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A prospect theory-driven three-way decision framework: Integrating prior probability tolerance and dominance relations in fuzzy incomplete information systems

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ABSTRACT

This study proposes a novel method that integrates Prospect Theory (PT) with a Three-Way Decision (TWD) framework to enhance decision quality in Fuzzy Incomplete Information Systems (FIIS). The method first applies prior probability tolerance dominance relations to manage and evaluate binary relationships among decision options, laying the groundwork for subsequent calculations. Based on this foundation, PT is used to construct objective weight and value functions, effectively reducing uncertainties due to subjective factors in the decision-making process. Unlike traditional methods, this PT-based TWD approach can address incomplete data while fully considering decision-makers' psychological preferences under risk, enabling more precise and scientific decision support. Extensive experimental studies and comparative analyses demonstrate that this method outperforms existing approaches in terms of stability, effectiveness, and superiority, highlighting its potential for applications in fields with high data diversity and uncertainty, such as healthcare.

1. Introduction

In modern society, the complexity and diversity of decision-making problems are increasing, especially in the medical field. The myriad of diseases and individual patient differences make medical decisions particularly complicated. Cardiovascular diseases represent one of the most significant health challenges today, profoundly impacting human health and quality of life. In response to this global issue, the healthcare industry is continually seeking more effective diagnostic and treatment methods to reduce patient disease risks and improve survival rates. In this context, Fuzzy Incomplete Information Systems (FIIS) have been widely studied and applied as crucial tools for managing medical information on cardiovascular patients, assisting doctors in making precise diagnoses and treatment plans. However, due to the incompleteness and diversity of medical data, as well as the subjective preferences and psychological behaviors of decision-makers, traditional methods still face challenges in addressing medical decision-making issues.

Current FIIS face difficulties in handling data incompleteness and diversity. To tackle these issues, researchers have begun to integrate Prospect Theory with Fuzzy Incomplete Information Systems, aiming to enhance the accuracy and efficiency of medical decision-making. Over the past few decades, significant progress has been made in the research of fuzzy set theory and incomplete information systems. Fuzzy set theory provides effective methods for addressing uncertainties and fuzziness, while incomplete information systems focus on handling missing data and diversity. For instance, Zadeh (1965) proposed fuzzy set theory, which offers a mathematical framework for addressing fuzzy and uncertain problems. Meanwhile, Pawlak (1982) introduced the concept of incomplete information systems, offering a new approach for handling incomplete and diverse data. With the rapid advancements in data collection, communication, and storage technologies, information systems frequently encounter situations where data may be missing or temporarily inaccessible.

Scholars have proposed various strategies to address missing values in incomplete information systems. Kryszkiewicz (1998) introduced the rough set reasoning technique for incomplete information systems, proposing that arbitrary attribute evaluation values can be substituted for missing data. Liang and Shi (2020) presented fuzzy clustering methods based on membership and similarity, estimation methods based on probabilistic statistics (Liu & Yu, 2018), and an interpolation method based on fuzzy relation matrices (Wang & Liu, 2008). However, these methods often overlook critical details and fail to fully exploit the complex relationships among data. Additionally, traditional incomplete information systems frequently neglect the impact of decision-makers' psychological states and subjective preferences on their choices.

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In this context, the importance of researching Fuzzy Incomplete Information Systems (FIIS) has become increasingly pronounced. FIIS integrates fuzzy set theory and rough set theory, addressing the shortcomings of traditional approaches in managing complex decision-making problems. Compared to conventional methods, FIIS not only effectively manages data vagueness but also tackles data incompleteness, demonstrating broader applicability and flexibility in real-world scenarios. Furthermore, FIIS preserves the information of original data while providing more flexible and accurate decision support, enabling decision-makers to better understand and resolve intricate decision-making challenges.

However, it is crucial to note that these systems and technologies have not considered the psychological emotions of decision-makers. This research provides a theoretical foundation for developing relevant technologies and applications. Traditional decision-making methods often view decision-makers as fully rational individuals focused solely on maximizing utility, neglecting the behavioral aspects of decision-making. Prospect Theory (Kai-Ineman & Tversky, 1979) examines psychological behaviors in risk decision-making and introduces concepts such as loss aversion and risk preference, thus providing a basis for a deeper analysis of decision-making behaviors. Prospect Theory indicates that individuals tend to avoid losses rather than pursue gains, with heightened sensitivity to losses compared to equivalent gains. Therefore, in medical decision-making, Prospect Theory aids doctors in understanding patients' psychological states and preferences, facilitating more accurate decisions.

