.36	88.50±9.30	50.00±70.71	94.00±5.44	**94.63**±5.07
D3	2.55±0.82	70.89±0.90	2.48±0.10	88.61±0.83	70.79±6.09	54.58±2.99	**92.32**±0.68	87.26±1.45	77.51±0.81	49.29±6.79	85.98±0.98	87.32±0.96
D4	10.09±3.57	81.98±4.49	0.01±0.01	1.62±0.72	52.37±4.03	46.35±4.37	67.51±4.03	61.89±2.39	58.16±2.74	85.00±0.00	84.92±2.03	**85.04**±2.00
D5	17.58±0.41	81.75±0.82	32.42±0.41	**83.97**±0.95	0.08±0.00	82.78±1.62	83.88±1.13	72.17±1.31	75.69±0.49	70.62±35.41	77.99±1.03	73.50±1.00
D6	23.42±9.27	93.92±2.20	1.74±0.30	79.18±3.64	42.01±8.38	45.14±6.03	97.58±1.20	70.28±7.97	91.73±1.54	85.71±20.20	97.65±1.94	**97.97**±1.34
D7	2.94±1.71	70.41±10.02	0.48±0.23	4.83±3.31	32.89±6.63	49.59±9.46	45.33±13.75	71.43±6.03	58.66±8.02	0.00±0.00	82.03±3.81	**83.06**±4.03
D8	68.49±7.81	82.74±8.35	37.66±5.09	76.36±9.15	56.58±17.86	45.47±11.06	88.22±10.49	83.06±14.33	90.64±5.01	83.43±0.67	87.39±12.35	**90.84**±8.02
D9	32.83±16.91	87.85±4.83	11.11±4.67	91.64±5.81	87.62±7.68	90.36±10.85	92.13±6.17	85.28±10.37	53.68±19.23	77.94±31.20	85.25±4.51	**95.19**±4.96
D10	21.16±8.70	93.47±1.92	2.04±0.37	94.24±1.33	70.63±4.37	66.44±5.46	93.81±2.10	83.70±3.20	89.07±1.63	42.81±24.71	92.90±2.33	**94.97**±2.61
D11	24.00±8.78	25.23±2.40	0.00±0.00	44.58±5.32	25.83±11.48	24.62±2.24	32.98±9.17	**47.24**±4.45	21.15±9.40	0.00±0.00	33.48±3.07	39.19±3.82
D12	51.83±26.64	63.37±26.32	17.86±6.47	59.20±21.35	37±17.49	59.83±26.24	37.48±16	46.08±18.09	91.20±15.29	75.00±35.36	61.67±29.41	62.5±29.94
D13	83.58±17.98	86.54±11.01	17.32±9.38	80.32±21.31	53.33±20.30	64.42±18.95	89.58±11.66	73.67±23.09	82.00±17.91	40.86±15.36	92.33±10.01	**92.33**±10.01
D14	11.10±9.82	92.10±9.90	19.10±9.67	61.32±22.47	66.49±19.01	50.62±16.58	82.32±14.80	64.48±19.87	–	88.18±2.57	83.32±14.80	**83.32**±14.80
D15	20.60±11.38	43.24±12.56	0.66±0.69	7.00±4.80	31.44±8.16	39.26±11.12	23.47±9.40	24.12±9.67	50.20±14.54	29.17±5.89	47.82±14.56	**47.82**±14.56
Ave	36.53±8.89	75.50±7.15	11.85±2.77	62.3±7.47	51.41±9.41	57.22±9.81	73.36±7.37	66.78±9.06	72.02±8.47	55.20±21.31	78.81±7.30	**80.56**±7.05
Rank	8.73	4.93	11.60	6.07	8.33	7.53	4.40	6.67	5.80	8.13	3.87	1.80

