



Matrix-based multi-granularity multi-source fuzzy information fusion for three-dimensional dynamic data

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Abstract

Multi-source information fusion is a key technique in big data technology, essential for data mining and knowledge discovery. When merging traditional multi-source systems into single systems, data loss is a common issue. Multi-granularity information fusion methods are designed to solve this problem. However, there is limited research on how to use these methods dynamically to extract valuable information when there are changes in three dimensions: objects, information sources, and the number of attributes. Additionally, information fusion in multi-source fuzzy information systems can better handle uncertainties in data. This paper proposes a method for calculating multi-granularity fusion operators based on a similarity relationship matrix and creates an incremental update fusion mechanism for six different scenarios, such as adding or removing objects, information sources, and attributes. Experimental results on 12 public datasets show that our proposed dynamic fusion method has clear advantages in dealing with complex changes and can update the fusion operator more efficiently.

Keywords Multi-source information fusion · Knowledge discovery · Incremental update mechanism · Matrix-based multi-granularity computing

1 Introduction

The application of information fusion in fuzzy information systems is particularly important, especially when dealing with multi-source data. Its core goal is to extract more comprehensive [1] and detailed information than that from a single information source by integrating data from multiple sources, thereby improving the accuracy and reliability of decision-making. Fuzzy information systems can effectively deal with the uncertainty and vagueness [2] in data, compensate for the biases introduced by a single source of information, and make decision-making more stable and comprehensive. One of the advantages of information fusion is that it can eliminate the uncertainty and vagueness in data and thereby ensure the precision of decision-making.

In the face of data with more noise and errors, information fusion can provide more stable decision support. At the same time, information fusion integrates data from different sources and perspectives to provide a comprehensive description [3] of events from multiple levels, helping decision-makers make more reasonable decisions in complex and dynamically changing environments. Different sources of information usually provide different perspectives, and by integrating complementary information, the decision-making can be further enhanced in terms of reliability.

Multi-source data integration involves complex data formed by combining information from multiple sources or datasets, with a notable feature being that different sources contain varying knowledge structures and describe and depict samples and their relationships from different perspectives. In multi-source fuzzy information systems, designing dynamic update fusion methods is of great significance [4], especially in the face of complex and dynamic data environments. Multi-source information systems typically involve information from multiple information sources, attributes, and objects, which may constantly update as they change over time, environment, or external factors. In this context, traditional static fusion methods

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are difficult to cope with the addition or removal of information sources, changes in object attributes, and data noise and uncertainty. Therefore, designing dynamic update fusion methods can effectively address these changes, enhance system flexibility, and improve decision accuracy. In multi-source fuzzy information systems, designing dynamic update fusion methods can significantly enhance system flexibility, adaptability, and robustness, enabling information fusion to address the challenges posed by changes in objects, information sources, and attributes. By updating fusion operators in real-time, dynamic update methods not only improve decision accuracy and reliability but also optimize system efficiency and scalability. As the complexity and scale of information systems continue to increase, dynamic update fusion methods will play an important role in intelligent decision support, risk assessment, resource management, and other fields.

Although several dynamic update strategies for multi-source information systems in different scenarios have been proposed in existing studies [5], research on multi-source fuzzy information systems with multi-granularity fusion operators based on dynamic update [6–9] is still rare, and no set of update mechanisms suitable for updating fusion operators in multi-source information systems when the number of objects, information sources, and attributes changes has been proposed. The goal of this study is to propose a multi-granularity fusion method based on a similarity relationship matrix, aiming to achieve an update of the fusion operator through the proposed dynamic mechanism. Figure 1 presents the motivation of our work.

The overarching structure of our research is delineated in Fig. 2. This study focuses on the following issues: (1) How to effectively handle information fusion in multi-source fuzzy information systems; (2) How to implement

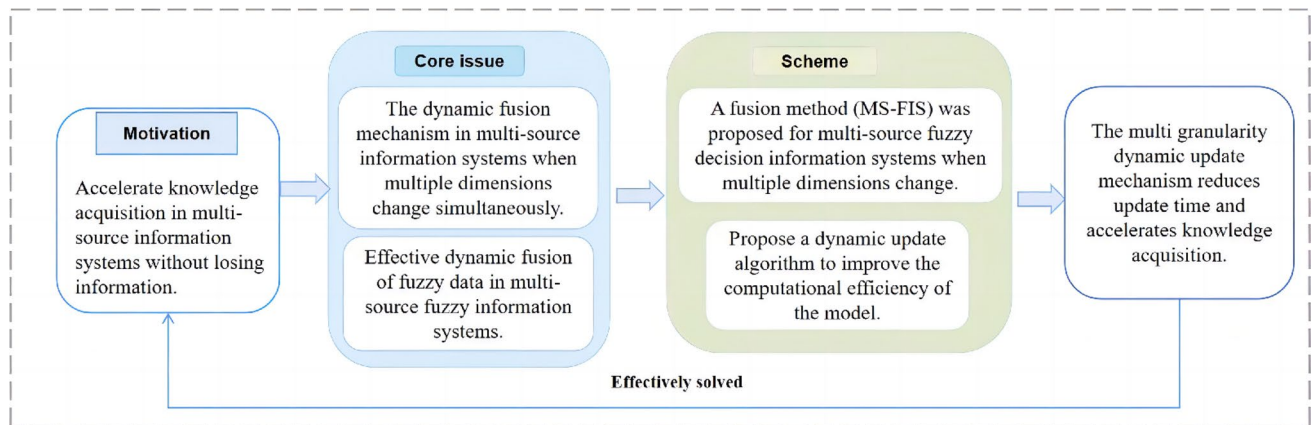


Fig. 1 Motivation of our work

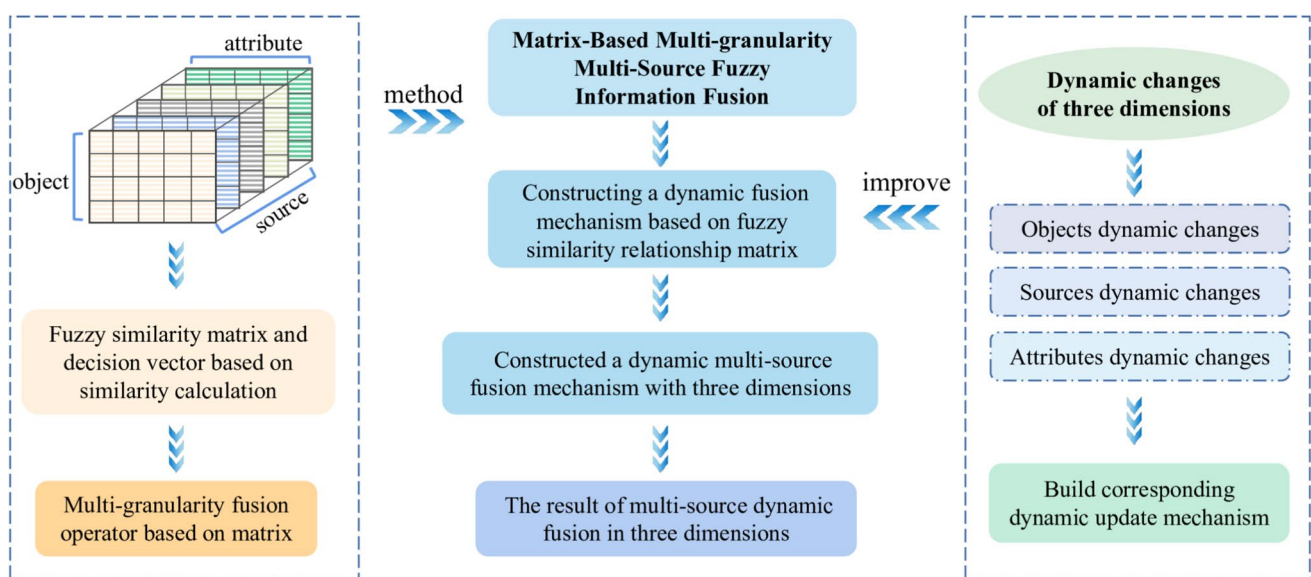


Fig. 2 The overarching structure of our research

multi-granularity fusion methods based on similarity relationship matrices in multi-source fuzzy information systems; (3) How to efficiently update the fusion results through dynamic algorithms when the number of objects, information sources, and attributes in the multi-source fuzzy information system changes.

This paper presents three main contributions to the dynamic updating of multi-source fuzzy information fusion: (1) It introduces a static calculation approach for multi-granularity fusion operators based on similarity relationship matrices; (2) It develops a dynamic fusion mechanism grounded in fuzzy similarity relationship matrices, which accommodates the addition and removal of objects, information sources, and attributes within multi-source information systems; (3) Through experimental validation, the proposed dynamic method demonstrates superior performance over static update mechanisms when multiple structures in a multi-source information system are altered, particularly in updating multi-granularity fusion operators.

The rest of this paper is structured as follows. In Section 2, existing research in the field of information fusion is categorized and reviewed. Section 3 presents the core concepts of fuzzy rough set theory, multi-source fuzzy information systems, and multi-granularity information fusion. Section 4 introduces a multi-granularity fusion method based on fuzzy similarity relationship matrices and elaborates on the static fusion mechanism. Additionally, it provides specific examples of update mechanisms for different scenarios involving the addition or deletion of objects, information sources, and attributes in multi-source fuzzy decision information systems, with the objective of enhancing fusion operator updates through dynamic algorithms. In Section 5, a series of experiments using the UCI dataset are conducted to compare and assess the performance of static versus dynamic algorithms. This section also compares the proposed methods with related algorithms and machine learning techniques, verifying the superiority of the proposed method through hypothesis testing. Finally, the paper concludes by summarizing the work, outlining the limitations of the research, and suggesting potential future research directions.

2 Related works

In the introduction, no specific information about the various studies is mentioned, and this information will be elaborated in the following section in detail. Table 1 categorizes the different fusion methods according to the different fusion methods and the corresponding characteristics.

- (1) Multi-granularity and rough set-based fusion methods. To address the challenges of uncertainty, ambiguity, and

Table 1 Summary of existing fusion techniques

Methods	Research content	Insufficient
Multi-granularity rough set fusion	Decision-making problems [10]	The application of multi-granularity to multi-source data is lacking.
Information entropy fusion method	Direct knowledge acquisition [11]	Loss of information can affect the outcome of the fusion.
	Based on the theory of evidence [12–15]	
Dynamic information fusion method	Based on conditional entropy [16–18]	Lacking efficient methods for handling changes in all three dimensions.
	Based on generalized information entropy [19]	
	Enlarged information entropy [20, 21]	
Dynamic information fusion method	Dynamic updating of object changes [22–26]	Lacking efficient methods for handling changes in all three dimensions.
	Dynamic updating of information sources [27, 28]	
	Dynamic update of attribute set changes [29–33]	

incomplete information [34], Xu et al. [35] introduced a fuzzy feature selection method that combines composite entropy uncertainty measure and data distribution. Zhang et al. [36] developed an uncertainty measure and feature selection technique based on fuzzy neighborhood rough set [37–39]. To flexibly manage different constraints and achieve information fusion while effectively minimizing information loss in the process of multi-source data fusion, Ma et al. [10] devised a multi-granularity decision-making method based on inclusion degree. Yang et al. [11] presented a method for directly extracting knowledge from multi-source information systems using multi-granularity rough set. However, research on the effective application of multi-granularity models [40, 41] to multi-source data remains relatively limited.

- (2) Information entropy-based fusion methods. Additionally, there has been progress in the application of evidence theory [12–14] in uncertain data decision-making, though the solution accuracy and reliability of these methods still require further enhancement. Another category of methods extracts important information through specific selection rules, typically using the minimum information entropy [15] as the selection criterion, and combining fusion rules for information fusion. For example, Xu et al. [16] developed a method of interval value similarity degree measurement information entropy fusion based on statistical distribution

and the K-L divergence principle; Zhang et al. [17] devised a conditional entropy based on tolerance relationships specifically for the fusion of incomplete information systems; Xu et al. [18] defined a new fuzzy support relationship to construct conditional entropy and achieved information fusion on ordinal information systems; Han et al. [19] suggested a method of handling incomplete information system fusion based on generalized information entropy. Furthermore, there are extended information entropy [20] models, such as rough entropy and composite entropy, which are effective to some extent [21], though they may also result in the loss of some information, thereby affecting the final fusion outcome.

- (3) **Dynamic and incremental information fusion methods.** Another research approach is based on granule computing for dynamic multi-source information fusion. In real-world scenarios, data often changes due to various factors. For example, the detection of a patient's health status in a hospital can be influenced by hospital facilities, software and hardware, and the professional level of doctors. As a result, efficiently updating knowledge dynamically becomes essential [42]. Designing effective multi-source information fusion algorithms has become a research hotspot, particularly in terms of how to gradually update and maintain knowledge when data is added or deleted. Incremental learning [43] methods not only integrate new and old information efficiently but also reduce storage and computation time. Regarding incremental update methods for multi-source information systems with dimensional changes, research has primarily concentrated on the following areas: Firstly, Ge et al. [22] introduced an approximate update model for object changes, while Zhang et al. [23] developed an incremental learning method for information fusion based on equivalence relation matrices. Hu et al. [24] presented a multi-granularity approximation strategy for object changes, and Zhang et al. [25] designed a dynamic update algorithm based on generalized multi-granularity neighborhood advantage rough sets. Xu et al. [26] advanced an incremental feature selection method for object changes. Secondly, Huang et al. [27] described a matrix-based approach for incremental feature selection using conditional entropy for multi-source data with changing information sources. Li et al. [28] devised a dynamic update method for fuzzy information granules with changing information sources. Finally, changes in attribute sets affect the addition or deletion of data dimensions and information. Chen et al. [29] explored the conceptual approximate update when adding attributes and objects. Wang et al. [30] developed a method to

reduce the dimension of attribute sets. Zhang et al. [31] discussed the approximate dynamic update method in multi-granularity interval hesitant fuzzy information systems when deleting or inserting attributes. Hu et al. [32] introduced a dynamic technique based on matrix updates for attribute approximation, designed to avoid reconstructing the induced matrix in composite sequential decision systems.