In practical decision-making processes, individuals often confront problems that are diverse and not easily categorized into risks and gains. To effectively navigate these complexities, the Three-Way Decision (TWD) method has emerged. TWD (Wang & Yao, 2017; Zhan, Wang, Ding, & Yao, 2022) is a decision-making framework based on uncertain information that categorizes decision objects into positive, negative, and uncertain classes, addressing the challenges of incomplete and uncertain information. Within the TWD framework, decision-makers can more effectively manage incomplete information and uncertainty, thereby enhancing decision accuracy and robustness. Compared to traditional binary decision-making, TWD better utilizes uncertain information, providing decision-makers with more options and support. This foundation has led to numerous studies related to generalized TWD. For example, Zhang, Li, and Wang (2014) proposed a three-way decision method based on hesitant fuzzy linguistic term sets, achieving effective results in addressing uncertainty and vagueness through defined similarity measures and decision rules. He, Liu, Zhang, and Li (2018) introduced an innovative probability-based three-way decision method to tackle classification problems by employing probability models to describe uncertainty, partitioning the decision space into deterministic, probabilistic, and aversion zones while proposing corresponding decision strategies. Experimental results demonstrated that this method exhibits high accuracy and robustness in classification tasks. Xu, Zhang, and Wei (2016) proposed a three-way decision method based on fuzzy rough sets, integrating fuzzy set theory and rough set theory to address uncertainties and incompleteness in decision-making, effectively achieving decision space partitioning and support through defined fuzzy rough sets and associated decision rules.

The concept of probability dominance relationship is one of the important concepts in the theory of three-way decisions, describing the degree of advantage of a decision object relative to other objects under uncertain conditions. By studying the probability dominance relationship, one can better understand the relationships between different decision objects, thus providing decision-makers with more accurate and reliable decision support. In the context of generalized three-way decision theory, probability dominance relationships are widely used in modeling and analyzing decision problems. By analyzing probability dominance relationships, researchers can determine the dominance order of different decision objects, thereby guiding decision-makers to make better decisions. Therefore, the probability dominance relationship is of great significance in decision theory and practice.

For example, Li and Zhang (2016) studied a three-way decision method based on probability dominance relationships, exploring how to use probability dominance relationships to rank and classify decision objects. Liu and Wang (2018) studied a three-way decision method for handling decision problems in incomplete decision tables based on probability dominance relationships. Wei and Wang (2020) studied a three-way decision method based on probability dominance relationships, applied to decision problems in uncertain decision systems. Zhang and Ma (2019) discussed the application of decision methods based on probability dominance relationships in e-commerce credit evaluation, providing an effective method for credit evaluation decisions.

In summary, we combine probability dominance relationships, Prospect Theory (PT), and the Three-Way Decision (TWD) to establish a new PT-TWD-PPTDR method within fuzzy incomplete information systems (FIIS). The motivations for this study are as follows:

(1) Current TWD techniques overly idealize decision-making by treating managers as completely rational and neglecting psychological influences on their behavior. Wang, Li, Qian, Huang, and Zhou (2020) note that previous TWD techniques rely on decision-makers independently providing result vectors. This study addresses this gap by integrating PT and FIIS to create a novel decision-making approach.
(2) Existing methods for handling missing data in decision problems, such as those proposed by Zhan, Ye, Ding, and Liu (2021) and Liu, Liang, and Wang (2016) have limitations due to their requirements for specific types of fuzzy decision attributes and interval loss functions. This paper uses prior probability knowledge to establish probability dominance relationships, effectively addressing decision problems involving missing data and binary relationships between alternative solutions.
(3) TWD methods based on Bayesian theory categorize all alternative solutions into two states. This paper introduces a method that applies to generalized TWD methods, allowing for comprehensive classification of all alternative solutions.

In light of these motivations and current limitations, this study presents the following advances:

(1) This study defines PPTDR in FIIS, enabling broader handling of binary relationships among all alternative solutions.
(2) This research proposes a novel TWD approach based on prior probability in FIIS that facilitates both classification and ranking of potential solutions.
(3) We link PT and TWD in the context of FIIS to form the unique PT-TWD-PPTDR method, which accounts for decision-makers' psychological behaviors and minimizes their subjectivity to scientifically derive the value function and weight function in Prospect Theory.

The organization of this paper is as follows: Section 2 introduces the fundamental concepts of Fuzzy Incomplete Information Systems (FIIS), Prospect Theory (PT), and Three-Way Decision (TWD). Section 3 presents a novel PT-TWD-PPTDR model and discusses the method for deriving the value function. In Section 4, we apply this model to address issues based on echocardiography data. Section 5 compares and analyzes this strategy with alternative methods. Section 6 conducts an experimental study to verify the effectiveness and logic of this approach. Finally, Section 7 summarizes key research findings and suggests future research directions. Additionally, Fig. 1 displays the general schematic of this study.

2. Relate work

We went over the pertinent ideas of theory of prospects, three-way decision-making, and fuzzy incomplete information systems in this part.

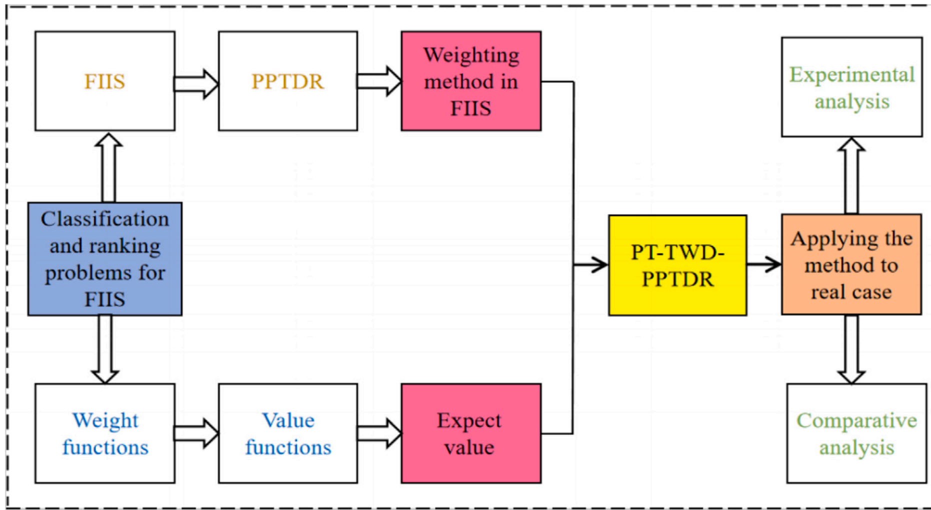

Fig. 1. The framework of this paper.