TABLE IV
CLASSIFICATION RECALL(%) COMPARISON WITH OTHER ELEVEN POPULAR METHODS ON 15 DATASETS

No.s	MEEFI	FENN	ssODM	RF	WFCCL	M-FCCL	R-FCCL	GB-KNN	CM-CCL	GBCT	GB-CCLF	GB-CCL
D1	69.71±4.43	74.54±3.41	64.58±2.16	75.49±5.35	55.19±6.57	35.35±3.67	64.85±4.10	61.28±4.91	58.36±2.86	0.12±0.17	79.13±2.07	**80.92**±2.60
D2	94.13±2.90	82.88±6.52	17.33±1.41	86.55±7.69	92.93±4.36	90.47±4.24	93.41±4.81	64.16±10.36	88.34±9.11	22.22±31.43	94.37±5.53	**94.84**±4.97
D3	16.88±0.68	69.29±0.89	16.67±0.00	89.37±0.83	64.74±5.19	42.56±1.22	**91.91**±0.74	86.69±1.47	68.46±2.30	65.49±47.45	85.94±0.95	87.29±1.07
D4	16.68±3.32	80.41±4.17	1.86±0.06	1.84±0.62	53.12±3.42	47.69±4.64	69.08±3.28	58.39±3.78	56.18±2.88	75.00±35.36	85.40±1.75	**85.64**±1.63
D5	50.00±0.00	75.25±0.86	50.00±0.00	**77.72**±0.76	0.07±0.00	62.22±1.41	75.65±0.80	74.19±1.44	65.72±0.73	28.37±39.18	68.83±0.74	73.83±1.06
D6	34.4±7.96	93.67±2.22	12.50±0.00	75.95±3.20	33.07±6.41	38.41±3.34	97.57±1.31	67.52±7.35	90.96±1.72	5.09±2.23	97.70±2.08	**98.12**±1.33
D7	11.47±3.09	68.17±9.81	7.16±0.42	5.44±2.93	37.71±6.38	48.03±10.70	47.47±12.46	69.3±8.46	59.74±8.04	0.00±0.00	79.90±4.06	**81.36**±4.35
D8	71.88±9.33	77.54±11.45	50.00±0.00	81.46±10.08	57.37±21.94	46.56±16.93	81.50±9.60	76.05±16.47	70.70±9.09	99.32±0.96	92.07±7.53	92.35±8.34
D9	45.6±12.40	88.12±3.78	33.33±0.00	92.25±6.36	86.06±6.37	90.26±10.36	91.83±7.39	84.26±10.94	66.00±4.42	89.06±15.47	83.16±7.61	**93.45**±6.77
D10	32.16±4.64	93.58±1.83	14.29±0.00	93.52±1.12	68.17±5.27	64.08±3.50	93.47±2.13	80.85±4.14	88.95±2.00	75.13±28.70	92.61±2.59	**94.96**±3.02
D11	20.62±1.99	24.88±2.73	0.00±0.00	36.44±5.04	20.42±2.71	23.73±6.08	27.49±2.22	39.73±7.16	20.64±1.96	0.00±0.00	37.09±3.92	**40.02**±4.57
D12	59.00±15.66	61.75±24.79	50.00±0.00	61.92±17.2.	41.42±16.00	63.25±24.01	50.67±6.44	50.25±10.27	86.37±18.53	14.93±5.40	60.08±29.31	61.33±30.27
D13	80.89±25.01	81.92±12.67	45.00±15.81	75.24±20.73	51.77±22.96	68.33±22.72	84.83±14.33	71.92±20.58	80.67±17.90	71.43±40.41	91.33±9.99	**91.33**±9.99
D14	39.77±20.45	85.21±10.58	27.00±8.23	56.71±17.37	72.93±18.03	50.09±20.12	81.00±13.75	62.51±18.44	–	92.72±1.81	82.28±16.60	82.28±16.60
D15	29.09±10.51	43.26±12.47	6.74±4.68	7.67±5.54	36.03±7.62	40.41±11.43	29.3±12.30	25.54±8.44	50.53±15.13	19.64±7.58	48.22±13.73	**48.22**±13.73
Ave	44.82±8.16	73.36±7.21	26.43±2.18	61.17±6.99	51.40±8.88	54.10±9.62	72.00±6.38	64.84±8.95	67.97±6.91	43.90±17.08	78.54±7.23	**80.40**±7.35
Rank	8.47	4.73	10.87	5.87	8.80	7.87	4.60	6.67	6.53	8.00	3.73	1.67

achieves better classification performance in most cases. These results also show the validity and necessity of characterizing concepts from both positive and negative information perspectives in fuzzy decision formal contexts.

- Compared with the other ablation experiments on similarity measure, granular ball generation, concept construction, and prediction, the proposed GB-CCL achieves the highest accuracy and lowest ranks, indicating the reasonability of the