Selecting efficient and feasible fusion methods remains a significant challenge. This paper addresses the issue of information fusion in multi-source fuzzy information systems from the perspective of multi-granularity fusion operators and dynamic algorithms, with the aim of achieving efficient information fusion and minimizing the loss of useful information.

3 Preliminaries

This subsection primarily lays the foundation for the rest of the paper by introducing important concepts such as fuzzy rough sets and multi-source fuzzy information systems (MS-FIS). Before proceeding with this subsection, it is necessary to clarify that this paper studies multi-source information systems within the framework of fuzzy classical decision making, rather than within the framework of fuzzy or classical fuzzy decision making. In other words, the condition attributes are fuzzy data, while the decision attributes are discrete data.

3.1 Fuzzy rough set theory

Let a $FIS = (U, AT, D)$ is designated as a fuzzy decision system with a decision, where $U = \{x_1, \dots, x_n\}$ is a non-empty finite object set, $AT = \{a_1, \dots, a_m\}$ is a conditional attributes set, $D = \{d\}$ denotes decision attribute set, and $AT \cap D = \emptyset$. Furthermore, $\forall a \in AT$, and $\forall x \in U$, $a(x)$ represents the value of object x under the attribute a . Also, if $a(x) \in [0, 1]$ for all conditional attributes, The corresponding information system is called a fuzzy decision information system, and Table 1 shows a fuzzy decision information dataset. The classical rough set model, due to its strict equivalence relation, can only precisely handle classification data, lacking inclusiveness in numerical data. To address this limitation, we use the euclidean distance ΔDis to calculate the similarity to elucidate the relationship between any two objects in the universe of fuzzy decision data sets.

For any $a \in AT$, $x_i, x_j \in U$, the formula for calculating the fuzzy euclidean distance ΔDis between x_i and x_j can be defined as below:

$$\Delta Dis = \sqrt{\sum_{k=1}^p (x_{ik} - x_{jk})^2}$$

The formula for calculating the similarity of x_i and x_j can be defined as below:

$$Sim_{(x_i, x_j)} = \frac{1}{1 + \Delta Dis}$$

where p is the number of attributes. If the fuzzy similarity $Sim_{(x_i, x_j)}$ exceeds the predefined threshold θ , then objects x_i and x_j are considered similar, and the corresponding position in the relationship matrix is assigned a value of 1; otherwise, it is set to 0.

After calculating the similarity, the objects can be divided into different similarity classes based on the similarity relationships. Suppose FIS_i is the i th MS-FIS, for condition attribute subset $A_i \subseteq AT$, the fuzzy similarity relation is expressed as follows:

$$R_{A_i} = (\bigwedge_{a \in A_i} Sim_a^i(x_i, x_j) \geq \theta), (\forall x \in U)$$

where $[x]_{A_k}$ denotes the similarity class of object x under the similarity condition R_{A_i} .

The support characteristic function derived from the similarity classes is as follows [11]:

$$S_X^{A_i}(x) = \begin{cases} 1, & \text{if } [x]_{A_i} \subseteq X, \\ 0, & \text{otherwise,} \end{cases} \quad (i \leq 2^{|AT|})$$

where the support feature function $S_X^{A_i}(x)$ is used to describe the inclusion relationship between the similarity class and the concept X , indicating whether the object x precisely supports the concept X through the attribute A_i .

For any $x \in U, \beta \in [0, 1]$, multigranulation approximation operators of X under relation $\sum_{i=1}^q A_i$ are expressed as follows [11]:

$$\overline{GM}_{\sum_{i=1}^q A_i}(X) = \left\{ x \in U \mid \frac{\sum_{i=1}^q S_X^{A_i}(x)}{q} \geq \beta \right\},$$

$$\underline{GM}_{\sum_{i=1}^q A_i}(X) = \left\{ x \in U \mid \frac{\sum_{i=1}^q (1 - S_X^{A_i}(x))}{q} > 1 - \beta \right\}$$

3.2 Multi-source fuzzy information system

A multi-source fuzzy information system can be denoted as $MS - FIS = \{FIS_k | FIS_k = (U, AT, D, V_A, f_A),$

$k = 1, 2, \dots, q\}$, where U is a nonempty finite set of samples; AT is the nonempty finite set of condition attributes; V_A is the domain of $AT, f_A : U \times AT \rightarrow V_A$ is an information function. For $D_j \in U/D, D$ is the set of decision attributes, and $AT \cap D = \emptyset$. For $x \in U, a \in AT, a(x)$ represents the value of object x under the attribute a and $a(x) \in [0, 1]$.

Next, we introduce fundamental concepts related to multi-granulation fusion methods for multi-source data, covering two characteristic functions and two multi-granulation fusion operators. In the context of an MS-FIS, two characteristic functions are defined as follows. For every $D_j \in U/D$, the decision support and decision-related characteristic functions of an object x are [11]:

$$SV_{D_j}^{S_i}(x) = \begin{cases} 1, & \text{if } [x]_{S_i} \subseteq D_j \quad (i \leq q), \\ 0, & \text{otherwise} \end{cases}$$

$$RV_{D_j}^{S_i}(x) = \begin{cases} 1, & \text{if } [x]_{S_i} \cap D_j \neq \emptyset \quad (i \leq q) \\ 0, & \text{otherwise} \end{cases}$$

where $[x]_{S_i}$ represents the similarity class of object x under the information source i , and D_j denotes the j -th class in the division $U/D = \{D_1, D_2, \dots, D_s\}$.

For an MF-FIS, given any $D_j \in U/D, (\alpha, \beta)$ represents a pair of thresholds [11].

$$MSV(D_j)_\alpha = \left\{ x \in U \mid \frac{\sum_{i=1}^q SV_{D_j}^{S_i}(x)}{q} \geq \alpha \right\},$$

$$MRV(D_j)_\beta = \left\{ x \in U \mid \frac{\sum_{i=1}^q RV_{D_j}^{S_i}(x)}{q} > \beta \right\}$$

where $\alpha + \beta = 1$ and $0 < \alpha \leq 1$. The fixed aggregation operator $MSV(D_j)_\alpha$ provides more rigorous decision support, ensuring the fusion results' stability and consistency. Meanwhile, the possibility aggregation operator $MRV(D_j)_\beta$ offers flexibility in decision-making, allowing for more lenient criteria and accounting for the diversity of information sources.

4 Matrix-based fusion operator fusion mechanism

In this section, we explore in detail the multi-granularity information fusion operator within the MS-FIS framework. We introduce a static fusion approach for MS-FIS based on similarity relation matrices, demonstrate the matrix representation of the fusion operator, and outline the key concepts associated with multi-granularity fusion techniques. Furthermore, based on the static approach, in order to cope

with various scenarios involving changes in the number of objects, sources and attributes, we propose a dynamic fusion mechanism based on this fusion mechanism, aiming to improve the fusion efficiency in the face of such changes. Figure 3 illustrates the framework of this study, including the construction process of MS-FIS, the proposed matrix-based multi-particle fusion method and its various derived variants.

4.1 Matrix-based fusion of static information in multi-source fuzzy value system

This subsection focuses on the multi-granulation information fusion operator in the MS-FIS, where we present a matrix-based fusion strategy. This includes the matrix forms of two operators and several fundamental concepts related to multi-granulation fusion approaches. The fuzzy values of an object x_i under an attribute a_j can be represented as a fuzzy set A_j , with a membership function that indicates the degree of membership of object x_i under attribute a_j .

Definition 4.1 Let S be an integrated system. For any $x \in U$ ($i = 1, 2, \dots, n$) and $D_j \in U/D$, the decision vector $V(D_j) = [v_{ij}]_{n \times 1}$ for D_j is defined as follows:

$$v_{ij} = \begin{cases} 1, & \text{if } x_i \in D_j, \\ 0, & \text{otherwise} \end{cases}$$

Definition 4.2 Let $MS - FIS = \{FIS_k \mid FIS_k = (U, AT, D, V_A, f_A), k = 1, 2, \dots, q\}$ be a multi-source fuzzy information system. The fuzzy similarity relationship between two objects $x, y \in U$ under each source q can be represented as:

$$m_{ij} = \begin{cases} 1, & \text{if } Sim(x_i, x_j) \geq \theta \\ 0, & \text{otherwise} \end{cases}$$

where x_i and x_j are the fuzzy values of objects x and y , respectively, and $Sim(x_i, x_j)$ denotes the similarity measure between them. For each $a \in AT$, the distance and similarity between any two objects $x, y \in U$ are calculated as described in Section 3.1. Based on this definition, the similarity relationship matrix for each source can be expressed as $SM^k = [m_{ij}^k]_{n \times n}$, where n is the number of objects in the system. This matrix is symmetric, i.e., $m_{ij}^k = m_{ji}^k$, and the diagonal elements satisfy $m_{ii}^k = 1$ for all i . The threshold θ is chosen within the range $(0.5, 1]$, and in this paper, we set $\theta = 0.7$. To determine the reasonable value of the similarity threshold θ , this paper conducted parameter scanning experiments under the condition of fixed $\alpha = 0.7$ ($\theta \in [0.50, 0.90]$, step size 0.05). The results indicate that when $\theta = 0.7$, the fusion accuracy of most datasets reaches its optimal level and the stability is good. Therefore, $\theta = 0.7$ is adopted in subsequent experiments in this paper.

If the value obtained exceeds the set threshold θ , it is determined that there is a similarity relationship between

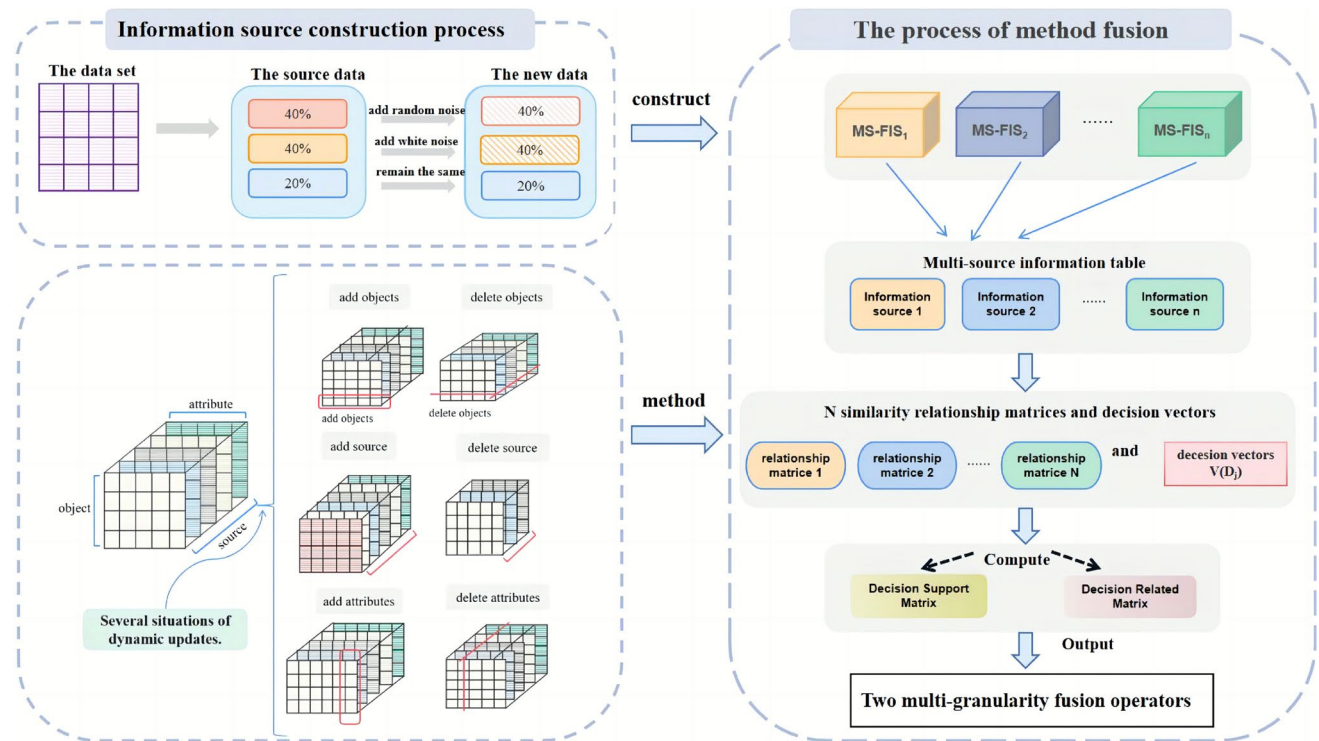


Fig. 3 The framework diagram of the work in this paper

the two objects. In this case, the corresponding value in the similarity matrix is set to 1. Otherwise, it is set to 0. Then, the similarity relationship between the objects in each source can be obtained, and the tolerance relationship matrix of the multi-source information system can be obtained. At the same time, the similarity class and the upper and lower approximations of the information system can be obtained.

Specifically, the similarity class $Sim_a(x_i)$ of object x_i under source S_k is represented by the i -th column $[m_i^k]_{1 \times n}$ or the i -th row $[m_i^k]_{n \times 1}$ of the similarity relationship matrix SM^k . A multi-granularity fusion operator based on multi-source data is then defined using the similarity relationship matrix SM^k and the decision vector $V(D_j)$. In this fusion approach, the operator is formed by the support feature functions from the decision support matrix $SV(D_j)$ and the matrix $RV(D_j)$, which is related to the decision-making process.