Table 1
FIIS $F^{*}=\{A,B,X,f\}$.

U/B	b_1	b_2	⋯	b_m	d
a_1	a_{11}	a_{12}	⋯	*	d_1
a_2	*	a_{22}	⋯	a_{2m}	d_1
⋮	⋮	⋮	⋯	⋮	⋮
a_n	a_{n1}	a_{n2}	⋯	a_{nm}	d_n

2.1. Fuzzy Incomplete Information Systems (FIIS)

Fuzzy Incomplete Information Systems refer to information systems designed to handle fuzzy and incomplete information, aiming to provide effective decision support and information management in the face of uncertainty and fuzziness. In traditional information systems, data is often precise and complete. However, in real-life situations, much of the information we encounter is fuzzy and incomplete. This necessitates the use of fuzzy incomplete information systems to manage such information.

Definition 2.1 (*Wang, Zhan, Zhang, Herrera-Viedma, & Kou, 2023*)**.** The representation of a fuzzy information system (FIS) is $F=\{A,B\cup\{d\},X,f\}$. A non-empty finite group of n objects is represented by $A=\{a_1,a_2,\dots,a_n\}$, and a non-empty finite set of m condition characteristics is denoted by $B=\{b_1,b_2,\dots,b_m\}$. In addition to $d\in F(A)$ and $d(x_i)\in[0,1]$, the conditional attribute b_j has a value range denoted by X_{b_j}, and the power collection $X=\cup_{b_j\in B}X_{b_j}$ is represented by $F(U)$. The function $A\times B\rightarrow X$ is $f:A$. It is specified that for every $i\in N=\{1,2,\dots,n\}$ and $j\in M=\{1,2,\dots,M\}$, $f(a_i,b_j)=o_{ij}\in X_{b_j}$.. A symbol $*$ in the FIS indicates an uncertain evaluation value for a conditional attribute. In other words, if there exists some $i\in N$ and $j\in M$, we have $o_{ij}=*$, indicating that certain values are missing from the value set under the conditional attribute. Table 1 illustrates how we transform an FIS into an FIIS. This article defines an FIIS as $F^{*}=\{A,B\cup D,X,f\}$, with $X=\cup_{b_j\in B}X_{b_j}$.

A specific fuzzy incomplete information system is shown in Table 2.

2.2. Prospect theory

The decision theory known as prospect theory, which was put out by psychologists Daniel Kahneman and Amos Tversky (Kai-Ineman & Tversky, 1979), explains how humans make decisions when faced with risk and uncertainty. The theory suggests that people's decisions are not based on maximizing absolute utility, but rather on weighting subjective expectations and values of different outcomes.

In prospect theory, people tend to avoid risks in the domain of losses and are more willing to take risks in the domain of gains. Specifically, individuals are more sensitive to potential losses, leading them to adopt a cautious approach and prefer conservative options when facing potential losses. Conversely, when faced with potential gains, they are more willing to take risks and pursue greater benefits. For example, the impact of a decrease in total assets from 1200 to 1100 is perceived as smaller than a decrease from 200 to 100.

Currently, prospect theory is a significant theory for describing behavior involving uncertainty and risk. Its core concepts include value function and weighting function, expressed in Eqs. (1) and (2) respectively.

$$E(\Delta G_k)=\begin{cases}(\Delta G_k)^{\beta}, & \Delta G_k\geq 0,\\ -\sigma(-\Delta G_k)^{\gamma}, & \Delta G_k<0.\end{cases} \tag{1}$$

The outcome's subjective value is represented by ΔG_k, where a gain is defined as $\Delta G_k\geq 0$ and a loss as $\Delta G_k<0$. While β and γ quantify the reactivity of the value of the function to gains and losses, respectively, and both satisfy $\beta,\gamma\in[0,1]$, α denotes the diminutive fear parameter and satisfies . This formula reveals people's asymmetric perception of profits and losses as well as their risk preferences during the decision-making process by reflecting the subjective value evaluation process of possible outcomes.

$$U_k=\begin{cases}U^{+}(p)=\dfrac{p^{\varphi}}{(p^{\varphi}+(1-p)^{\varphi})^{\frac{1}{\varphi}}},\\ U^{-}(p)=\dfrac{p^{\theta}}{(p^{\theta}+(1-p)^{\theta})^{\frac{1}{\theta}}}.\end{cases} \tag{2}$$

The weighting formula involves the following symbols: U^{+} for holding, U^{-} for selling, p for the probability value, and φ and θ for the value of the weight size's impact on the profit or loss. Up to now, some scholars have studied the values of these coefficients. For example, Tversky and Kahneman (1992) found, and . Abdellaoui, Bleichrodt, and Paraschiv (2007) found. Xu, Zhou, and Xu (2011) found, and $\varphi=\theta=0.74$. The goal of PT's decision-making process is to choose the target with the highest anticipation value. Assuming that object a_i has k outcomes and that we know the values of $E(\Delta G_k)$ and U_k $(k=1,2,\dots,n)$, we may use Eq. (3) to find the object a_i's prospect value.

$$\mathbb{P}=\sum_{k=1}^{n}E(\Delta G_k)U_k. \tag{3}$$

Table 2
The relative loss functions.