TABLE V
CLASSIFICATION F1-SCORE(%) COMPARISON WITH OTHER ELEVEN POPULAR METHODS ON 15 DATASETS

No.s	MEEFI	FENN	ssODM	RF	WFCCL	M-FCCL	R-FCCL	GB-KNN	CM-CCL	GBCT	GB-CCLF	GB-CCL
D1	76.17$_{\pm 4.45}$	76.01$_{\pm 3.83}$	40.44$_{\pm 3.27}$	74.12$_{\pm 4.97}$	53.79$_{\pm 5.20}$	39.41$_{\pm 8.15}$	71.60$_{\pm 3.72}$	63.55$_{\pm 3.80}$	67.25$_{\pm 6.59}$	22.98$_{\pm 0.2}$	77.21$_{\pm 2.45}$	**80.78**$_{\pm 2.40}$
D2	93.91$_{\pm 3.20}$	81.99$_{\pm 5.67}$	8.12$_{\pm 1.38}$	87.62$_{\pm 6.63}$	92.16$_{\pm 4.56}$	91.32$_{\pm 3.32}$	93.51$_{\pm 4.73}$	64.11$_{\pm 8.20}$	88.39$_{\pm 9.06}$	42.67$_{\pm 5.49}$	94.18$_{\pm 5.47}$	**94.73**$_{\pm 5.00}$
D3	4.39$_{\pm 1.18}$	70.08$_{\pm 0.88}$	4.32$_{\pm 0.15}$	88.99$_{\pm 0.83}$	67.55$_{\pm 5.07}$	47.82$_{\pm 1.82}$	**92.12**$_{\pm 0.70}$	86.98$_{\pm 1.44}$	72.69$_{\pm 0.94}$	23.57$_{\pm 6.84}$	85.96$_{\pm 0.94}$	87.31$_{\pm 1.00}$
D4	12.38$_{\pm 3.72}$	81.18$_{\pm 4.26}$	0.03$_{\pm 0.01}$	1.70$_{\pm 0.67}$	52.72$_{\pm 3.62}$	46.94$_{\pm 4.08}$	68.27$_{\pm 3.55}$	60.06$_{\pm 2.93}$	57.13$_{\pm 2.53}$	19.73$_{\pm 0.39}$	85.16$_{\pm 1.84}$	**85.34**$_{\pm 1.77}$
D5	26.01$_{\pm 0.45}$	78.36$_{\pm 0.82}$	39.33$_{\pm 0.30}$	**80.72**$_{\pm 0.78}$	0.07$_{\pm 0.00}$	71.04$_{\pm 1.51}$	79.55$_{\pm 0.90}$	73.17$_{\pm 1.36}$	70.36$_{\pm 0.63}$	49.53$_{\pm 13.31}$	73.13$_{\pm 0.80}$	73.66$_{\pm 1.02}$
D6	27.56$_{\pm 9.16}$	93.80$_{\pm 2.16}$	3.05$_{\pm 0.46}$	77.53$_{\pm 3.36}$	36.75$_{\pm 6.70}$	41.30$_{\pm 3.47}$	97.58$_{\pm 1.21}$	68.81$_{\pm 7.40}$	91.34$_{\pm 1.5}$	11.71$_{\pm 7.49}$	97.67$_{\pm 1.99}$	**98.05**$_{\pm 1.33}$
D7	4.56$_{\pm 2.36}$	69.20$_{\pm 9.68}$	0.90$_{\pm 0.40}$	4.95$_{\pm 2.94}$	34.92$_{\pm 6.07}$	48.68$_{\pm 9.90}$	46.28$_{\pm 13.12}$	70.27$_{\pm 6.96}$	59.09$_{\pm 7.68}$	7.38$_{\pm 5.23}$	80.92$_{\pm 3.63}$	**82.16**$_{\pm 4.02}$
D8	70.06$_{\pm 8.19}$	79.3$_{\pm 6.05}$	42.78$_{\pm 3.37}$	78.71$_{\pm 9.00}$	56.59$_{\pm 19.64}$	45.83$_{\pm 13.65}$	84.50$_{\pm 9.00}$	79.03$_{\pm 14.35}$	79.62$_{\pm 6.96}$	73.26$_{\pm 0.92}$	89.41$_{\pm 9.51}$	**91.54**$_{\pm 7.84}$