Algorithm 1 Static fusion algorithm in multi-source fuzzy information systems.

```

Input :
1.  $MS - FIS = (S_1, S_2, \dots, S_q)$ , where a fuzzy decision dataset,
    $FIS = (U, A, D), U/D = \{D_1, D_2, \dots, D_s\}$ ;
2. Threshold  $\alpha$  and  $\beta$ .

Output : Fixed aggregation vector  $MSV_\alpha$ ,
           possible aggregation vector  $MSR_\beta$ .

1 begin
2   for  $j = 1 : s$  do
3     for  $i = 1 : n$  do
4        $v_{ij} = 1$  if  $x_i \in D_j$  else 0;
5     end
6   end
7   for  $k = 1 : s$  do
8     #  $s$  is the number of information sources.
9     for each  $x_i, x_j$  in  $U$  do
10      Calculate the distance and similarity degree
11       $Sim(x_i, x_j)$ ;
12      Calculate the similarity relationship matrix
13       $SM^{k,t}$ ;
14    end
15     $SM^{k,t} = [m_{ij}^{k,t}]_{n \times n}$ 
16  end
17  for each  $SM^{k,t}$  do
18    Calculate the decision support matrix  $SV(D_j)^t$  and
19    decision related matrix  $RV(D_j)^t$ 
20    induced by  $SM^{k,t}$ ;
21    // According to definition 4.3.
22    Calculate fixed fusion operators  $MSV_\alpha$  and possible
23    fusion operators  $MSR_\beta$ .
24    // According to definition 4.5.
25  end
26 end

```

Definition 4.3 Given an $MS - FIS = \{FIS_k | FIS_k = (U, AT, D, V_A, f_A), k = 1, 2, \dots, q\}$ and $D_j \in U/D$, for any $a \in AT$, the decision vector $V(D_j)$ for D_j is represented as $V(D_j) = [v_{ij}]_{n \times 1}$. The similarity relation matrix of S_k , denoted as SM^k , is given by $SM^k = [m_{ij}^k]_{n \times n}$. The

decision support vector $SV^k(D_j) = [sv_{ij}^k]_{n \times 1}$ and the decision-related vector $RV^k(D_j) = [rv_{ij}^k]_{n \times 1}$ for x in D_j are defined as follows:

$$SV_{ij}^k = \begin{cases} 1, & \text{if } [m_i^k]_{n \times 1} \vee V(D_j) = V(D_j) \\ 0, & \text{otherwise} \end{cases},$$

$$RV_{ij}^k = \begin{cases} 1, & \text{if } [m_i^k]_{n \times 1} \wedge V(D_j) \neq 0 \\ 0, & \text{otherwise} \end{cases}$$

where " \vee " and " \wedge " represent the maximum and minimum values of the corresponding elements in the two vectors, respectively.

Definition 4.4 Given an $MS - FIS = \{FIS_k | FIS_k = (U, AT, D, V_k, f_k), k = 1, 2, \dots, q\}$ and $D_j \in U/D$, $SV^k(D_j)$ and $RV^k(D_j)$ represent the decision support and decision-related vectors for S_k . The decision support and decision-related matrices in the multi-source information system are defined as $SV(D_j) = [sv^1(D_j), sv^2(D_j), \dots, sv^q(D_j)]_{n \times q}$ and $RV(D_j) = [rv^1(D_j), rv^2(D_j), \dots, rv^q(D_j)]_{n \times q}$, respectively.

Definition 4.5 Given an $MS - FIS = \{FIS_k | FIS_k = (U, AT, D, V_A, f_A), k = 1, 2, \dots, q\}$, (α, β) is a pair of thresholds. For any $D_j \in U/D$, the definitions for fixed aggregation vector $MSV(D_j)_\alpha = [MSV(D_j)_\alpha^i]_{n \times 1}$ and possible aggregation vector $MRV(D_j)_\alpha = [MRV(D_j)_\alpha^i]_{n \times 1}$ of D_j are as follows:

$$MSV(D_j)_\alpha^i = \begin{cases} 1, & \text{if } \frac{|SV(D_j)(x_i)|}{q} \geq \alpha \\ 0, & \text{otherwise} \end{cases},$$

$$MRV(D_j)_\beta^i = \begin{cases} 1, & \text{if } \frac{|RV(D_j)(x_i)|}{q} \geq \beta \\ 0, & \text{otherwise} \end{cases}$$

where $|SV(D_j)(x_i)| = \sum_{k=1}^q sv_{ij}^k$, $|RV(D_j)(x_i)| = \sum_{k=1}^q rv_{ij}^k$, and $\alpha + \beta = 1, 0 < \alpha \leq 1$.

The fixed aggregation operator reflects the feedback from the majority of information sources, where x_i supports decision D_j in a fixed manner. The possible aggregation operator, on the other hand, accounts for feedback from multiple sources, allowing x_i to potentially support decision D_j . To better illustrate the static multi-granulation fusion strategy in multi-source data contexts, additional examples are provided to explain the detailed calculation steps. Furthermore, Algorithm 1 outlines the corresponding static multi-granulation fusion algorithm, detailing the steps involved in utilizing multi-granularity methods for extracting information from multi-source data. The time complexity of Algorithm

1 is $O(U^2 \times A \times S)$, where the main computational time arises from the calculation of the similarity relationship matrix.

4.2 Matrix-based multi-granularity multi-source fuzzy fusion for dynamic data

This subsection focuses on the dynamic fusion technology of multi-source fuzzy information system based on matrices, and introduces the corresponding update mechanisms when the objects, sources, and attributes change.

4.2.1 Dynamic multi-granularity multi-source fuzzy fusion with objects variation

This subsection mainly explains the situation when objects change in a multi-source fuzzy information system, mainly referring to the increase and decrease of objects. When objects increase or decrease, the dimension of the similarity relationship matrix corresponding to each information source will change, and the corresponding decision vector will also change. Therefore, it is necessary to update the relationship matrix and decision vector according to the specific changes.

Proposition 4.1 *Let $MS - FIS^t$ and $MS - FIS^{t+1}$ are multi-source fuzzy decision information systems before and after adding objects, and $U_t = \{x_i | i = 1, 2, \dots, n\}$, $U_{t+1} = \{x_i | i = 1, 2, \dots, n + \Delta n\}$ represent the number of objects before and after the change, respectively. The similarity relationship matrix after adding objects includes $M^{k,t} = [m^{k,t}]_{n \times n}$, $M^{k,t+1}_{(U, \Delta U)} = [m^{k,t+1}_{(U, \Delta U)}]_{n \times \Delta n}$, $M^{k,t+1}_{(\Delta U, U)} = [m^{k,t+1}_{(\Delta U, U)}]_{\Delta n \times n}$ and $M^{k,t+1}_{(\Delta U, \Delta U)} = [m^{k,t+1}_{(\Delta U, \Delta U)}]_{\Delta n \times \Delta n}$, defined as follows:*

$$SM^{k,t+1} = \begin{bmatrix} M^{k,t} & M^{k,t+1}_{(U, \Delta U)} \\ M^{k,t+1}_{(\Delta U, U)} & M^{k,t+1}_{(\Delta U, \Delta U)} \end{bmatrix}$$

When an existing object x_p is deleted from the system, the updated object set becomes $U_{t+1} = U_t - \{x_p\}$. The corresponding similarity matrix can be obtained by deleting the p -th row and the p -th column of the original matrix $M^{k,t}$, resulting in a new $(n - 1) \times (n - 1)$ similarity matrix:

$$M^{k,t+1} = M^{k,t} - \{m^{k,t}_{p*}, m^{k,t}_{*p}\},$$

where $m^{k,t}_{p*}$ and $m^{k,t}_{*p}$ represent the p -th row and column associated with object x_p . This deletion operation effectively adjusts the dimensionality of the similarity relationship matrix without recomputing the entire matrix, thereby maintaining the structural consistency of the fusion model. Overall, the dynamic updating process for

object variation can be summarized as follows. When new objects are added, the similarity relationship matrix is expanded by appending the cross-similarity submatrices between the new and existing objects. When objects are deleted, the corresponding rows and columns in the matrix are removed to maintain dimensional consistency. This mechanism allows the proposed dynamic multi-source fusion model to efficiently adapt to incremental or decremental changes in object sets while ensuring the stability of the decision results.

Proof The above proposition can be easily proved from Definition 4.2. Definition 4.2 describes the similarity relationship matrix obtained between objects in an information system. When objects are added and subtracted, only the similarity relationship matrix for the changed part of the object needs to be updated. \square

Algorithm 2 Dynamic information fusion for object changes.

```

Input :
1.  $MS - FIS = (S_1, S_2, \dots, S_q)$ , fuzzy decision dataset
    $FIS = (U, AT, D)$ ,  $U/D = \{D_1, D_2, \dots, D_s\}$ .
2. Thresholds  $\alpha$  and  $\beta$ .
3. Similarity relationship matrix  $SM^t$  from Algorithm 1.
4. Object change  $\Delta X$ : positive for addition, negative for removal.

Output: Fixed aggregation vector  $MSV_\alpha$ , possible aggregation
vector  $MSR_\beta$ 

1 begin
2   Initialize similarity matrix  $SM^t$ ;
3   if  $\Delta X > 0$  then
4     for  $k = x + 1 : x + \Delta X$  do
5       Compute similarity for new objects with existing
        ones;
6       Update global matrix  $SM^{t+1}$  based on
        proposition 4.1;
7     end
8   end
9   else if  $\Delta X < 0$  then
10    for  $k = 1 : x + \Delta X$  do
11      Delete the rows and columns corresponding to
         $\Delta X$  in the similarity matrix;
12      Update  $SM^t$  based on proposition 4.1.
13    end
14  end
15  for each  $SM^{t+1}$  do
16    Compute decision support matrix  $SV(D_j)^{t+1}$  and
        related matrix  $RV(D_j)^{t+1}$ ;
17    Compute  $MSV_\alpha$  and  $MSR_\beta$  from  $SV(D_j)^{t+1}$  and
         $RV(D_j)^{t+1}$ ;
18    //According to definition 4.3. and 4.5.
19  end
20  return  $MSV_\alpha, MSR_\beta$ ;
21 end

```

Proposition 4.2 *Let $MS - FIS^t$ and $MS - FIS^{t+1}$ be two multi-source fuzzy decision information systems, for any $D_j \in U/D$, the decision vector are defined as $V(D_j)^t = [v_{ij}^t]_{n \times 1}$ and $V(D_j)_{\Delta U}^{t+1} = [v_{ij}^{t+1}]_{\Delta n \times 1}$:*

According to the above definition, the decision vector after adding objects is as follows:

$$V(D_j)^{t+1} = \begin{bmatrix} V(D_j)^t \\ V(D_j)_{\Delta U}^{t+1} \end{bmatrix}$$

$V(D_j)^t$ and $V(D_j)^{t-1}$ represent the decision vector in the original information system and the decision vector after deleting the object, respectively. According to the above definition, the decision vector after deleting objects is as follows:

$$V(D_j)^t = \begin{cases} \begin{bmatrix} V(D_j)^{t-1} \\ 0 \end{bmatrix}_{n \times 1}, & \text{if } x_i \notin D_j, \\ \begin{bmatrix} V(D_j)^{t-1} \\ 1 \end{bmatrix}_{n \times 1}, & \text{if } x_i \in D_j. \end{cases}$$

where $V(D_j)^{t-1}$ is the decision vector before deleting the object, and 0 or 1 represents whether the object is in the decision set D_j , 0 represents not, and 1 represents present.

Proof The above proposition can be easily proved from Definition 4.1. In light of Definition 4.1, it is readily observable that the decision vector can be split into two subsections. Each subsection can be directly acquired in accordance with Definition 4.1.

By performing deletion operations on the original similarity relationship matrix, reducing the number of rows and columns of the object, we can obtain the similarity relationship matrix after deleting the object, and obtain the decision vector in a corresponding manner. Based on this, the equivalence relationship matrix and decision vector are updated. $MSV_{\alpha}^t(D_j)$ and $MRV_{\beta}^t(D_j)$ represent the fixed aggregation operator and possible aggregation operator before adding an object, respectively. $MSV_{\alpha}^{t+1}(D_j)$ and $MRV_{\beta}^{t+1}(D_j)$ represent the fixed aggregation operator and the possible aggregation operator after changing objects, respectively. After the similarity relationship matrix and the decision vector have been updated, the fusion operator is calculated according to the definition used earlier. Algorithm 2 is a matrix-based multi-granularity fusion method when adding or removing objects from a multi-source fuzzy information system. If Algorithm 1's static update is used, the time complexity is $O(|U + \Delta X|^2 \times A \times S)$, resulting in a total time complexity of $O(|U|^2 \times |\Delta U|^2 \times A \times S)$ for Algorithm 2. \square

4.2.2 Dynamic multi-granularity multi-source fuzzy fusion with sources variation

The key transformation in the information source lies in the change in its quantity. When the number of information sources increases, more information is provided about the same object,

introducing new content and altering the overall representation of the object. Conversely, a decrease in the number of information sources reduces the available information, leading to changes and discrepancies in the object as represented by those sources. To address the variability in information source changes, it is necessary to develop methods that can effectively adapt to the fluctuating availability of information, ensuring that the object's representation remains as accurate and consistent as possible despite the changes in source quantity. This may involve utilizing dynamic fusion techniques, which can combine data from multiple sources in a way that accounts for their variability, ensuring robust and reliable results even in the face of incomplete or changing information.