	$O(P)$	$\neg O(N)$
B_p	τ_{PP}	τ_{PN}
B_b	τ_{BP}	τ_{BN}
B_n	τ_{NP}	τ_{NN}

2.3. Three Way Decision(TWD)

The Three-Way Decision (TWD) concept was proposed by Yao (2010) in 2010 after he analyzed the criteria for decisions for both decisions-theoretic and conventional rough groups. Research on the TWD concept has steadily advanced in the last few years. From the notion of equivalency classes [23], conditioned probability estimation possesses, strictly speaking, extended to rough sets beyond classes (Zhan, Jiang, & Yao, 2020), probability dominance classes (Wang, Zhan, & Zhang, 2021), and other related areas (Liang, Fu, Xu, & Tang, 2021; Zhang, Dai, & Xu, 2021). To create sequential TWD models, researchers have combined subjective and objective dynamics (Yang, Chen, Fujita, Liu, & Li, 2022). Concurrently, they have introduced the idea of related functions for loss and integrated loss functions with evaluation value (Jia & Liu, 2019). Table 2 displays the particular form, which is a 3×2 matrix with three states and three actions. The two states for the conditional attribute b_j are, indicating whether the object belongs to O or not. The three actions are to accept B_p, maintain B_b, and reject B_n, representing, and respectively. τ_{PP}, τ_{BP}, τ_{NP} represent the losses incurred when the object belongs to O and the three actions B_p, B_b, and B_n are executed, while τ_{PN}, τ_{BN}, τ_{NN} represent the losses incurred when the object does not belong to O and the three actions B_p, B_b, and B_n are executed.

τ represents the relative loss value, and the relative loss function allows decision-makers to balance risks and rewards among different decision options. Lower relative loss values indicate relatively smaller risks or greater rewards, providing an intuitive way to compare the degree of loss between different decision options, making the decision-making process more transparent, and enabling decision-makers to understand the risks and impacts of each option more clearly.

3. The model PT-TWD-PPTDR

To resolve the multi-attribute decision-making (MADM) problem, we will define a new PT-TWD-PPTDR model in this part.

3.1. Maximum deviation weights and PPTDR

We suggest using a maximum deviation weight and a PPTDR to handle the binary relationships between items that have no values (Wang et al., 2023).

Definition 3.1. Some predicted previous information is embodied in a FIIS $F^* = (A, B, V, f)$, based on the distribution of probability concept. As a result, by taking into account the probabilities that are currently known inside the evaluation range, we can represent unknown assessment values. Drawing from previous information, the unknown evaluation value under attribute MB_j receives probabilities for all objects a_j, in such a way that $P(=v_1) = P_1$, $P(=v_2) = P_2$, …, and $P(=v_k) = P_k$. Here, $\forall b_j \in B$, $V(b_j) = v_1, v_2, \dots, v_k$ represents a finite set of values for attribute b_j, satisfying $v_1 \le v_2 \le \cdots \le v_k$. $P(b_j) = P_1, P_2, \dots, P_k$ is the vector containing the probabilities of all values in $V(b_j)$. The prior probability is defined as P_f $(f \in 1, 2, \dots, k)$. The core semantics of it are derived from attribute b_j, which is the ratio of evaluation value v_f occurrences to all objects $(n-1)$. The evaluation values of each student for each course are shown by the data in Table 3, where $A = a_1, a_2, \dots, a_7$ represents seven students and $B = \{b_1, b_2, b_3, b_4\}$ represents four courses. For instance, the previous likelihood of $P(= 0.21)$ for attribute b_1 is $1/7$. The amount with the highest probability is usually assumed by the unidentified assessment value. This may be represented as $P_1 \ge P_2 \Rightarrow P(=v_1|p(v_1) = P_1) \ge P(=v_2|p(v_2) = P_2)$ in terms of conditional probabilities.

Table 3
A scores evaluation degree table of students.

	b_1	b_2	b_3	b_4
a_1	0.11	0.32	0.65	0.54
a_2	0.23	*	0.59	0.36
a_3	*	0.25	0.058	0.98
a_4	0.21	0.85	*	0.27
a_5	0.53	0.52	0.59	0.19
a_6	0.74	*	0.25	0.85
a_7	0.61	0.18	*	0.52