D9	37.12$_{\pm 16.4}$	87.96$_{\pm 4.04}$	16.16$_{\pm 5.42}$	91.92$_{\pm 5.88}$	86.81$_{\pm 6.92}$	90.31$_{\pm 10.60}$	91.94$_{\pm 6.60}$	84.76$_{\pm 10.63}$	57.97$_{\pm 11.9}$	71.43$_{\pm 23.19}$	84.11$_{\pm 5.83}$	**94.29**$_{\pm 5.79}$
D10	24.83$_{\pm 7.65}$	93.52$_{\pm 1.86}$	3.56$_{\pm 0.58}$	93.88$_{\pm 1.16}$	69.37$_{\pm 4.79}$	65.19$_{\pm 4.09}$	93.64$_{\pm 2.10}$	82.24$_{\pm 3.63}$	89.01$_{\pm 1.78}$	51.04$_{\pm 9.95}$	92.75$_{\pm 2.45}$	**95.02**$_{\pm 2.80}$
D11	21.69$_{\pm 4.65}$	25.05$_{\pm 2.54}$	0.00$_{\pm 0.00}$	40.05$_{\pm 4.99}$	22.26$_{\pm 5.67}$	23.79$_{\pm 2.90}$	29.52$_{\pm 4.47}$	**42.86**$_{\pm 4.62}$	20.33$_{\pm 5.1}$	17.43$_{\pm 1.08}$	35.18$_{\pm 3.38}$	39.59$_{\pm 4.16}$
D12	53.78$_{\pm 21.02}$	62.32$_{\pm 25.31}$	25.68$_{\pm 7.45}$	60.18$_{\pm 18.92}$	37.28$_{\pm 16.15}$	61.17$_{\pm 25.04}$	42.19$_{\pm 11.37}$	47.36$_{\pm 14.32}$	88.57$_{\pm 16.75}$	52.62$_{\pm 1.28}$	60.75$_{\pm 29.38}$	61.78$_{\pm 30.11}$
D13	81.52$_{\pm 22.51}$	83.85$_{\pm 10.74}$	24.30$_{\pm 11.52}$	77.35$_{\pm 20.97}$	52.25$_{\pm 21.56}$	66.19$_{\pm 20.46}$	87.36$_{\pm 12.40}$	72.65$_{\pm 21.62}$	81.10$_{\pm 17.3}$	50.31$_{\pm 7.23}$	91.63$_{\pm 9.21}$	**91.63**$_{\pm 9.21}$
D14	16.81$_{\pm 13.02}$	90.50$_{\pm 9.71}$	22.16$_{\pm 9.28}$	58.67$_{\pm 19.29}$	69.44$_{\pm 18.42}$	49.37$_{\pm 16.42}$	82.93$_{\pm 13.92}$	63.29$_{\pm 18.73}$	–	64.91$_{\pm 27.05}$	82.74$_{\pm 15.65}$	**82.74**$_{\pm 15.65}$
D15	23.45$_{\pm 10.32}$	42.88$_{\pm 12.09}$	1.17$_{\pm 1.16}$	7.14$_{\pm 4.99}$	33.47$_{\pm 7.76}$	39.72$_{\pm 11.07}$	25.87$_{\pm 10.41}$	24.24$_{\pm 8.16}$	50.26$_{\pm 14.77}$	39.71$_{\pm 6.40}$	47.92$_{\pm 13.89}$	**47.92**$_{\pm 13.89}$
Ave	38.28$_{\pm 8.55}$	74.40$_{\pm 6.64}$	15.47$_{\pm 2.98}$	61.57$_{\pm 7.03}$	51.03$_{\pm 8.81}$	55.21$_{\pm 9.10}$	72.46$_{\pm 6.55}$	65.56$_{\pm 8.54}$	69.51$_{\pm 7.39}$	39.89$_{\pm 7.74}$	78.58$_{\pm 7.09}$	**80.44**$_{\pm 7.07}$
Rank	8.87	4.53	11.60	5.93	8.27	7.53	4.33	6.40	6.13	9.27	3.53	1.53