Proposition 4.3 After adding the information sources $\{S_{q+1}, S_{q+2}, \dots, S_{q+p}\}$, $MSV(D_j)_{q+p}^{\alpha} = [msv_1^{\alpha}, msv_2^{\alpha}, \dots, msv_n^{\alpha}]^T$ and $MRV(D_j)_{q+p}^{\beta} = [mrv_1^{\beta}, mrv_2^{\beta}, \dots, mrv_n^{\beta}]^T$ represents the updated vector. $SV_k(D_j) = [SV_k^1, SV_k^2, \dots, SV_k^m]^T$ and $RV_m(D_j) = [RV_m^1, RV_m^2, \dots, RV_m^n]^T$ ($m = 1, 2, \dots, p$) are decision support and decision related vectors for added information sources S_m . $SV_{adding}(D_j) = [SV_1, SV_2, \dots, SV_p]^T$ and $RV_{adding}(D_j) = [RV_1, RV_2, \dots, RV_p]^T$ represent decision support and decision matrix after deleting information sources. Let U/D denote the partition of the universal set U induced by the set of properties D , the fixed aggregation operator and the possible aggregation operator satisfy the following:

$$MSV^i(D_j)_{\alpha} = \begin{cases} 1, & \text{if } \frac{|SV(D_j)(x_i)| + |SV(x_i)_{adding}|}{q+p} \geq \alpha, \\ 0, & \text{otherwise} \end{cases},$$

$$MRV^i(D_j)_{\beta} = \begin{cases} 1, & \text{if } \frac{|RV(D_j)(x_i)| + |RV(x_i)_{adding}|}{q+p} > \beta, \\ 0, & \text{otherwise} \end{cases}$$

Proof It can be easily obtained from Definition 4.5 by simply calculating the similarity relationship matrix for the newly added information sources. \square

Proposition 4.4 After deleting the information sources $\{S_1, S_2, \dots, S_p\}$, $MSV(D_j)_{q-p}^{\alpha} = [msv_1^{\alpha}, msv_2^{\alpha}, \dots, msv_n^{\alpha}]^T$ and $MRV(D_j)_{q-p}^{\beta} = [mrv_1^{\beta}, mrv_2^{\beta}, \dots, mrv_n^{\beta}]^T$ represents the updated vector. $SV_k(D_j) = [SV_k^1, SV_k^2, \dots, SV_k^m]^T$ and $RV_m(D_j) = [RV_m^1, RV_m^2, \dots, RV_m^n]^T$, ($m = 1, 2, \dots, p - q$) are decision support and decision related vectors for deleted information sources S_m . $SV_{deleting}(D_j) = [SV_1, SV_2, \dots, SV_p]^T$ and $RV_{deleting}(D_j) = [RV_1, RV_2, \dots, RV_p]^T$ represent decision support and decision matrix after deleting information sources. Let U/D denote the partition of the universal set U induced by the set of properties D , the fixed aggregation operator and the possible aggregation operator satisfy the following:

$$MSV^i(D_j)_\alpha = \begin{cases} 1, & \text{if } \frac{|SV(D_j)(x_i)| - |SV_{deleting}(x_i)|}{q-p} \geq \alpha \\ 0, & \text{otherwise} \end{cases},$$

$$MRV^i(D_j)_\beta = \begin{cases} 1, & \text{if } \frac{|RV(D_j)(x_i)| - |RV_{deleting}(x_i)|}{q-p} > \beta \\ 0, & \text{otherwise} \end{cases}$$

Proof It can be easily obtained from Definition 4.5 by simply calculating the similarity relationship matrix for the newly deleted information sources. \square

Algorithm 3 is a matrix-based multi-granularity fusion method when adding or removing sources from a multi-source fuzzy information system. If the static update method of algorithm 1 is used, the time complexity is $O(U^2 \times A \times (S + \Delta S))$, resulting in a total time complexity of $O(U^2 \times A \times \Delta S)$ for Algorithm 3.

Algorithm 3 Dynamic information fusion for changes in information sources.

Input :

- $MS - FIS = (S_1, S_2, \dots, S_q)$, fuzzy decision dataset
 $FIS = (U, AT, D), U/D = \{D_1, D_2, \dots, D_s\}$.
- Thresholds α and β .
- Similarity relationship matrix SM^t from Algorithm 1.
- Information source change ΔS : positive for addition, negative for removal.

Output: Fixed aggregation vector MSV_α , possible aggregation vector MSR_β

```

1 begin
2   if  $\Delta S > 0$  then
3     for  $k = s + 1 : s + \Delta S$  do
4       Compute similarity matrix  $SM^{k,t}$  for new source  $S_k$ ;
5       Update global matrix  $SM^{t+1}$ ;
6     end
7   end
8   else if  $\Delta S < 0$  then
9     for  $k = 1 : S + \Delta S$  do
10      Delete the similarity relation matrix corresponding to source  $\Delta S$ ;
11      Update global matrix  $SM^{t+1}$ ;
12    end
13  end
14  for each  $SM^{t+1}$  do
15    Compute decision support matrix  $SV(D_j)^{t+1}$  and decision related matrix  $RV(D_j)^{t+1}$ ;
16    Compute  $MSV_\alpha$  and  $MSR_\beta$  based on  $SV(D_j)^{t+1}$  and  $RV(D_j)^{t+1}$ ;
17    //According to definition 4.3. and 4.5.
18  end
19  return  $MSV_\alpha, MSR_\beta$ ;
20 end
```

4.2.3 Dynamic multi-granularity multi-source fuzzy fusion with attributes variation

In a multi-source information system, attributes are an important foundation for representing the characteristics

and relationships of objects. As the system scale continues to grow and the data dimension keeps increasing, how to ensure the stability and decision-making ability of the system under the condition of changing attributes has become a key problem that needs to be solved urgently. This subsection will focus on how to effectively implement information fusion methods using fusion operators when the number of attributes changes.

Let $MS - FIS^t = \{FIS_i^t | FIS_i^t = (U, AT_t, D, F_t^i), i = 1, 2, \dots, q\}$ represent the multi-source fuzzy decision information systems at time t , and let $MS - FIS^{t+1} = \{FIS_i^{t+1} | FIS_i^{t+1} = (U, AT_{t+1}, D, F_{t+1}^i), i = 1, 2, \dots, q\}$ represent the corresponding systems at time $t + 1$, where $A^{t+1} = A^t \cup \Delta A$ for the case of adding attributes, and $A^{t+1} = A^t - \Delta A$ for the case of removing attributes. Here, $MSV_\alpha^t(D_j)$ and $MRV_\beta^t(D_j)$ denote the fixed and possible aggregation operators before adding attributes, respectively. $MSV_\alpha^{t+1}(D_j)$ and $MRV_\beta^{t+1}(D_j)$ represent the fixed and possible aggregation operators after adding attributes, respectively.

Algorithm 4 Dynamic information fusion for changes in attributes.

Input :

- $MS - FIS = (S_1, S_2, \dots, S_q)$, fuzzy decision dataset
 $FIS = (U, AT, D), U/D = \{D_1, D_2, \dots, D_s\}$.
- Thresholds α and β .
- Similarity relationship matrix SM^t from Algorithm 1.
- Attribute change ΔA .

Output: Fixed aggregation vector MSV_α , possible aggregation vector MSR_β .

```

1 begin
2   for each attribute change  $\Delta A$  do
3     Compute similarity matrix for the changed attribute(s):  $SM^{\Delta A}$ ;
4     for each object pair  $(x_i, x_j) \in U$  do
5       if  $SM^t(x_i, x_j) = 0$  then
6          $SM^{t+1}(x_i, x_j) \leftarrow 0$ ;
7       end
8       else if  $SM^t(x_i, x_j) = 1$  and  $SM^{\Delta A}(x_i, x_j) = 1$  then
9          $SM^{t+1}(x_i, x_j) \leftarrow 1$ ;
10      end
11      else
12        Recompute similarity for  $A + \Delta A$ :
13         $SM^{A+\Delta A}(x_i, x_j)$ ;
14         $SM^{t+1}(x_i, x_j) \leftarrow SM^{A+\Delta A}(x_i, x_j)$ ;
15      end
16    end
17  end
18  // According to proposition 4.5.
19  Compute decision support matrix  $SV(D_j)^{t+1}$  and related matrix  $RV(D_j)^{t+1}$  based on  $SM^{t+1}$ ;
20  Compute  $MSV_\alpha$  and  $MSR_\beta$  from  $SV(D_j)^{t+1}$  and  $RV(D_j)^{t+1}$ .
21  //According to definition 4.3. and 4.5.
22  return  $MSV_\alpha, MSR_\beta$ 
```

Proposition 4.5 Let $SM^{A,t+1} = [m^{A,t+1}]_{n \times n}$ denote the similarity relationship matrix of FIS, when attributes change, the similarity matrix is updated according to the following method.

$$m^{A,t+1} = \begin{cases} m^{A,t+1} = m^{A,t}, & \text{if } m^{A,t} = 0; \\ m^{A,t+1} = m^{A,t}, & \text{if } m^{A,t} = 1 \text{ and } m^{\Delta A,t+1} = 1; \\ m^{A \cup \Delta A,t+1}, & \text{if } m^{A,t} = 1 \text{ and } m^{\Delta A,t+1} = 0; \end{cases}$$

where ΔA represents the change in the number of attributes, which includes the addition and removal of attributes. $m^{\Delta A,t+1}$ represents the similarity relationship matrix computed for the changed attributes (whether added or removed parts), $m^{A \cup \Delta A,t+1}$ represents the similarity relationship matrix recalculated after changing the number of attributes, $m^{A,t}$ represents the original similarity relationship matrix. Meanwhile, changes in the number of attributes will not cause any change in the decision vector $V(D_j)$. After updating the similarity matrix, the fusion operator is calculated according to Definition 4.5.

Proof If $m^{A,t} = 0$, according to Definition 4.2, it is shown that in the original system there is no similarity relationship between objects and that adding or removing individual attributes does not change the similarity relationship. If $m^{A,t} = 1$ and $m^{\Delta A,t+1} = 1$, which follows $x_i \in Sim_a(x_i)$ indicating that the similarity remains unchanged after the attribute change. If $m^{A,t} = 1$ and $m^{\Delta A,t+1} = 0$, it follows that $x_i \notin Sim_a(x_i)$, suggesting that the change in attributes impacts the original similarity relationship. \square

Algorithm 4 is a matrix-based multi-granularity fusion method when adding or removing attribute sources from a multi-source fuzzy information system. If the static update method of Algorithm 1 is used, the time complexity is $O(U^2 \times S \times (A + \Delta A))$, while the time complexity of Algorithm 4 is $O(U^2 \times S \times (\Delta A + 1))$.

4.3 Examples and algorithm analysis

In real-world medical systems, patient information is often distributed across multiple hospitals, where each source provides distinct data types such as laboratory test results, medical imaging, and clinical records. These heterogeneous data sources are continuously updated as new examinations or hospital visits occur. Using the proposed dynamic multi-source fuzzy fusion framework, each hospital's dataset can be treated as an independent information source. When new diagnostic data are added (e.g., a new CT scan) or outdated records are removed, the similarity relationship matrix is dynamically updated without recalculating all pairwise similarities. This enables efficient, real-time integration of

evolving medical data and produces a unified and up-to-date patient assessment to assist clinical decision-making.

To further illustrate the multi-source information fusion process based on the similarity relationship matrix, we present an example from the healthcare domain. Suppose we have multiple medical information sources providing patient data, including basic indicators such as weight, height, and age, as well as disease severity evaluations categorized into mild, moderate, and severe levels. Different hospitals may provide varying and sometimes fuzzy assessments of the same patient's condition. For instance, one hospital may classify a patient as having a mild condition, while another may consider it severe. To accurately integrate these uncertain and heterogeneous assessments and provide unified diagnostic support, the proposed MS-FIS can be employed. By constructing and dynamically updating the similarity relationship matrices for each information source, the framework effectively fuses medical opinions from multiple hospitals, achieving a comprehensive and consistent diagnostic evaluation for each patient.

Example 4.1 This example illustrates the computational flow of the initial multi-source fuzzy information system (MS-FIS) fusion based on Algorithm 1, followed by dynamic updating mechanisms for changes in objects, information sources, and attributes. Specifically, case 1, case 2, and case 3 highlight the core computational steps for each scenario. Figure 4 shows a multi-source fuzzy information system, where each row represents a patient and each column represents fuzzy evaluations of attributes (e.g., weight, height, age) from different hospitals. The decision attribute d is used to determine the final disease diagnosis. Here, U is the object set, A is the set of condition attributes, and S is the initial set of information sources. Algorithm 1 describes the process of initial multi-source information system fusion.

Figure 4 shows the process of obtaining similarity relationship matrices and decision vectors from the initial MS-FIS, followed by the calculation of the decision support matrix and decision dependency matrix, ultimately leading to the calculation flow of the two fusion operators.

Case 1 Figure 5 explores how to handle the situation where objects (such as patient data) are added or deleted in an information system. In practical applications, new patient information may be added over time, or certain patient data may be deleted. To ensure the accuracy of the information fusion process, the similarity relationship matrix must be dynamically updated and the fusion result must be recalculated. When the number of objects changes, the primary time complexity arises from calculating the similarity and updating the matrix.

Case 2 Figure 6 explores how to handle the increase and decrease of information sources in practical

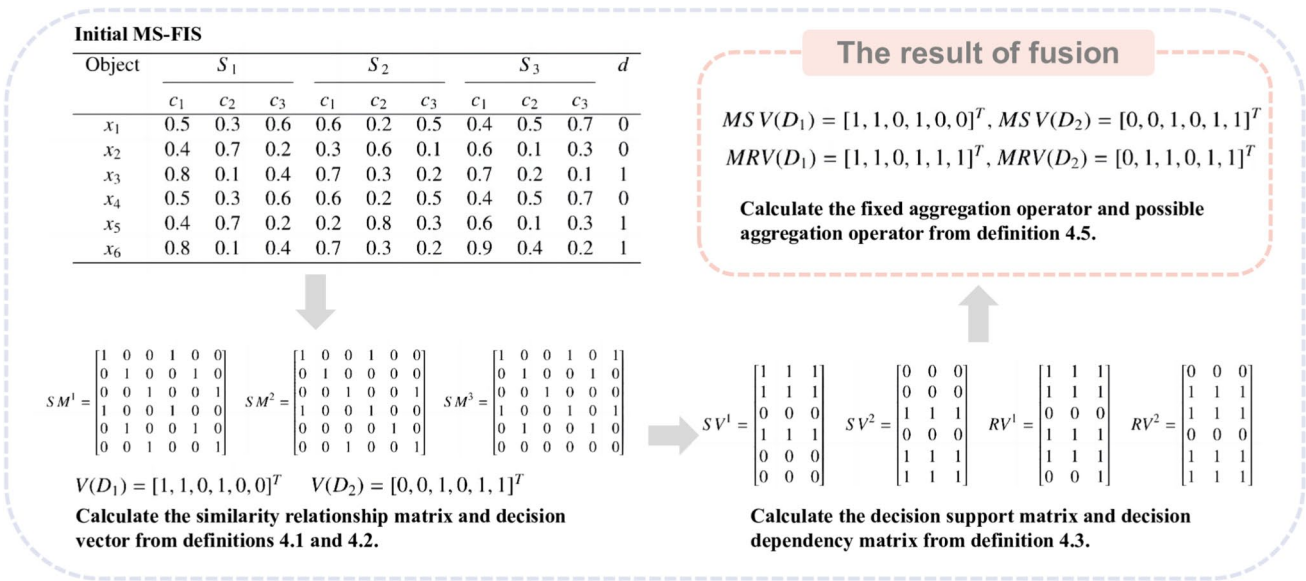


Fig. 4 Multi-granulation fusion operator solution flow chart (where $\alpha = 0.7$ and $\beta = 0.3$)