Definition 3.2 (*Wang et al., 2023*). Suppose that $F^* = \{A, B, V, f\}$ is a FIIS. For any $a_i, a_l \in A$ $(i, l \in \{1, 2, \dots, n\})$, $b_j \in B$ $(j \in \{1, 2, \dots, m\})$, and $v_f \in V(c_j)$ $(f \in \{1, 2, \dots, k\})$, the probability that the item a_i is poorer compared to the item a_l is as follows:

$$R_{b_j}(a_i, a_l) = \begin{cases} 0 & \text{if} a_{ij} \ge a_{lj} \wedge a_{ij} \ne * \wedge a_{lj} \ne *; \\ \frac{a_{lj} - a_{lj}}{a_{lj}} & \text{if} a_{ij} < a_{lj} \wedge a_{ij} \ne * \wedge a_{lj} \ne *; \\ \sum_{f=1}^{k} P_f^2 & \text{if} a_{ij} = * \wedge a_{l\Delta j} = *; \\ \sum_{1 \le h \le f} P_h & \text{if} a_{lj} = * \wedge a_{lj} = v_f; \\ \sum_{f \le h \le k} P_h & \text{if} a_{ij} = v_f \wedge a_{lj} = * . \end{cases} \tag{4}$$

According to the definition, we can use prior probabilities to fully utilize the knowledge of incomplete information systems in order to determine the dominance probability between two objects as well as the likelihood of an unknown attribute value *. This is more consistent with how people interpret and understand unclear information in their gut feelings (Li, Liang, & Pang, 2017).

Definition 3.3. The prior to probability tolerance dominance level in regard to the characteristic set C is as follows for every $C \subseteq B$ and $\forall a_i, a_l \in A$, given a FIIS $F^* = \{A, B, V, f\}$.

$$R_C(a_i, a_l) = \frac{\sum_{b_j \in C} R_{b_j}(a_i, a_l)}{|C|}, \tag{5}$$

where $|C|$ represents the cardinality of attribute set C. Subsequently, under F^*, the prior probability tolerance dominance relation concerning characteristic set C is expressed as:

$$R_C^{\le *\alpha} = \{(a_i, a_l) \in A \times A | R_C(a_i, a_l) \le \alpha\}, \tag{6}$$

where the tolerance level for characteristic set C is defined as α, satisfying $0 \le \alpha \le 1$. R_α is called PPTDR in this context.

Additionally, the prior probability tolerance dominance class is shown as follows relying on the conditional characteristic set C:

$$[a_i]_C^{\le *\alpha} = \{a_l \in A | R_C(a_l, a_i) \in R_C^{\le *\alpha}\}. \tag{7}$$

We standardize the data in the data table in order to make it easier to promote the best mutual interests within a certain range. This procedure of normalization is stated as:

$$e_{ij} = \frac{a_{ij}}{\sqrt{\sum_{i=1}^{n} a_{ij}^2}}, \quad i = 1, 2, \dots, n; j = 1, 2, \dots, m. \tag{8}$$

Example 3.1. Table 4 displays the students' A scores assessment degree table; $A = \{a_1, a_2, \dots, a_{10}\}$ denotes the student collection, and $B = \{b_1, b_2, b_3, b_4\}$ represents the B scores evaluation degree table. The collection of courses is denoted by the math scores degree by b_1, the Chinese scores degree by b_2, the English scores degree by b_3, and

Table 4
A scores evaluation degree table of students.

	b_1	b_2	b_3	b_4	D
a_1	3	2	4	1	4
a_2	4	1	2	2	2
a_3	2	*	3	3	1
a_4	2	2	1	2	1
a_5	*	3	2	1	2
a_6	1	2	3	2	4
a_7	1	1	*	2	3
a_8	2	3	2	1	1
a_9	2	2	1	*	3
a_{10}	3	*	1	3	1

Table 5
Weight calculation for each attribute.

	b_1	b_2	b_3	b_4
W_j	0.2534	0.2466	0.2603	0.2397

the history scores degree by b_4. Utilizing Formula (7), we standardize the information in Table 4. For every student a_i, we can compute the previous likelihood tolerance dominant group in the following way, per Definitions 3.1–3.3:

$[a_1]_B^{(\leq * 0.2)} = \{a_1\}$,
$[a_2]_B^{(\leq * 0.2)} = \{a_1, a_2, a_3, a_6\}$,
$[a_3]_B^{(\leq * 0.2)} = \{a_3\}$,
$[a_4]_B^{(\leq * 0.2)} = \{a_1, a_2, a_3, a_4, a_6, a_8, a_9, a_{10}\}$,
$[a_5]_B^{(\leq * 0.2)} = \{a_5, a_8\}$,
$[a_6]_B^{(\leq * 0.2)} = \{a_1, a_3, a_4\}$,
$[a_7]_B^{(\leq * 0.2)} = \{a_1, a_2, a_3, a_6, a_7\}$,
$[a_8]_B^{(\leq * 0.2)} = \{a_1, a_2, a_3, a_5, a_8\}$,
$[a_9]_B^{(\leq * 0.2)} = \{a_4, a_9\}$,
$[a_{10}]_B^{(\leq * 0.2)} = \{a_3, a_{10}\}$.

Definition 3.4. Defining the jth characteristic of a FIIS with $F^* = \{A, B, V, f\}$, the equation calculating the highest bias weight w_j of the prior likelihood tolerance dominance group is as follows:

$$w_j = \frac{\sum_{i=1}^{n} \sum_{k=1}^{n} |E_{ij} - E_{kj}|}{\sum_{j=1}^{m} \sum_{i=1}^{n} \sum_{k=1}^{n} |E_{ij} - E_{kj}|}, \tag{9}$$

where E_{ij} denotes the value of attribute j for the i-th decision option, $E_{ij} = 1 - \frac{[a_i]_B^{(\leq * \alpha)}(b_j)}{n}$, $\quad i \in \{1, 2, \ldots, n\}$, and n represents the number of decision options, $0 \leq w_j \leq 1$, $\sum_j w_j = 1$.