TABLE VI
CLASSIFICATION ACCURACY OF ABLATION EXPERIMENTS

No.s	Ours_{cos}	Ours_{RBF}	Ours_{GB}	Ours_{Con}	Ours_{Pre}	Ours
D1	79.77$_{\pm 2.60}$	80.06$_{\pm 1.84}$	51.03$_{\pm 3.77}$	77.66$_{\pm 2.91}$	78.88$_{\pm 1.79}$	**81.52**$_{\pm 2.34}$
D2	93.72$_{\pm 2.60}$	94.26$_{\pm 3.74}$	95.52$_{\pm 2.38}$	83.33$_{\pm 6.09}$	90.46$_{\pm 5.59}$	**95.65**$_{\pm 3.18}$
D3	86.58$_{\pm 1.02}$	86.72$_{\pm 1.01}$	62.30$_{\pm 2.23}$	81.90$_{\pm 1.61}$	86.44$_{\pm 1.07}$	**86.72**$_{\pm 1.01}$
D4	**86.67**$_{\pm 1.08}$	86.39$_{\pm 0.96}$	13.68$_{\pm 2.28}$	79.28$_{\pm 2.08}$	84.52$_{\pm 0.78}$	86.39$_{\pm 0.96}$
D5	75.59$_{\pm 0.70}$	75.80$_{\pm 0.96}$	54.1$_{\pm 4.06}$	71.24$_{\pm 1.04}$	75.23$_{\pm 0.71}$	**75.8**$_{\pm 0.96}$
D6	97.96$_{\pm 1.23}$	96.84$_{\pm 1.46}$	97.31$_{\pm 2.46}$	94.52$_{\pm 2.49}$	97.68$_{\pm 1.86}$	**97.96**$_{\pm 1.56}$
D7	81.11$_{\pm 6.25}$	80.56$_{\pm 5.71}$	83.61$_{\pm 4.62}$	79.72$_{\pm 6.68}$	83.33$_{\pm 4.90}$	**83.89**$_{\pm 4.86}$
D8	90.87$_{\pm 9.34}$	91.37$_{\pm 7.06}$	77.03$_{\pm 11.50}$	85.68$_{\pm 8.41}$	91.37$_{\pm 4.15}$	**93.26**$_{\pm 6.57}$
D9	90.00$_{\pm 4.74}$	90.00$_{\pm 7.26}$	88.10$_{\pm 6.04}$	86.19$_{\pm 10.15}$	90.95$_{\pm 7.60}$	**93.81**$_{\pm 6.37}$
D10	**94.55**$_{\pm 2.09}$	**94.55**$_{\pm 2.09}$	66.97$_{\pm 2.9}$	86.67$_{\pm 3.37}$	92.73$_{\pm 3.03}$	93.98$_{\pm 2.84}$
D11	61.25$_{\pm 2.35}$	61.49$_{\pm 2.50}$	33.18$_{\pm 3.77}$	61.23$_{\pm 2.13}$	60.55$_{\pm 2.00}$	**61.74**$_{\pm 1.84}$
D12	60.71$_{\pm 27.84}$	71.90$_{\pm 21.20}$	70.95$_{\pm 13.03}$	70.00$_{\pm 20.76}$	68.33$_{\pm 21.71}$	**76.43**$_{\pm 15.41}$
D13	86.25$_{\pm 11.23}$	67.86$_{\pm 20.48}$	91.43$_{\pm 12.05}$	81.79$_{\pm 11.93}$	90.54$_{\pm 13.88}$	**91.61**$_{\pm 9.89}$
D14	96.73$_{\pm 3.76}$	96.20$_{\pm 5.04}$	93.52$_{\pm 5.30}$	91.17$_{\pm 5.95}$	94.44$_{\pm 5.86}$	**97.78**$_{\pm 3.88}$
D15	60.70$_{\pm 10.32}$	62.50$_{\pm 8.55}$	61.84$_{\pm 9.78}$	62.50$_{\pm 8.55}$	61.25$_{\pm 8.44}$	**62.87**$_{\pm 9.74}$
Ave	82.83$_{\pm 5.81}$	82.43$_{\pm 5.99}$	69.37$_{\pm 5.74}$	79.53$_{\pm 6.28}$	83.11$_{\pm 5.56}$	**85.29**$_{\pm 4.76}$
Rank	3.20	2.67	4.60	5.00	3.80	**1.20**

Note: Ours_{cos}, Ours_{RBF}, Ours_{GB}, Ours_{Con}, and Ours_{Pre} denote the ablation experiments about the similarity cosine measure, RBF measure, granular ball generation, concept construction, and prediction.