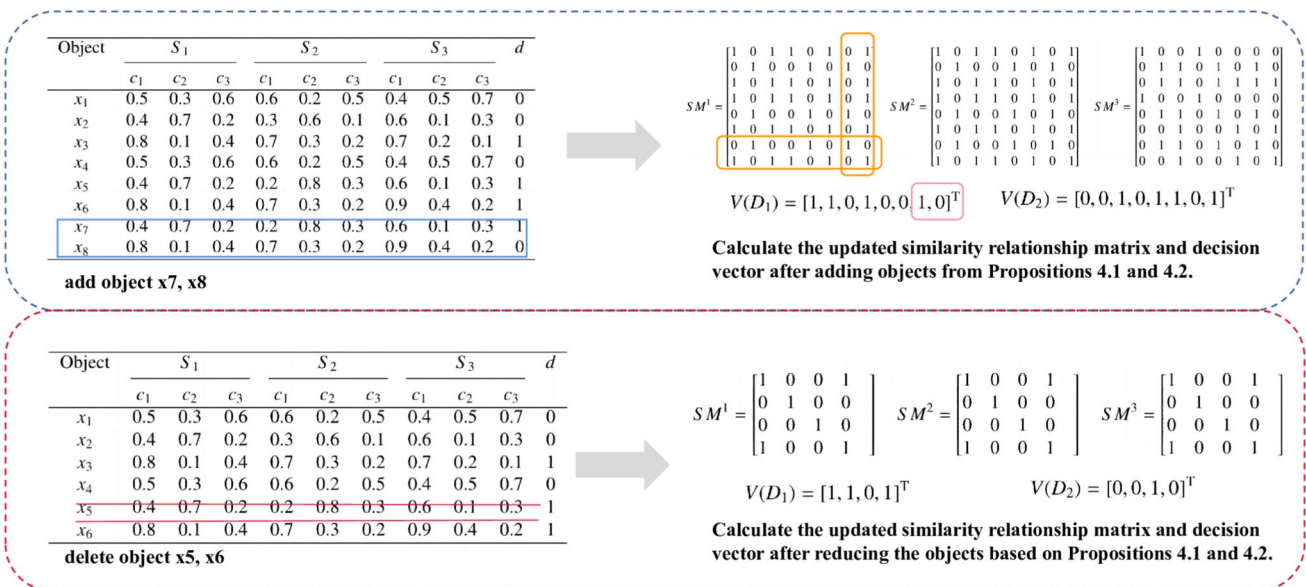


Fig. 5 Calculation flowchart for adding and deleting objects (where $\alpha = 0.7$ and $\beta = 0.3$)

applications, where medical diagnostic institutions may be added or removed continuously. In this case, we need to update the number of similar relationship matrices to recalculate the decision support matrix and fusion operator. Therefore, the core of information source changes lies in the update of the number of similar relationship matrices, which requires the calculation of the relationship matrix of the new information source s_4 and the reduction of the relationship matrix of the information source s_3 . The core calculation steps are illustrated by taking the example of

adding information source s_4 and reducing information source s_3 . When the number of sources changes, the main time complexity comes from calculating the similarity relationship matrix for the new sources.

Case 3 Figure 7 explores how to handle the increase and decrease of attributes, where in practical applications, the number of indicators that need to be detected may increase or decrease. In this case, we need to update the similarity relationship matrix without recalculating it. When the number of attributes changes, the main time complexity comes from updating the similarity relationship matrix.

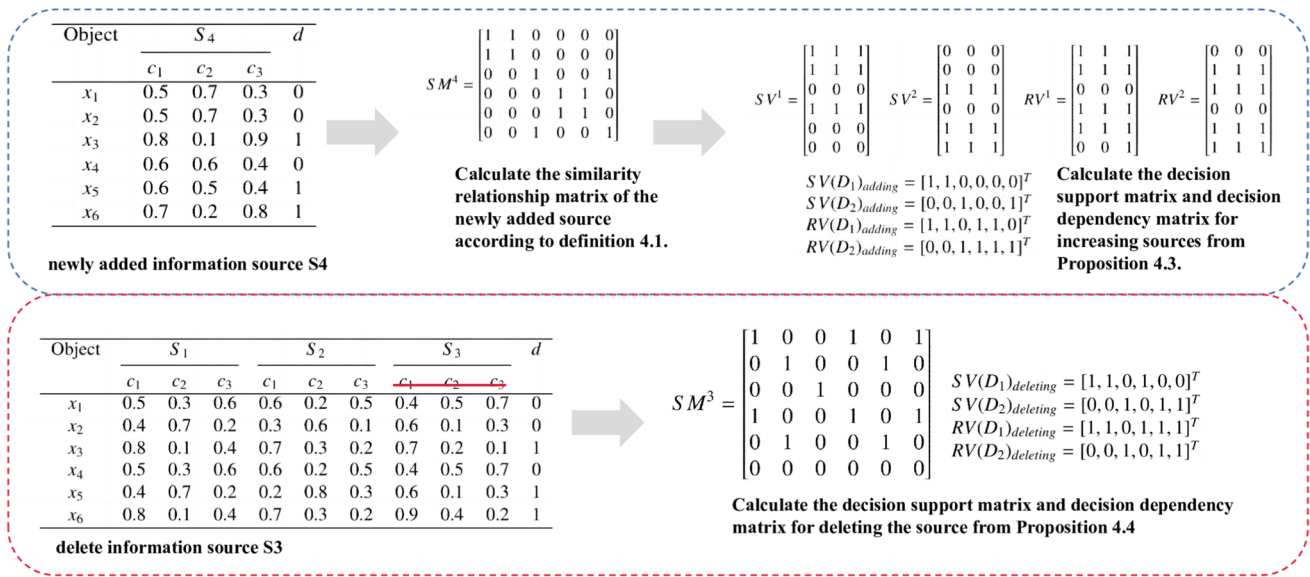


Fig. 6 Calculation flowchart for adding and deleting sources (where $\alpha = 0.7$ and $\beta = 0.3$)

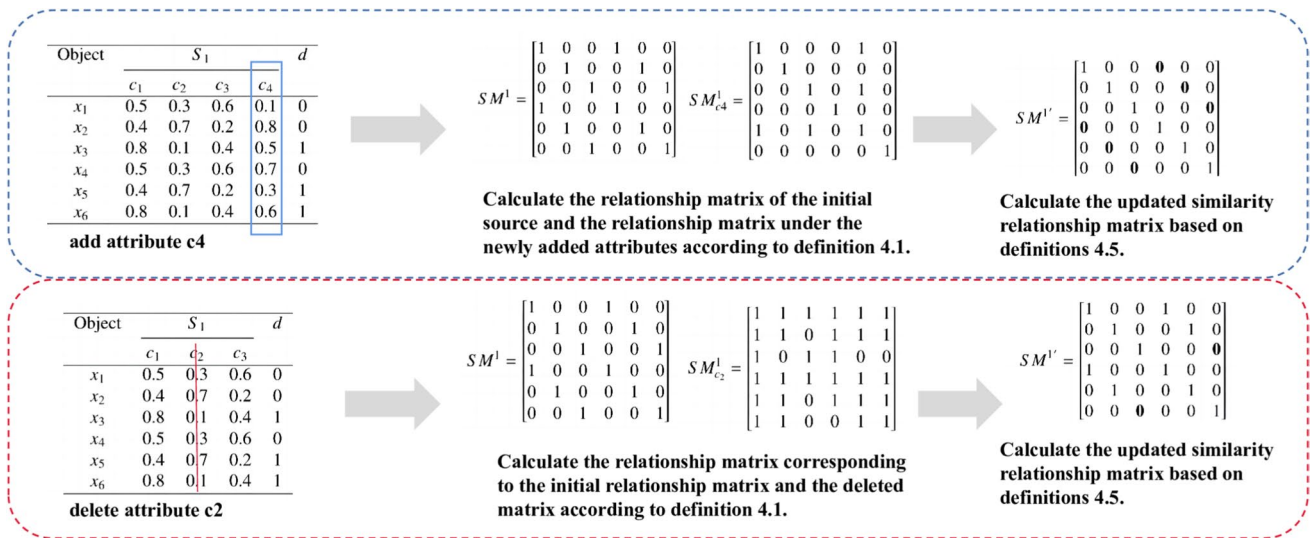


Fig. 7 Calculation flowchart for adding and deleting attributes (where $\alpha = 0.7$ and $\beta = 0.3$)

Through the three examples presented above, we demonstrated both the static fusion process and the dynamic fusion mechanism of the matrix-based multi-granularity fusion method.

5 Experimental design and analysis

To verify the effectiveness of the proposed fusion method, comparative experiments were conducted on 12 datasets obtained from the UCI database (<https://archive.ics.uci.edu/ml/index.php>). Table 2 provides detailed information about the dataset, including the dataset name, sample size (i.e. number of objects), number of attributes in the dataset, and total number

of categories of data in the dataset (which can be divided into several categories). The computer configuration environment used was an Intel Core i5-12500H 2.50 GHz of the 12th generation, and the operating system was Windows x 64.

5.1 Methods of data preprocessing

Before starting the experiment, we used the maximum minimum scaling normalization technique to pre fuzzify the data, rescaling it to the range of 0 to 1. The program is as follows:

$$a(x_j) = \frac{a(x_j) - \min(a(x))}{\max(a(x)) - \min(a(x))}$$

Table 2 Dataset description

No.	Datasets	Abbreviation	Samples	Attributes	Decision classes
1	Wine	Wine	178	14	3
2	Yacht Hydrodynamics	YH	308	6	3
3	Blood Transfusion Service Center	BT	748	4	2
4	Concrete	Concrete	1029	8	10
5	Contraceptive Method Choice	CMC	1473	9	3
6	Wireless Indoor Localization	WIL	2000	7	4
7	Customer Churn	CC	3150	13	2
8	Abalone	Abalone	4177	7	3
9	Wilt	Wilt	4839	5	2
10	Shill Bidding	Shill	6321	13	2
11	Combined Cycle Power Plant	CP	9569	4	10
12	Dry Bean	DB	13611	16	7

where $\min(a(x))$ and $\max(a(x))$ represent the maximum and minimum values of attribute a , respectively.

Next, due to the difficulty of directly obtaining multi-source fuzzy information system (MS-FIS) from the machine learning database, we generated experimental data by adding noise. In the process of data set processing, 40% of the data set was injected with white noise first; then, 20% of the data set was injected with random noise; the remaining 40% of the data set was kept unchanged to ensure the accuracy of the data [11].

By generate q random numbers (n_1, n_2, \dots, n_q) that follow a normal distribution, and then add them to the original data. The specific steps are as follows:

$$FIS_i(x, a) = \begin{cases} FIS(x, a) + n_i, & \text{if } 0 \leq n_i \leq 1, \\ FIS(x, a), & \text{otherwise} \end{cases}$$

Then, using the same method as white noise, random noise is generated and added to the data, as follows:

$$FIS_i(x, a) = \begin{cases} FIS(x, a) + r_i, & \text{if } 0 \leq r_i \leq 1, \\ FIS(x, a), & \text{otherwise} \end{cases}$$

Finally, in order to further explore the impact of noise on information systems and verify the effectiveness of subsequent processing algorithms, we created ten independent information sources based on the above strategy on the

original dataset. In the initial information system architecture, the value of object x under attribute a is marked as (x, a) . This method introduces noise to simulate the uncertainty and diversity in the data set, thereby enhancing the robustness of subsequent processing algorithms.

5.2 Analysis of fusion efficiency of the model

In this subsection, we present a series of experiments designed to evaluate the computational performance of the proposed matrix dynamic algorithm in comparison to static methods. Our study uses fuzzy datasets, with numerical data being preprocessed using the maximum-minimum scaling normalization technique. This ensures that the data is rescaled to fall within the range of 0 to 1, thereby constructing a multi-source fuzzy information system. To assess the effectiveness of the proposed algorithm, we compare the running time of the dynamic algorithm against the static update mechanism. The experiments are based on 12 datasets, as outlined in Table 3. First, we describe Algorithm 1, which outlines a static algorithm for calculating multi-granularity fusion operators based on the similarity relationship matrix. This algorithm provides a detailed explanation of the calculation process, starting with the generation of the initial similarity relationship matrix. The subsequent incremental algorithm updates the matrix based on this initial framework. We then analyze the time performance by comparing the running times of both algorithms.

In this study, for experiments involving the addition and deletion of objects and attribute changes, we constructed a system with 10 information sources. For the experiments involving adding and deleting information sources, we constructed a system containing 20 information sources. In addition, our experiments were based on fuzzy contexts on fuzzy datasets generated by preprocessing and the experiments were analyzed using $\alpha = 0, 7, \beta = 0.3$. For the experiment on changes in the number of objects (Algorithm 2), we select 50% of the data as the base set and gradually introduce the remaining objects over five iterations, adding 10% of the objects at each step. In the object reduction experiment, we begin with the full dataset as the base set and conduct five iterations of object removal, each time randomly removing 10% of the objects. Tables 3 and 4 indicate the running time of the static and dynamic algorithms when different proportions of objects are added and subtracted, respectively.