Example 3.2. We use Definition 3.4 to determine the greatest possible variance weight for every conditional characteristic b_j based on Example 3.1. The findings are displayed in Table 5.

Definition 3.5 (*Wang, Zhan, & Herrera-Viedma, 2022*). Let $F^* = \{A, B, V, f\}$ be an FIIS, where $A = \{a_1, a_2, \ldots, a_n\}$ and $D = \{d_1, d_2, \ldots, d_n\}$. Thus, $Y = \{O, \neg O\}$ is represented as:

$$O = \{a_i \in A \mid d_i > |VD|\}, \neg O = \{a_i \in A \mid d_i \leq |VD|\}. \tag{10}$$

where $|V_D|$ indicates the domain of all objects under the choice characteristic, and d_i indicates the integer value of the choice characteristic corresponding to item a_i, which increases from 1.

Example 3.3. Building upon Example 3.1, according to Definition 3.1, the two states, O and $\neg O$, for Table 4 are calculated as follows: $O = \{a_1, a_2, a_5, a_6, a_7, a_9\}$; $\neg O = \{a_3, a_4, a_8, a_{10}\}$.

Remark 3.1. For an FIIS $F^* = \{A, B, V, f\}$, the set $Y = \{O, \neg O\}$ must satisfy the following two conditions:

$O \cup \neg O = A, \quad O \cap \neg O = \emptyset.$

The classical conditional probability formula $Pr(O|[a_i]_B^{\leq\alpha}) = \frac{|O \cap [a_i]_{\leq B}\alpha|}{|[a_i]_{\leq B}\alpha|}$, is known to us based on Definition 3.5. The weighting function of the theory of prospects and the conditional likelihood allow us to construct the weighted function of this article as follows:

$$\omega_i = \begin{cases} \omega^+\left(Pr(O|[a_i]_B^{\leq\alpha})\right) = \dfrac{Pr(O|[a_i]_B^{\leq\alpha})^{\varphi}}{\left(Pr(O|[a_i]_B^{\leq\alpha})^{\sigma} + (1 - Pr(O|[a_i]_B^{\leq\alpha}))^{\sigma}\right)^{1/\varphi}}, \\ \omega^-\left(Pr(\neg O|[a_i]_0^{\leq\alpha})\right) = \dfrac{Pr(-O|[a_i]_B^{\leq\alpha})^{\theta}}{\left(Pr(O|[a_i]_B^{\leq\alpha})^{\delta} + (1 - Pr(-O|[a_i]_B^{\leq\alpha}))^{\delta}\right)^{1/\theta}}. \end{cases} \tag{11}$$

where ϕ and θ represent the effects of increasing and decreasing weights on gains, and $\phi, \theta \in (0, 1)$. This weighting function is calculated based on the decision attributes in the information, thus obtaining the target weighting function. The advantage of this weighting function is that it is more objective than existing weighting functions, avoiding decision risks brought about by subjectivity.

Based on the classical TWD theory of prospect theory, we consider that different decision options have their own unique value functions, and different attributes have different weight values. As a result, we have the more thorough formula that follows to show the value functions for every attribute:

$$U_{\Delta\partial}^j(a_i) = \begin{pmatrix} U_{PP}^j(a_i) & U_{PN}^j(a_i) \\ U_{BP}^j(a_i) & U_{BN}^j(a_i) \\ U_{NP}^j(a_i) & U_{NN}^j(a_i) \end{pmatrix} = \begin{pmatrix} \left(\frac{a_{ij} - a_{\min}^j}{a_{\max}^j - a_{\min}^j}\right)^{\omega} & 0 \\ \mu\left(\frac{a_{ij} - a_{\min}^j}{a_{\max}^j - a_{\min}^j}\right)^{\omega} & \mu\left(\frac{a_{\max}^j - a_{ij}}{a_{\max}^j - a_{\min}^j}\right)^{\omega} \\ 0 & \left(\frac{a_{\max}^j - a_{ij}}{a_{\max}^j - a_{\min}^j}\right)^{\omega} \end{pmatrix} \tag{12}$$

In Formula (12), a_{ij} represents the assessment outcome for every object a_i beneath every conditional characteristic b_j. The minimum and maximum evaluation values for each condition attribute are denoted, respectively, by a_{min}^j and a_{max}^j. μ denotes the loss aversion coefficient and fulfills $\mu \in [0, 1]$, whereas ω represents the sensitivity declining level of the value function and fulfills $\omega \in (0.5, 1)$.