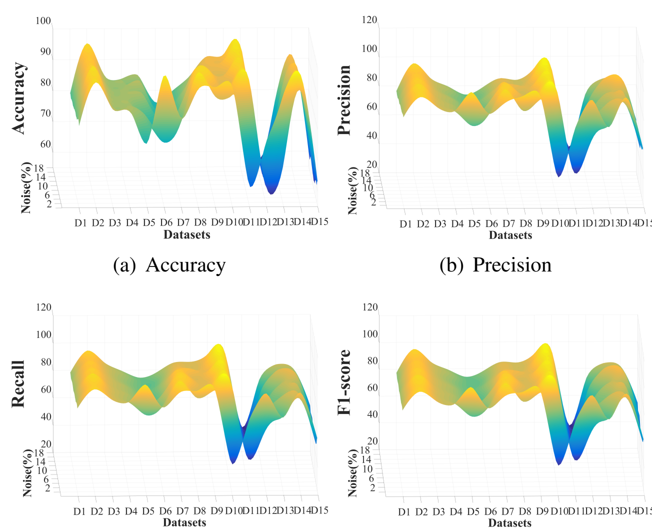
TABLE VII
CONCEPT LEARNING TIME(S) COMPARISON OF FIVE CCL METHODS

Dataset	WFCCL	M-FCCL	R-FCCL	CM-CCL	GB-CCL
D1	452.56	1066.80	282.31	552.36	**28.26**
D2	16.91	16.70	6.06	102.78	**0.78**
D3	18262.23	15577.00	3967.60	210.36	**545.27**
D4	4494.16	1272.30	570.32	4540.00	**233.00**
D5	123145.00	45901.85	23055	1088.60	**2958.88**
D6	107.67	80.50	35.27	210.20	**1.93**
D7	20.77	10.00	**6.95**	223.10	10.38
D8	26.37	10.80	3.95	62.00	**2.63**
D9	45.93	7.20	3.10	44.50	**2.90**
D10	473.89	356.40	130.96	577.90	**58.20**
D11	2611.55	3482.80	960.01	2368.40	**204.25**
D12	0.83	0.83	5.82	45605.00	**0.58**
D13	12.29	30.90	15.10	278.75	**1.55**
D14	1.39	15.25	96.11	–	**0.53**
D15	0.81	0.63	8.40	5917.30	**0.56**
Ave	9978.16	4522.00	435.14	4412.95	**269.98**

designed framework.

2) Comparison of concept learning time: The results of comparing the concept learning time of GB-CCL with all the involved CCL methods are recorded in Table VII. From the comparison results, we have three observations as follows.

- The proposed GB-CCL consumes the least concept learning time among all concept-cognitive learning methods except on the Move dataset of R-FCCL. The main reason is that our concept learning is based on justifiable granular-ball generation space, and in the whole process, there is no parameter optimization involved.
- WFCCL shows the worst performance, and CM-CCL and M-FCCL also show poor performance in most cases, mainly because they rely on neighborhood-based information granules and parameter optimization. Moreover, the introduction of attribute weights computation in W-FCCL increases computational costs.
- GB-CCL achieves the lowest concept learning time and best classification performance among all the CCL methods, which indicates that GB-CCL has a better concept learning performance than the current state-of-the-art CCL methods.



(a) Accuracy (b) Precision

(c) Recall (d) F1-score

Fig. 6. Classification results comparison on diffenent noise levels.