For the experiment on changes in the number of information sources (Algorithm 3), we create a dataset containing 20 information sources. In the source increment experiment, we use the first 10 information sources as the base set to construct the original multi-source information system. The remaining 10 sources are then gradually incorporated into

Table 3 Running time of static and dynamic methods when increasing objects of different proportions

Data sets	10%		20%		30%		40%		50%	
	Static	Dynamic	Static	Dynamic	Static	Dynamic	Static	Dynamic	Static	Dynamic
Yach Hydrodynamics	0.44	0.13	0.65	0.52	1.92	0.95	2.30	1.69	3.73	2.52
Wine	0.40	0.14	0.56	0.24	0.86	0.39	1.20	0.55	1.56	0.69
Wireless Indoor Localization	6.74	4.18	12.89	5.73	18.80	9.24	25.40	13.60	31.55	15.94
Wilt	15.69	3.14	36.86	10.41	51.50	20.88	144.45	28.73	242.03	53.27
Shill Bidding	27.36	4.61	55.73	8.11	168.33	18.67	286.86	56.73	357.58	104.70
Dry Bean	892.21	166.30	1102.54	259.68	1423.76	317.21	1701.02	389.43	2458.03	458.08
Concrete	2.36	1.44	4.44	2.44	6.58	3.14	9.08	3.94	10.88	6.33
Contraceptive Method Choice	3.12	1.84	5.56	2.93	8.06	3.53	11.39	5.05	15.99	7.73
Customer Churn	6.88	2.27	16.67	6.47	26.55	12.38	41.05	18.48	62.03	26.47
Combined Cycle Power Plant	757.81	110.55	925.31	185.06	1122.34	224.47	1376.35	275.89	1720.10	354.89
Blood Transfusion Service Center	1.34	0.67	2.44	1.21	3.12	1.81	4.34	2.29	5.59	3.37
Abalone	101.98	67.97	97.39	67.53	90.91	66.75	82.84	66.06	73.95	64.92

Table 4 Running time of static and dynamic methods when deleting objects of different proportions

Data sets	10%		20%		30%		40%		50%	
	Static	Dynamic	Static	Dynamic	Static	Dynamic	Static	Dynamic	Static	Dynamic
Yach Hydrodynamics	3.27	0.82	2.67	0.70	2.17	0.61	1.75	0.48	1.17	0.37
Wine	2.22	0.34	1.89	0.34	1.56	0.33	1.12	0.31	0.72	0.30
Wireless Indoor Localization	14.86	4.77	13.47	4.67	11.69	4.45	9.52	4.23	6.63	3.89
Wilt	179.81	112.27	176.95	112.01	173.02	111.61	166.86	111.08	161.00	110.53
Shill Bidding	106.70	35.94	99.19	35.42	87.33	34.52	73.56	32.70	53.75	31.50
Dry Bean	1868.33	311.43	1621.67	289.94	1348.91	259.66	1189.56	176.58	786.57	157.61
Concrete	9.05	3.17	8.03	2.98	6.77	2.84	5.44	2.64	4.11	2.47
Contraceptive Method Choice	12.02	2.88	11.03	2.80	9.78	2.70	8.09	2.67	5.72	2.64
Customer Churn	86.09	24.69	75.55	24.22	62.53	23.55	47.39	22.66	35.27	21.44
Combined Cycle Power Plant	647.35	82.32	532.21	75.36	466.58	66.21	379.32	52.21	299.21	47.36
Blood Transfusion Service Center	6.86	1.78	5.92	1.75	5.03	1.69	4.16	1.64	3.00	1.60
Abalone	101.98	67.97	97.39	67.53	90.91	66.75	82.84	66.06	73.95	64.92

the system in steps of 2, 4, 6, 8, and 10 sources per iteration. In the source reduction experiment, we begin with the full 20 information sources and reduce the number of sources stepwise, removing 2, 4, 6, 8, and 10 sources at each iteration. Tables 5 and 6 indicate the running time of the static and dynamic algorithms when different numbers of information sources are added and subtracted, respectively.

In the number of attributes variation experiment (Algorithm 4), we use an initial 10 sources of information. The applicability of this approach may vary from dataset to dataset due to the small number of attributes in some datasets. For example, in a dataset with a number of attributes of 6, we adjust for the percentage of increase or decrease. Specifically, when increasing the number of attributes, the original number of attributes would be increased by 10%, 20%, 30%, 40%, and 50%, respectively. In this case, we round the increase to an integer. Assuming the original number of attributes is 6, a 10% increase in the number of attributes is 6.6, rounded to 7; a 20% increase in the number of attributes is 7.2, rounded to 7, and so on. Ensure that the incremental change in the number of attributes does not exceed one-half of the initial number of attributes. Tables 7 and 8 indicate the running time

of the static and dynamic algorithms when different numbers of attributes are added and subtracted, respectively.

A statistical analysis was conducted on the running time of the static update process and the dynamic update process under the above different scenarios, the results of which are summarized in Tables 5, 6, 7, 8, 9 and 10 presenting the comparison of the running time of the two algorithms under different scenarios of object, source, and attribute numbers. As shown in Tables 5 to Table 10, the running time of the dynamic algorithm is significantly lower than that of the static algorithm on the 12 datasets. Figure 8 shows the acceleration ratio (acceleration ratio = $T_{Static}/T_{Dynamic}$) of the static algorithm and the dynamic algorithm under scenarios of increasing or decreasing object, source, and attribute numbers on the 12 datasets. It is clearly shown from the figure that the acceleration ratio is significantly greater than 1 in all datasets.

5.3 Comparative experiments with other related algorithms.

To verify the effectiveness of the fusion method proposed in this paper in terms of time and accuracy, we selected

Table 5 Running time of static and dynamic methods when increasing sources of different proportions

Data sets	10%		20%		30%		40%		50%	
	Static	Dynamic	Static	Dynamic	Static	Dynamic	Static	Dynamic	Static	Dynamic
Yach Hydrodynamics	1.25	0.69	2.16	0.97	3.22	1.28	3.88	1.75	5.34	2.28
Wine	0.75	0.47	1.41	0.61	2.02	0.72	2.13	0.92	2.22	0.99
Wireless Indoor Localization	8.14	4.31	12.13	5.52	17.91	8.36	23.08	10.92	30.25	13.70
Wilt	251.98	88.47	538.44	144.69	917.56	239.95	1289.78	369.95	1664.59	820.58
Shill Bidding	654.27	231.38	871.30	336.03	1193.06	418.34	1531.05	631.09	2115.84	724.59
Dry Bean	3621.09	543.05	3876.12	745.38	4131.39	902.15	4467.76	1282.34	4959.68	1455.08
Concrete	7.20	3.86	13.06	5.36	19.63	7.64	27.17	11.06	35.69	15.22
Contraceptive Method Choice	9.16	4.86	12.94	6.22	18.06	7.25	25.55	11.11	33.08	14.64
Customer Churn	46.11	24.16	73.02	32.30	98.88	41.30	122.88	55.36	163.59	76.55
Combined Cycle Power Plant	2757.81	390.63	3122.20	503.43	3489.23	767.16	3676.53	1062.94	3889.21	1347.65
Blood Transfusion Service Center	3.23	1.56	4.33	1.81	5.95	2.47	7.22	3.48	9.81	4.27
Abalone	153.48	50.17	274.13	56.42	474.92	70.45	635.28	97.36	758.00	118.66

Table 6 Running time of static and dynamic methods when deleting sources of different proportions

Data sets	10%		20%		30%		40%		50%	
	Static	Dynamic	Static	Dynamic	Static	Dynamic	Static	Dynamic	Static	Dynamic
Yach Hydrodynamics	3.21	2.05	2.83	1.55	2.45	1.40	2.02	1.17	1.45	1.08
Wine	0.98	0.63	0.94	0.55	0.80	0.52	0.73	0.36	0.42	0.20
Wireless Indoor Localization	25.91	14.34	23.63	12.38	18.92	10.09	16.22	8.88	12.42	7.22
Wilt	1134.87	369.35	988.35	303.23	746.80	264.95	562.06	202.05	418.98	162.54
Shill Bidding	1598.06	477.05	1283.21	395.26	952.03	305.35	813.32	223.98	668.36	185.45
Dry Bean	4132.98	945.87	3431.23	809.70	2991.45	729.20	2064.38	669.82	1802.11	564.83
Concrete	7.80	4.41	6.66	3.78	6.06	2.98	4.59	2.42	3.52	2.22
Contraceptive Method Choice	14.14	8.58	11.97	5.81	8.81	4.70	6.97	4.12	5.38	3.52
Customer Churn	90.83	37.19	71.42	30.11	59.80	23.91	43.55	19.50	36.70	14.45
Combined Cycle Power Plant	3623.63	725.08	2864.30	611.30	2053.34	522.12	1767.00	412.76	1289.38	328.72
Blood Transfusion Service Center	7.20	4.08	6.80	3.77	6.08	3.69	5.72	3.48	5.19	3.34
Abalone	776.14	108.45	656.48	95.78	587.86	84.45	472.47	72.81	382.47	66.39

Table 7 Running time of static and dynamic methods when increasing attributes of different proportions

Data sets	10%		20%		30%		40%		50%	
	Static	Dynamic	Static	Dynamic	Static	Dynamic	Static	Dynamic	Static	Dynamic
Yach Hydrodynamics	2.84	0.94	5.25	1.70	9.39	4.05	10.80	4.91	11.53	6.25
Wine	2.22	1.20	5.09	2.61	7.78	3.27	9.13	3.77	10.25	4.27
Wireless Indoor Localization	320.55	227.05	745.14	536.45	1281.31	697.73	1699.17	1020.95	2022.64	1192.31
Wilt	754.42	498.02	1235.85	621.12	1759.34	834.54	2238.91	1314.57	2849.05	1832.27
Shill Bidding	1052.72	643.98	1455.85	867.53	1832.05	1016.92	2455.87	1334.76	3256.96	2101.00
Dry Bean	3825.89	1623.09	4153.43	1978.70	4878.86	2449.20	-	2729.58	-	3164.78
Concrete	55.42	25.98	96.22	117.11	369.75	130.30	419.08	171.59	456.47	193.52
Contraceptive Method Choice	143.08	37.03	298.56	113.69	456.42	214.06	775.59	283.86	1042.45	368.61
Customer Churn	389.12	266.53	832.27	642.11	1389.71	778.63	2106.87	1102.32	2347.87	1327.09
Combined Cycle Power Plant	1501.75	584.28	2164.19	1170.63	3212.69	1764.42	3835.84	1905.34	4448.86	2392.47
Blood Transfusion Service Center	29.03	21.14	54.14	44.34	84.67	68.14	111.55	86.17	137.5	164.32
Abalone	607.98	426.94	1021.54	532.36	1665.75	728.41	1989.14	1021.16	2658.95	1572.62

The '-' symbol indicates a runtime exceeding 5000 s

traditional multi-source fusion methods and the latest multi-source fusion methods and compared them with the method proposed in this paper from different dimensions:

- (1) The traditional mean fusion method can be expressed as MeanF:

$$MEANF_a(x) = [\frac{1}{N} \sum_{i=1}^N f_i^L(x, a), \frac{1}{N} \sum_{i=1}^N f_i^U(x, a)]$$

where $f_i^L(x, a)$ and $f_i^U(x, a)$ denote the left and right endpoints of information source i in relation to attribute a .

Table 8 Running time of static and dynamic methods when deleting attributes of different proportions

Data sets	10%		20%		30%		40%		50%	
	Static	Dynamic	Static	Dynamic	Static	Dynamic	Static	Dynamic	Static	Dynamic
Yach Hydrodynamics	18.74	10.47	16.08	8.55	14.27	7.67	12.08	6.52	10.45	5.97
Wine	16.61	8.28	13.17	7.25	10.25	4.98	6.97	3.77	3.84	1.72
Wireless Indoor Localization	1282.31	626.45	1022.67	568.38	893.23	412.07	814.45	307.16	625.87	408.73
Wilt	2463.22	1257.76	2177.45	1022.61	1835.03	862.06	1692.07	768.87	1353.62	683.23
Shill Bidding	3038.23	1556.98	2634.98	1245.09	2455.17	1036.96	2176.07	823.83	1664.85	702.62
Dry Bean	-	4849.78	-	3970.09	-	3114.36	3814.45	2183.16	1922.98	1108.17
Concrete	614.73	314.16	519.97	255.56	415.03	208.88	252.39	145.89	124.23	68.33
Contraceptive Method Choice	1125.0	575.20	982.30	502.22	811.88	403.86	615.80	313.47	359.34	206.31
Customer Churn	1776.24	873.25	1523.45	741.67	1356.34	589.49	1123.09	496.56	924.45	461.23
Combined Cycle Power Plant	4269.43	2297.30	3656.91	1807.23	3038.25	1667.53	2061.78	1405.91	1871.53	957.00
Blood Transfusion Service Center	411.36	378.09	333.88	302.72	247.12	242.25	162.23	158.19	80.00	70.41
Abalone	2232.15	1024.43	1924.34	915.24	1709.02	721.51	1525.42	652.34	1143.00	533.37

The '-' symbol indicates a runtime exceeding 5000 s

- (2) The multi-granularity fusion method based on equivalence relation matrix can be expressed as MERM [23].
- (3) The multi-source information fusion method based on fuzzy approximate conditional entropy can be expressed as MSIEFM [16].

The selection of these methods is based on the following considerations: (1) Methodological representativeness: MERM represents the class of static equivalence-relation-based fusion methods, which use similarity or equivalence relations among objects as the basis for information integration. MSIEFM is a fuzzy-entropy-based fusion approach that measures inter-source uncertainty and is widely adopted in information fusion and rough set studies. These two methods thus reflect the mainstream paradigms of static multi-source fusion-relation-based and entropy-based approaches-providing strong and complementary baselines for comparison. (2) Relevance to the proposed method: Both MERM and MSIEFM assume a static information system, without considering dynamic variations of objects, attributes, or information sources. Therefore, comparing the proposed dynamic update mechanism with these static baselines can clearly demonstrate its advantages in handling evolving data environments.

In addition to these two baseline methods, a raw data accuracy benchmark was designed to serve as a lower-bound reference. Specifically, we evaluated the classification accuracy of the raw data without any dynamic update mechanism and compared it with the results of the proposed method and the two baseline methods (MERM and MSIEFM). This setup ensures that the raw-data accuracy benchmark reflects the inherent data precision, while the two static baselines represent state-of-the-art static fusion techniques. The proposed method's superior performance across dynamic test cases validates its ability to maintain high accuracy and robustness under continuous data changes.