In cases where an object a_i lacks its evaluation value, the object's general value function can be assessed using the weight values assigned to the given assessment attribute and the evaluation values that are currently available. The following figure illustrates this value function's expression:

$$U_{\Delta\partial}(a_i) = \begin{pmatrix} U_{PP}(a_i) & U_{PN}(a_i) \\ U_{BP}(a_i) & U_{BN}(a_i) \\ U_{NP}(a_i) & U_{NN}(a_i) \end{pmatrix} = \begin{pmatrix} \sum_j^m \mathbf{W}_j \left(\frac{a_{IJ} - a_{\min}^j}{a_{\max}^j - a_{\min}^j}\right)^{\omega} & \mathbf{0} \\ \sum_j^m \mathbf{W}_j \mu \left(\frac{a_{IJ} - a_{\min}^j}{d_{\max}^j - d_{\min}^j}\right)^{\omega} & \sum_j^m \mathbf{W}_j \mu \left(\frac{d_{\max}^j - a_{ij}}{d_{\max}^j - d_{\min}^j}\right)^{\omega} \\ 0 & \sum_j^m \mathbf{W}_j \left(\frac{a_{\max}^j - a_{ij}}{a_{\max}^j - a_{\min}^j}\right)^{\omega} \end{pmatrix} \tag{13}$$

where $\mu \in (0.5, 1)$.

Table 6 lists the value functions for all objects:

Remark 3.2. According to the semantic interpretation of the value functions $U_{\Delta\partial}(a_i)$ under attribute b_j, where $\Delta = P, B, N$ and $\partial = P, N$, the value functions $U_{\Delta\partial}(a_i)$ satisfy the following two conditions: $U_{PP}^j \geq U_{BP}^j > U_{NP}^j$, $U_{PN}^j \geq U_{RN}^j > U_{NN}^j$.

Table 6
The value functions for all objects.

	$U_{PP}(a_i)$	$U_{BP}(a_i)$	$U_{NP}(a_i)$	$U_{PN}(a_i)$	$U_{BN}(a_i)$	$U_{NN}(a_i)$
a_1	$U_{PP}(a_1)$	$U_{BP}(a_1)$	$U_{NP}(a_1)$	$U_{PN}(a_1)$	$U_{BN}(a_1)$	$U_{NN}(a_1)$
a_2	$U_{PP}(a_2)$	$U_{BP}(a_2)$	$U_{NP}(a_2)$	$U_{PN}(a_2)$	$U_{BN}(a_2)$	$U_{NN}(a_2)$
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$
a_n	$U_{PP}(a_N)$	$U_{BP}(a_n)$	$U_{NP}(a_n)$	$U_{PN}(a_n)$	$U_{BN}(a_n)$	$U_{NN}(a_n)$

For all $a_i \in A$, $U(h_\Delta|[a_i])$ $(\Delta = P, B, N)$ represents the expected value of taking action h_Δ. The expression is as follows:

$$U(B_p|[a_i]) = U_{PP}(a_i)\omega^+(Pr(O|[a_i]_0^{\leq * \alpha})) + U_{PN}(a_i)\omega^-(Pr(\neg O|[a_i]_0^{\leq * \alpha}))$$
$$U(B_b|[a_i]) = U_{BP}(a_i)\omega^+(Pr(O|[a_i]_0^{\leq * \alpha})) + U_{BN}(a_i)\omega^-(Pr(\neg O|[a_i]_0^{\leq * \alpha}))$$
$$U(B_n|[a_i]) = U_{NP}(a_i)\omega^+(Pr(O|[a_i]_0^{\leq * \alpha})) + U_{NN}(a_i)\omega^-(Pr(\neg O|[a_i]_0^{\leq * \alpha})). \quad (14)$$

Based on Bayesian decision theory, the decision rules with the principle of maximum expected value are represented as follows:

(P1) Decide $a_i \in \mathrm{POS}(O)$, if $U(B_p|[a_i]) > U(B_b|[a_i])$ and $U(B_p|[a_i]) > U(B_n|[a_i])$.
(B1) Decide $a_i \in \mathrm{BOU}(O)$, if $U(B_b|[a_i]) > U(B_p|[a_i])$ and $U(B_b|[a_i]) > U(B_n|[a_i])$.
(N1) Decide $a_i \in \mathrm{NEG}(O)$, if $U(B_n|[a_i]) \geq U(B_b|[a_i])$ and $U(h_R|[a_i]) \geq U(B_p|[a_i])$.

Finally, we rank the decision categories according to priority and expected value as follows: $\mathrm{POS}(O) > \mathrm{BOU}(O) > \mathrm{NEG}(O)$.

3.2. PT-TWD-PPTDR model algorithm

The detailed algorithm of the PT-TWD-PPTDR model is presented in Algorithm 1.

Remark 3.3. The temporal complexity of the algorithm is described in the text below, where n denotes the number of elements in the object set and m denotes the number of members in the condition attribute set, in order to demonstrate the method's efficiency. With a temporal complexity of $O(1)$, we use Formula (7) to calculate the normalized data for each object in Step 1. The maximum time complexity in Step 2, based on Definitions 3.1-3.3, is $O(n^3m)$ for calculating the prior probability tolerance dominant class for each object a_i. The temporal complexity for calculating the maximum deviation weight ω_j for each attribute b_i's prior probability tolerance dominance class in Step 3 is $O(n^2m)$, per Definition 3.4. In Step 4, $O(n^2m)$ is the time complexity for calculating the weight function for each item based on Formula (8). The time complexity to acquire the value function for each item in Step 5 is $O(nm)$, based on Formula (11). The time complexity in Step 6 (using Formula (12)) is $O(nm)$ for calculating the anticipated value for each item in each domain. Step 7 has a time complexity of $O(n)$ for categorizing each item based on (P1)-(N1). In Step 8, $\mathrm{POS}(O) > \mathrm{BOU}(O) > \mathrm{NEG}(O)$ is the order of predicted values in the distinct domains, where $O(n \log n)$ is the time complexity for sorting every item. Therefore, the total time complexity of the algorithm proposed in this paper is $O(n^3m)$.