TABLE VIII
CLASSIFICATION ACCURACY(%) COMPARISON OF GB-CCL ON DIFFERENT NOISE LEVELS

Dataset	RAW	Noise Level								
		2%	4%	6%	8%	10%	12%	14%	16%	18%
D1	81.52$_{\pm 2.34}$	82.31$_{\pm 1.47}$	80.66$_{\pm 3.99}$	80.38$_{\pm 3.66}$	80.62$_{\pm 2.99}$	79.91$_{\pm 2.76}$	79.44$_{\pm 3.03}$	79.58$_{\pm 2.42}$	78.93$_{\pm 2.18}$	78.79$_{\pm 1.43}$
D2	95.65$_{\pm 3.18}$	95.56$_{\pm 2.99}$	95.55$_{\pm 3.26}$	96.64$_{\pm 2.57}$	94.13$_{\pm 4.07}$	94.14$_{\pm 2.75}$	93.32$_{\pm 4.37}$	93.60$_{\pm 2.92}$	93.87$_{\pm 3.40}$	95.27$_{\pm 3.94}$
D3	86.72$_{\pm 1.01}$	85.69$_{\pm 0.99}$	85.36$_{\pm 1.02}$	85.63$_{\pm 0.66}$	85.43$_{\pm 1.13}$	85.63$_{\pm 0.81}$	85.47$_{\pm 1.17}$	85.14$_{\pm 0.86}$	85.06$_{\pm 0.77}$	84.88$_{\pm 1.00}$
D4	86.39$_{\pm 0.96}$	85.55$_{\pm 1.51}$	85.06$_{\pm 0.98}$	85.31$_{\pm 1.24}$	84.92$_{\pm 1.08}$	84.77$_{\pm 0.84}$	84.60$_{\pm 1.54}$	84.06$_{\pm 1.04}$	84.73$_{\pm 0.74}$	83.70$_{\pm 1.21}$
D5	75.80$_{\pm 0.96}$	75.75$_{\pm 1.14}$	75.80$_{\pm 0.73}$	76.23$_{\pm 0.81}$	76.58$_{\pm 0.65}$	76.05$_{\pm 1.12}$	76.27$_{\pm 0.92}$	76.19$_{\pm 0.90}$	76.15$_{\pm 0.93}$	76.06$_{\pm 0.74}$
D6	97.96$_{\pm 1.56}$	80.56$_{\pm 7.17}$	78.89$_{\pm 8.30}$	82.22$_{\pm 10.49}$	77.22$_{\pm 9.60}$	78.61$_{\pm 7.53}$	80.28$_{\pm 7.46}$	79.44$_{\pm 9.18}$	79.72$_{\pm 7.41}$	79.72$_{\pm 10.15}$
D7	83.89$_{\pm 4.86}$	97.03$_{\pm 1.68}$	94.80$_{\pm 2.02}$	94.43$_{\pm 1.90}$	93.50$_{\pm 1.58}$	92.01$_{\pm 1.76}$	88.22$_{\pm 3.00}$	86.36$_{\pm 3.55}$	85.61$_{\pm 2.39}$	84.40$_{\pm 3.10}$
D8	93.26$_{\pm 6.57}$	89.79$_{\pm 6.34}$	89.71$_{\pm 5.91}$	92.79$_{\pm 7.42}$	92.84$_{\pm 5.48}$	91.34$_{\pm 5.30}$	90.26$_{\pm 7.11}$	90.26$_{\pm 6.98}$	90.71$_{\pm 9.18}$	90.79$_{\pm 7.54}$
D9	93.81$_{\pm 6.37}$	89.52$_{\pm 5.85}$	90.95$_{\pm 6.53}$	89.05$_{\pm 7.79}$	88.57$_{\pm 9.58}$	91.43$_{\pm 7.03}$	89.05$_{\pm 7.79}$	92.38$_{\pm 6.81}$	90.95$_{\pm 6.90}$	91.43$_{\pm 5.85}$
D10	93.98$_{\pm 2.84}$	94.20$_{\pm 1.83}$	93.72$_{\pm 1.91}$	93.98$_{\pm 1.46}$	92.55$_{\pm 1.06}$	93.25$_{\pm 1.02}$	93.77$_{\pm 1.84}$	93.94$_{\pm 2.24}$	94.68$_{\pm 1.73}$	93.77$_{\pm 1.46}$
D11	61.74$_{\pm 1.84}$	61.41$_{\pm 1.44}$	59.94$_{\pm 2.06}$	60.45$_{\pm 2.11}$	59.37$_{\pm 2.54}$	59.02$_{\pm 2.38}$	59.39$_{\pm 1.50}$	59.41$_{\pm 1.41}$	58.96$_{\pm 2.34}$	58.47$_{\pm 1.84}$
D12	66.19$_{\pm 25.09}$	66.19$_{\pm 25.09}$	72.62$_{\pm 7.63}$	74.52$_{\pm 16.84}$	72.62$_{\pm 17.47}$	69.29$_{\pm 12.32}$	69.05$_{\pm 16.84}$	75.71$_{\pm 16.14}$	67.62$_{\pm 15.84}$	75.71$_{\pm 14.10}$
D13	91.61$_{\pm 9.89}$	80.89$_{\pm 12.66}$	82.14$_{\pm 15.82}$	77.86$_{\pm 15.13}$	79.29$_{\pm 16.24}$	84.82$_{\pm 13.92}$	80.71$_{\pm 16.43}$	85.89$_{\pm 15.07}$	74.82$_{\pm 11.43}$	84.82$_{\pm 13.92}$
D14	93.64$_{\pm 6.98}$	92.12$_{\pm 6.69}$	93.07$_{\pm 4.86}$	91.10$_{\pm 8.12}$	93.52$_{\pm 6.69}$	91.07$_{\pm 7.79}$	93.05$_{\pm 9.47}$	91.19$_{\pm 6.85}$	91.10$_{\pm 6.98}$	91.10$_{\pm 6.98}$
D15	56.76$_{\pm 11.23}$	60.55$_{\pm 13.98}$	60.55$_{\pm 12.18}$	58.2$_{\pm 13.13}$	57.65$_{\pm 10.55}$	58.75$_{\pm 15.18}$	61.84$_{\pm 11.86}$	61.14$_{\pm 12.18}$	58.75$_{\pm 12.67}$	61.18$_{\pm 10.92}$
Ave	83.93$_{\pm 5.71}$	82.47$_{\pm 6.06}$	82.59$_{\pm 5.15}$	82.59$_{\pm 6.22}$	81.92$_{\pm 6.05}$	82.01$_{\pm 5.50}$	81.65$_{\pm 6.29}$	82.29$_{\pm 5.90}$	80.78$_{\pm 5.66}$	82.01$_{\pm 5.61}$