First, we evaluated the running time of our incremental method on different datasets and compared our method with MSIEFM at the time scale level. We iteratively changed the object, source, and attribute quantities by 10%, 20%, 30%, 40%, and 50% (covering increases and decreases) according to the changing conditions, and selected four datasets for each set of conditions. Tables 7, 8, and 9 show the comparison of fusion time when the object quantity is increased or decreased by different proportions in the Wine, YH, BT, and Concrete datasets, the comparison of fusion time when the source quantity is increased or decreased by different proportions in the Wine, YH, BT, and Concrete datasets, and the comparison of fusion time when the object quantity is increased or decreased by different proportions in the Wine, YH, BT, and CMC datasets, respectively. The results show that our method has significant advantages in terms of running time.

Additionally, we assessed the classifier performance using a ten-fold cross-validation method to ensure the reliability of the model evaluation. We systematically compared the performance of three commonly used classifiers-KNN, SVM, and MLP-across multiple methods, including the initial fuzzy dataset (RAW), the traditional mean fusion method (MEANF), the advanced multi-source multi-granularity fusion method (MERM), and MS-FIS.

For the KNN classifier, we adjusted the k value based on the characteristics of each dataset. The results, presented in Table 12, show that MS-FIS achieves the highest classification accuracy on most datasets, particularly on the YH, Wine and Wilt datasets, where its accuracy significantly surpasses that of other methods. The fusion strategy used in MS-FIS notably enhances classification performance, yielding outstanding results.

Similarly, for the SVM classifier, MS-FIS also demonstrates strong performance, especially on the YH and Wine datasets, where it achieves higher accuracy than

Table 9 Comparison of different fusion methods under KNN classifier

Method	YH	Wine	WIL	Wilt	Shill	DB	Concrete	CMC	CC	CP	BT	Abalone
RAW	61.19 ± 0.03	44.37 ± 0.04	43.13 ± 0.02	60.07 ± 0.05	70.46 ± 0.01	49.06 ± 0.03	61.48 ± 0.02	30.38 ± 0.04	68.35 ± 0.05	42.56 ± 0.02	60.41 ± 0.03	56.36 ± 0.05
MEANF	70.97 ± 0.03	50.70 ± 0.04	44.33 ± 0.01	92.15 ± 0.02	86.57 ± 0.03	39.10 ± 0.05	66.99 ± 0.01	38.69 ± 0.04	79.37 ± 0.03	50.08 ± 0.02	61.17 ± 0.05	57.97 ± 0.01
MERM	67.74 ± 0.04	64.40 ± 0.02	61.83 ± 0.05	92.91 ± 0.03	87.08 ± 0.02	56.52 ± 0.01	68.61 ± 0.01	42.99 ± 0.02	82.79 ± 0.03	50.08 ± 0.01	62.22 ± 0.04	63.72 ± 1.05
MS-FIS	72.04 ± 0.03	64.45 ± 0.04	67.67 ± 0.02	93.18 ± 0.05	89.93 ± 0.03	56.52 ± 0.01	68.93 ± 0.04	48.39 ± 0.02	81.79 ± 0.03	57.79 ± 0.02	62.22 ± 0.01	63.72 ± 1.05

Bold text means optimal values

Table 10 Comparison of different fusion methods under SVM classifier

Method	YH	Wine	WIL	Wilt	Shill	DB	Concrete	CMC	CC	CP	BT	Abalone
RAW	64.52 ± 0.03	38.28 ± 0.05	52.35 ± 0.02	80.52 ± 0.04	90.01 ± 0.01	52.31 ± 0.02	66.04 ± 0.03	36.53 ± 0.04	70.31 ± 0.01	39.24 ± 0.05	64.75 ± 0.02	58.79 ± 0.04
MEANF	76.91 ± 0.02	38.89 ± 0.03	30.50 ± 0.05	94.05 ± 0.01	92.04 ± 0.04	42.45 ± 0.02	70.04 ± 0.03	43.89 ± 0.05	78.56 ± 0.01	42.34 ± 0.03	73.33 ± 0.05	66.99 ± 0.02
MERM	77.42 ± 0.00	63.52 ± 1.14	55.67 ± 0.05	94.35 ± 0.04	89.98 ± 0.03	58.31 ± 0.23	71.20 ± 0.01	42.53 ± 0.03	82.86 ± 0.02	42.85 ± 0.01	72.33 ± 0.04	70.33 ± 0.05
MS-FIS	77.42 ± 0.00	63.52 ± 1.14	60.67 ± 0.02	94.35 ± 0.04	90.83 ± 0.01	58.31 ± 0.23	71.20 ± 0.01	43.89 ± 0.04	83.37 ± 0.01	57.10 ± 0.05	73.34 ± 0.02	70.33 ± 0.05

Bold text means optimal values

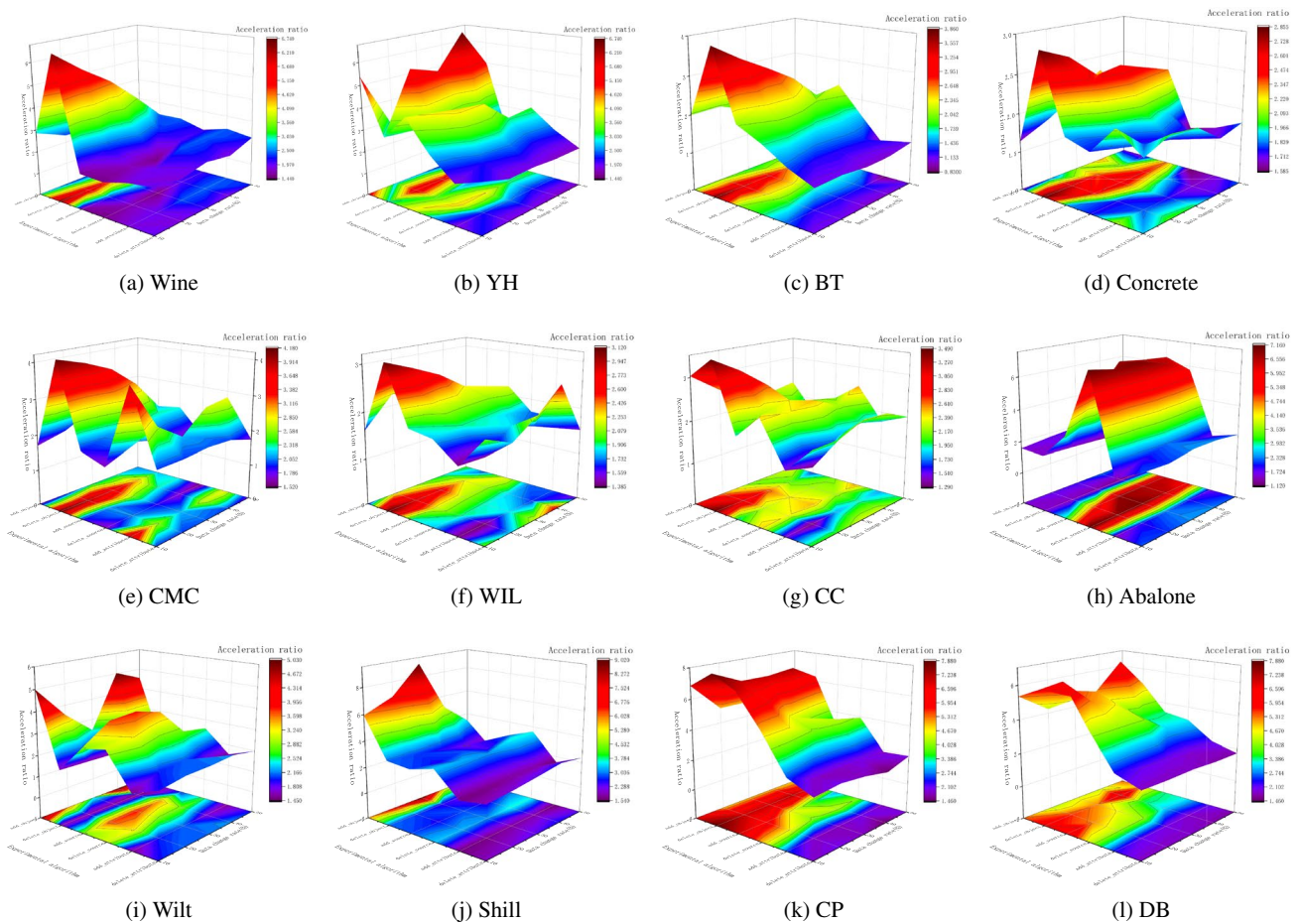


Fig. 8 Algorithm acceleration ratio of the dataset

other methods. Additionally, MS-FIS shows competitive performance on the CMC and Abalone datasets, further underscoring its adaptability and superiority across different datasets.

In the case of the MLP classifier, MS-FIS stands out as the top performer, particularly on the YH and Wine datasets, where its accuracy far exceeds that of other methods, delivering the best classification results. This performance highlights MS-FIS’s enhanced adaptability and effectiveness across multiple datasets. Although MEANF and MERM also perform well on certain datasets, they are generally outperformed by MS-FIS, which delivers the best results in most experiments, solidifying its significant advantage in classification performance.

5.4 Statistical analysis

We employed both *T*-tests and χ^2 tests to assess the effectiveness and validity of the model. For the MS-FIS and MERM methods, we performed χ^2 -tests on three different classifiers (KNN, SVM, and MLP) to evaluate the performance of these methods. The chi-square

statistic measures the deviation between the observed and expected data for each classifier. The results of these χ^2 tests are used to assess the outcomes of the statistical hypothesis test. A value below the significance level (typically 0.05) suggests that the null hypothesis cannot be rejected. In this case, all classifier values were 0.99, indicating that neither classifier was significantly influenced by the selected method. Although MS-FIS showed a statistical advantage over MERM, no significant difference was observed, suggesting that the two methods may exhibit complementary characteristics in practical applications, and their performance could potentially be enhanced by optimizing the fusion strategy (Table 11).

Additionally, we conducted *t*-tests on the original data (RAW) and the MS-FIS method for the three classifiers. The *P*-values for all classifiers were 0.0001, well below the typical significance levels (e.g., 0.05 or 0.01), allowing us to reject the null hypothesis. These findings indicate that the classification performance of the MS-FIS method is significantly superior to that of the RAW method, with a statistically meaningful difference. The fusion strategy employed by MS-FIS has clearly

Table 11 Comparison of different fusion methods under MLP classifier

Method	YH	Wine	WIL	Wilt	Shill	DB	Concrete	CMC	CC	CP	BT	Abalone
RAW	63.02 ± 0.03	45.16 ± 0.02	46.37 ± 0.05	63.32 ± 0.01	78.64 ± 0.03	47.42 ± 0.02	65.63 ± 0.04	37.67 ± 0.03	72.31 ± 0.03	40.65 ± 0.05	69.52 ± 0.01	58.64 ± 0.04
MEANF	73.41 ± 0.04	38.51 ± 0.03	49.00 ± 0.01	85.50 ± 0.05	84.73 ± 0.02	57.84 ± 0.03	71.52 ± 0.02	45.93 ± 0.04	82.86 ± 0.04	52.03 ± 0.03	73.33 ± 0.01	66.99 ± 0.02
MERM	77.42 ± 0.03	67.41 ± 0.03	55.17 ± 1.31	94.35 ± 0.01	90.83 ± 0.02	62.35 ± 0.04	70.55 ± 0.03	42.76 ± 0.01	83.07 ± 0.01	56.13 ± 0.04	73.45 ± 0.01	69.86 ± 0.02
MS-FIS	77.42 ± 0.03	68.15 ± 0.04	55.17 ± 1.31	92.15 ± 0.02	90.85 ± 0.86	59.34 ± 0.05	72.17 ± 0.00	43.89 ± 0.01	83.38 ± 0.05	56.13 ± 0.03	73.78 ± 0.01	69.94 ± 1.12

Bold text means optimal values

Table 12 Perform χ^2 - test on MS-FIS and MERM

Classifier	χ^2 statistic	<i>P</i> value	<i>F</i>
KNN	0.912	0.99	11
SVM	1.964	0.99	11
MLP	0.138	0.99	11

Table 13 Perform *T*- test on MS-FIS and RAW

Classifier	<i>T</i> -statistic	<i>P</i> value
KNN	-5.91	0.0001
SVM	-5.87	0.0001
MLP	-6.30	0.0001

enhanced the classifier’s performance, both numerically and statistically. Detailed results will be provided in Tables 12 and 13, while Figs. 9, 10 and 11 will offer a deeper analysis of the performance differences between the two methods (Fig. 12).

5.5 Parameter sensitivity analysis

To verify the stability and rationality of the multi-source fuzzy information fusion method proposed in this paper under different parameter settings, a sensitivity analysis was performed on the key parameters α and β . The primary objective of this analysis is to reveal the law governing how parameters influence fusion accuracy, thereby providing a basis for determining the optimal parameter combination in experiments. In the process of multi-source fuzzy information fusion, this paper employs fixed fusion operators and possible fusion operators to characterize two distinct logics of information fusion. The fixed fusion operator takes parameter α as a threshold, which represents the degree of consistent support for an object across multi-source information. The possible fusion operator uses parameter β as a threshold, indicating the degree of potential support for the object in multi-source information. These two parameters satisfy the complementary relationship $\beta = 1 - \alpha$. This implies that as α increases, the system tends to confirm support only when multi-source information is highly consistent; correspondingly, β decreases, making the system more cautious toward information sources with uncertainty or significant discrepancies. Conversely, when α is small, the fusion results are more inclusive but may also introduce noise. Therefore, α and β collectively determine the balance between the stringency and robustness of the fusion operator. Within the framework of this paper, studying the variation pattern of parameter α alone can reflect the variation trends of both fusion mechanisms simultaneously.

Table 14 presents the fusion accuracy across 12 datasets as the α value varies from 0.1 to 0.9. The experimental

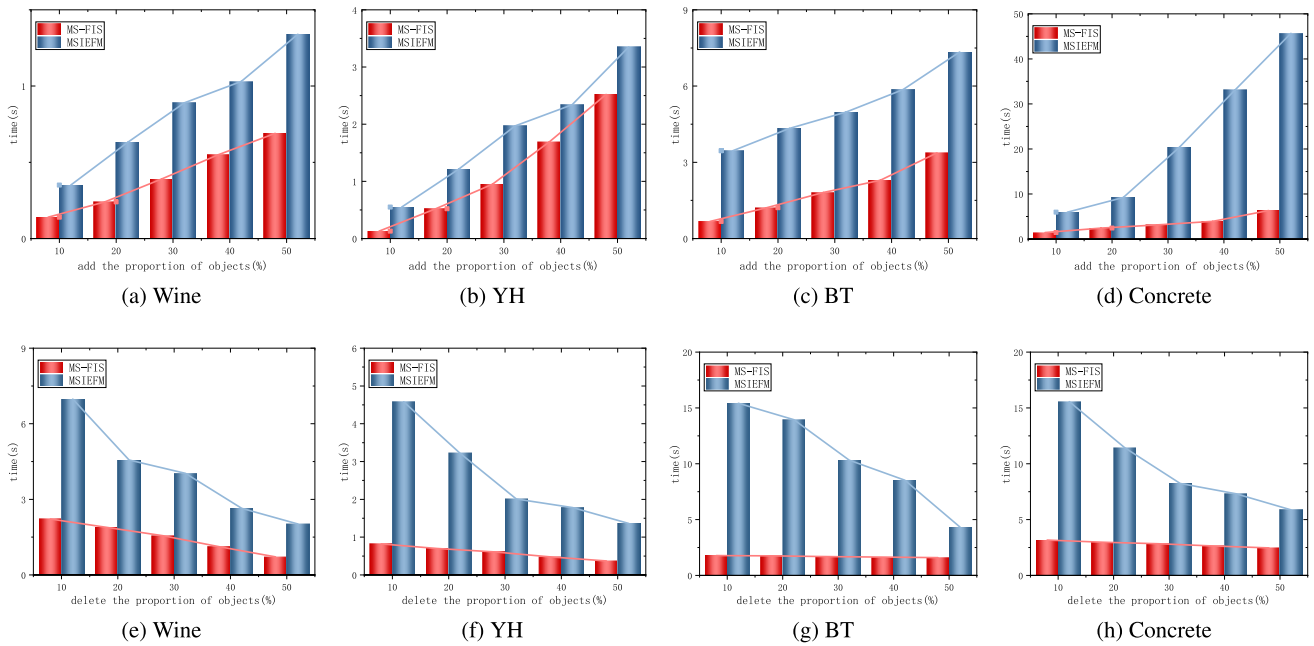


Fig. 9 Compare the running time with other algorithms when the number of objects changes

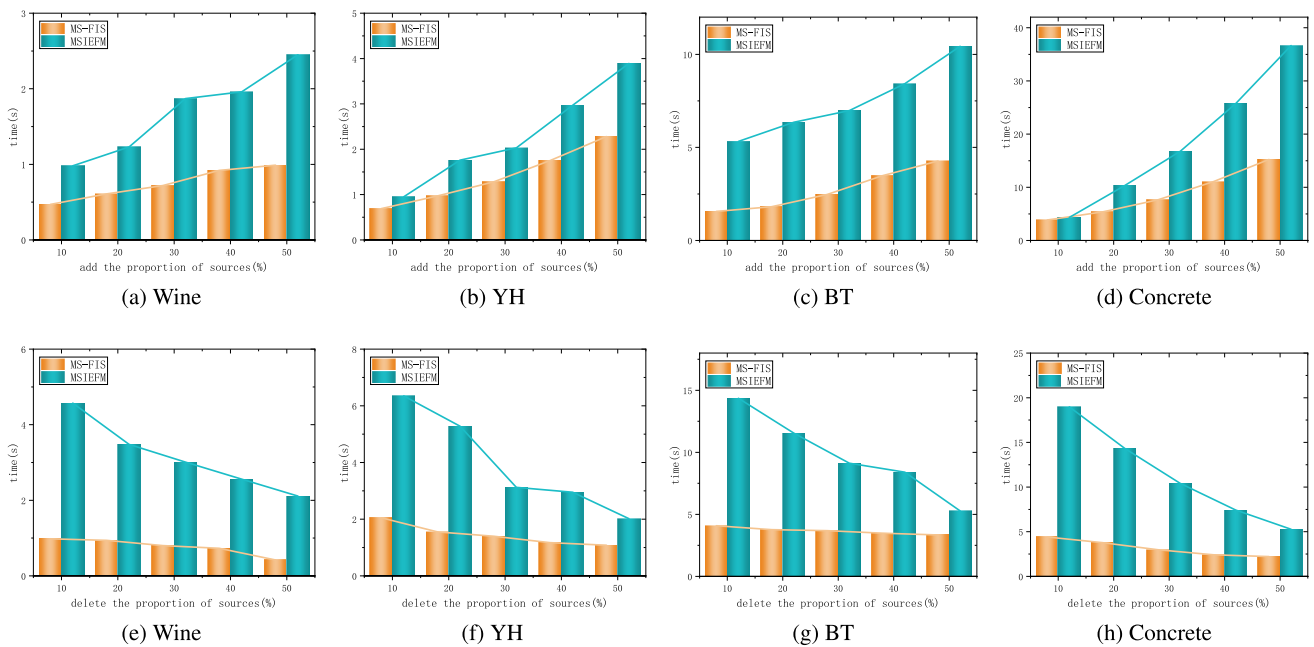


Fig. 10 Compare the running time with other algorithms when the number of information sources changes

results demonstrate that with the increase of α , the fusion accuracy generally exhibits a trend of first rising and then stabilizing. For most datasets, the fusion accuracy reaches the highest value or a value close to the highest when $\alpha = 0.7$. When α continues to increase to 0.8, the accuracy remains essentially stable; however, when $\alpha = 0.9$, the accuracy of some datasets decreases slightly.

This indicates that $\alpha = 0.7$ can ensure fusion consistency while avoiding overly strict condition constraints. When α is excessively small (such as 0.1-0.3), the fusion operator becomes too lenient, leading to interference from inconsistent information sources on the results. When α is overly large (such as 0.9), the fusion conditions are too stringent, which may result in the omission of some

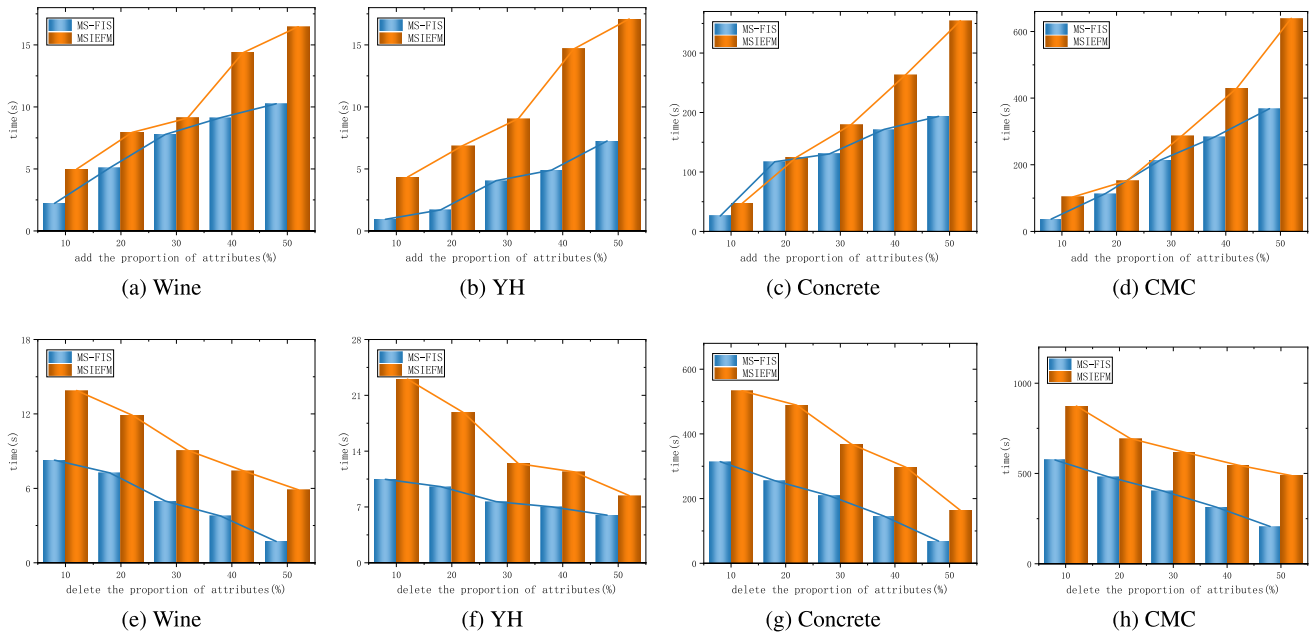


Fig. 11 Compare the running time with other algorithms when the number of attributes changes

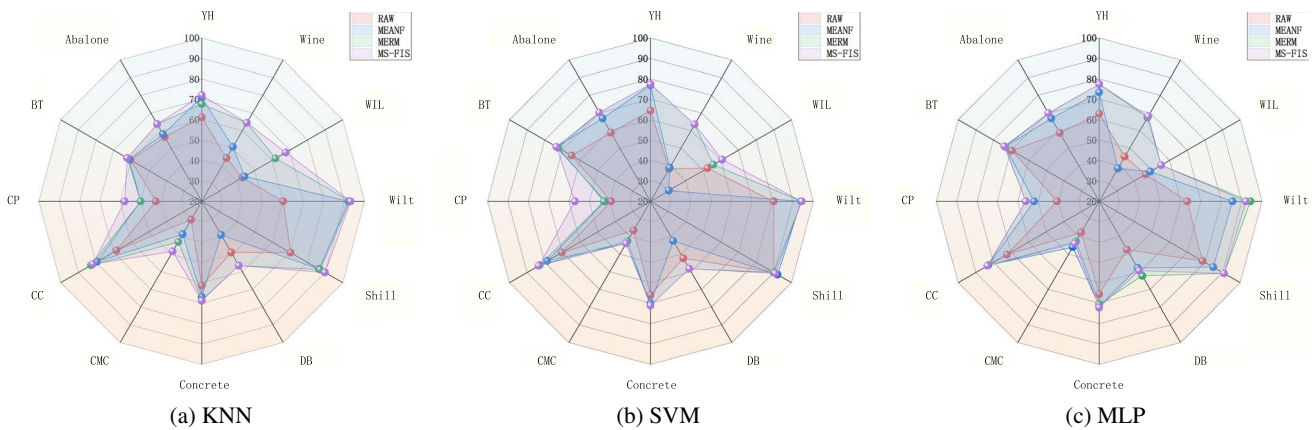


Fig. 12 Comparison of fusion of RAW, MEAN, MERM, and MS-FIS on three different classifiers

effective information. Considering the performance of all datasets comprehensively, this paper selects $\alpha = 0.7$ as the standard parameter value, as this setting achieves a favorable balance between accuracy and stability.

6 Conclusion and prospects

6.1 Conclusions

In the context of the ever-expanding scales of information data, this study proposed a matrix-based multi-granularity fusion approach to extract knowledge directly from multi-source fuzzy information systems. The proposed method

prevents information loss during the fusion process and significantly improves the computational efficiency of data fusion. By constructing a fuzzy similarity matrix, the relationships among objects across different information sources were clearly expressed. A dynamic update strategy was developed to efficiently handle variations in the number of objects, sources, and attributes, thereby reducing redundant computations and improving fusion speed. Experimental results confirmed that the proposed method achieves high computational efficiency while maintaining stability and accuracy.

Beyond algorithmic improvements, the proposed framework also demonstrates strong potential for practical applications. For instance, in the healthcare domain, it can

Table 14 Approximate accuracy of fusion under different values of α

Method	YH	Wine	WIL	Wilt	Shill	DB	Concrete	CMC	CC	CP	BT	Abalone
0.1	0.52	0.40	0.44	0.73	0.68	0.39	0.59	0.32	0.64	0.30	0.52	0.49
0.2	0.54	0.51	0.44	0.73	0.68	0.39	0.61	0.35	0.69	0.30	0.52	0.49
0.3	0.54	0.53	0.44	0.77	0.70	0.43	0.61	0.35	0.69	0.32	0.58	0.49
0.4	0.54	0.53	0.49	0.77	0.72	0.47	0.66	0.35	0.73	0.44	0.58	0.50
0.5	0.66	0.56	0.49	0.77	0.75	0.47	0.66	0.41	0.76	0.44	0.58	0.58
0.6	0.66	0.67	0.60	0.89	0.75	0.54	0.69	0.41	0.79	0.49	0.61	0.63
0.7	0.73	0.67	0.64	0.92	0.82	0.54	0.69	0.44	0.82	0.54	0.62	0.63
0.8	0.73	0.67	0.64	0.92	0.82	0.54	0.69	0.44	0.82	0.57	0.62	0.64
0.9	0.74	0.67	0.64	0.91	0.82	0.50	0.69	0.46	0.81	0.57	0.62	0.67

dynamically integrate multi-hospital patient information such as laboratory results, medical imaging, and clinical records to support real-time and accurate diagnosis. These applications verify that the proposed method not only enhances computational performance but also offers valuable support for real-world, dynamic, data-driven systems.

6.2 Future work and limitations

Although the proposed approach effectively manages dynamic changes in single dimensions within multi-source fuzzy information systems, several limitations remain. First, the current framework mainly focuses on independently handling changes in objects, attributes, and information sources, and provides processing mechanisms for these three dimensions. In future research, we plan to develop an efficient fusion mechanism for multi-dimensional simultaneous changes, enabling joint updates across multiple dimensions while avoiding redundant calculations. Second, the present work focuses mainly on numerical and fuzzy-quantitative data. Future extensions will explore applications in heterogeneous and multi-modal information systems, such as the integration of textual, visual, and temporal data, to assess the method’s scalability in complex environments. Finally, further studies will focus on improving computational scalability through distributed or parallel architectures to ensure real-time performance in large-scale systems. These efforts will enhance the generality, robustness, and practicality of the proposed dynamic multi-source fusion framework.

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