D. Influence of the proposed viewpoint(Rq.3)

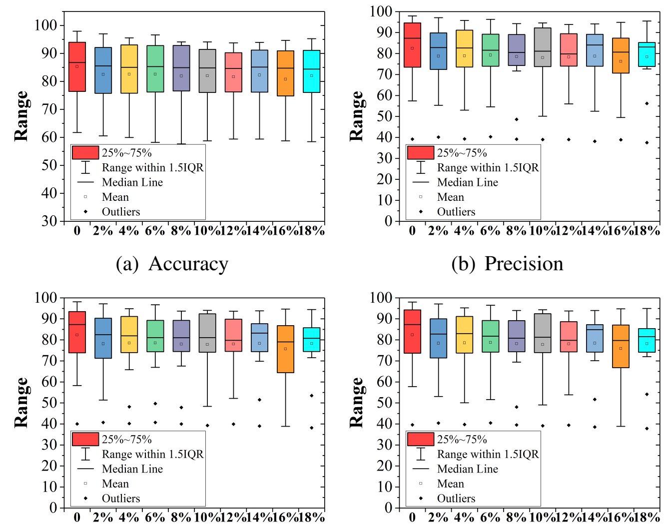
According to the above analysis, we know that GB-CCL is without any parameters, and the process of GB generation is an adaptive learning based on the fuzzy decision formal context. Hence, in order to verify the robustness of the proposed ideas, we add noise to the raw conditional attributes to show the changes in the classification performance of GB-CCL under different noise situations. Specifically, random noise is introduced from 2% to 18% with an increment of 2%. For each noise level, v random numbers $(g_1, g_2, \cdots, g_v)$ are generated within the range of $[-g, g]$, where g denotes the random error threshold. The method of adding noise is as follows:

$$\widetilde{I}_i(x,a) = \begin{cases} \widetilde{I}(x,a) + g_i & if(0 \leq \widetilde{I}(x,a) + g_i \leq 1) \\ \widetilde{I}(x,a) & else \end{cases}$$

where $\widetilde{I}(x,a)$ represents the value of x under a in fuzzy decision formal context $FC = (G, M, \widetilde{I}, D, J)$, $\widetilde{I}_i(x,a)$ represents the value of x under a by add i^{th} noise.

The classification accuracy of GB-CCL under different noise data levels is shown in Tables VIII. It can be seen from these tables and figures that the classification measures of GB-CCL fluctuate slightly in experimental datasets as the noise data increases. The more detailed comparison in accuracy, precision, recall, and F1-score is shown in Fig.6. To further observe the variation induced by noise, the Box-plot is employed to show the distribution central location and a spread range of GB-CCL under different noise data levels, which is shown in Fig.7. In these figures, the □, −, ⊥, and ⊤ represent the mean value, median line, first quantile, and third quartile, respectively. The difference between ⊥ and ⊤ is the quartile, which is also called the 1.5 interquartile range(1.5 IQR). The data outside the IQR is called an outlier.

All these measures could be used to analyze the differences among the compared methods. The compared methods are considered the same when the adopted measures differ little. From Fig.7, we could obtain the following point: 1) At different noise levels, the median line and mean values of the GB-CCL method are almost on the same level, indicating that the central location consistency of GB-CCL with noise data increases. 2) Except for the WiQW dataset, other results are all in the 1.5 IQR, and the 1.5 IQR is roughly the same in most situations. This robustness can be attributed to two key components of the GB-CCL framework. Firstly, the adaptive granulation generation mechanism improves local noise tolerance by controlling purity, thereby preventing the influence of noisy samples on the local data structure. Moreover, the fuzzy three-way concept learning method further restricts the scope of concept learning by integrating positive and negative context information. They all collaborate to ensure GB-CCL's stable and robust learning performance in noisy environments.



(a) Accuracy (b) Precision

(c) Recall (d) F1-score

Fig. 7. Box diagram of GB-CCL on different noise data levels.

VI. CONCLUSION

Two influential studies in granular computing and artificial intelligence, namely concept-cognitive learning and granular-ball computing, can achieve the same results through different emphases. The proposed GB-CCL is a fruitful marriage of granular-ball computing and concept-cognitive learning to study uncertain information and knowledge processing embedded in complex data. It can achieve effective, robust, and adaptive concept learning for knowledge discovery. It adopts threefold ideas: 1) justifiable granular-ball space adaptive generation, 2) fuzzy three-way concept to effectively characterize fuzzy ontology, 3) concept center based on GB for label predictor. In this article, the theoretical framework of GB-CCL has been fully developed, and the corresponding experiments have been thoroughly demonstrated.

Despite the proposed GB-CCL method showing good performance on knowledge discovery tasks, we acknowledge a range of limitations to our method thus far. For example, we have not yet tested a wider range of complex problem application scenarios, including cognitive and learning multimodal scenarios with various data types. The granular representation in GB-CCL provides a flexible mechanism for constructing concept intentions at different granularities, which offers a natural foundation for extending the framework to multimodal concept learning. In addition, another interesting issue for future work is to study CCL in an open environment, namely, environments that change over time. Owing to its dynamic concept learning mechanism and adaptive granular construction, GB-CCL is inherently suitable for incrementally updating concepts and accommodating drifting patterns. Finally, future work will involve exploring concept-cognitive mechanisms, optimizing cognitive systems with various fuzzy evaluation measures, and systematically modeling cognitive processes, particularly in large-scale, dynamic data environments.

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