Remark 3.4. In order to provide an objective weight calculation approach, this research first defines the prior probability tolerance dominance connection of FIIS. The influence of unique psychological variables on the decision outcomes is then considered in conjunction with PT, and ultimately the predicted values of every object are obtained, along with ranking and classification. Based on the aforementioned findings, our approach can effectively reduce choice risks by classifying all objects into three categories while taking delayed decision-making into account. This means that the approach presented in this research can handle choice issues with potential missing assessment values and take the psychological states of the decision-makers into account. Different decision-makers can obtain varying results by adjusting the settings, which demonstrates the broad applicability of the PT-TWD-PPTDR approach.

Algorithm 1: The PT-TWD-PPTDR Model in an FIIS

Input: A MADM problem and five parameters $\alpha, \phi, \theta, \omega, \mu$
Output: The sorting and classification of all alternatives
Initialization;
Given $\alpha \in [0,1]$, $\phi \in (0,1)$, $\theta \in \{0,1\}$, $\omega \in (0.5,1)$, $\mu \in [0,1]$;
for $i = 1$ *to* n, $j = 1$ *to* m **do**
  Calculate: normalized data for each object. // by Formula (7);
for $i = 1$ *to* n, $l = 1$ *to* n, $j = 1$ *to* m, $k = 1$ *to* `length(unique[`a_{ij}`])` **do**
  Calculate: every object's prior probability tolerance dominance category. // by Definitions 3.1–3.3;
for $i = 1$ *to* n, $j = 1$ *to* m, $k = 1$ *to* n **do**
  Calculate: max deviation weight ω_j based on prior probability tolerance dominance category for every characteristic b_j. // by Definition 3.4;
for $j = 1$ *to* m **do**
  Calculate: weight function of each object. // by Formula (9);
for $i = 1$ *to* n **do**
  Calculate: value function of each object. // by Formula (11);
for $i = 1$ *to* n **do**
  Calculate: expected value of each object. // by Formula (12);
Obtain $POS(O)$, $BOU(O)$, $NEG(O)$;
for $i = 1$ *to* n **do**
  Determine: domain where object is located according to decision rule (P1)-(N1) and calculate the expected values;
  if $U(B_p \mid [a_i]) > U(B_b \mid [a_i])$ *and* $U(B_p \mid [a_i]) > U(B_n \mid [a_i])$ **then**
    $a_i \in POS(O)$, $U(a_i) = U(B_p \mid [a_i])$;
  if $U(B_b \mid [a_i]) > U(B_p \mid [a_i])$ *and* $U(B_b \mid [a_i]) > U(B_n \mid [a_i])$ **then**
    $a_i \in BOU(O)$, $U(a_i) = U(B_b \mid [a_i])$;
  if $U(B_n \mid [a_i]) \geq U(B_b \mid [a_i])$ *and* $U(h_R \mid [a_i]) \geq U(B_p \mid [a_i])$ **then**
    $a_i \in NEG(O)$, $U(a_i) = U(B_b \mid [a_i])$;
for $i = 1$ *to* n **do**
  Prioritizing $POS(O) > BOU(O) > NEG(O)$, all alternatives are separated according to the predicted values of each domain's alternatives;
return classification and sorting of all objects;

4. Application case

In Section 3, we constructed a new PT-TWD-PPTDR model to solve the MADM problem. From Definition 3.2, it can be observed that this model mainly addresses the decision problem in the FIIS. In order to verify the practicality and applicability of this method, this section will analyze a real case of heart disease from the UCI repository.

Heart Disease: Heart disease (Heenan, Parks, Bärnighausen, Kado, Bloom, & Steer, 2020) refers to a series of diseases caused by abnormal heart structure and function. It encompasses various cardiovascular diseases, including coronary heart disease, myocardial disease, arrhythmia, and heart valve diseases. Nowadays, heart disease has become the leading killer endangering human health. According to the statistics from the World Health Organization, over one-third of global deaths are caused by heart disease. Therefore, it is crucial to prevent heart disease, and patients with heart disease need regular check-ups. Echocardiography (Hackett & Chin, 2021) is a non-invasive examination method that uses sound waves to observe and evaluate the structure and function of the heart. It plays a crucial role in diagnosing and monitoring heart disease. It can display the structure of the heart, helping doctors determine the type and severity of heart disease, as well as assess the contraction and relaxation function of the heart. Echocardiography can also be used for screening high-risk populations. By regularly undergoing echocardiography examinations, early signs of heart disease can be detected,

Table 7
Categorization of every option in echocardiography.

Domains	Classifications
POS(O)	$a_1, a_{12}, a_4, a_8, a_{10}, a_{11}, a_{18}, a_{23}, a_{25}, a_{27}, a_{29}, a_{32}, a_{34}, a_{35}, a_{36}, a_{38}, a_{39}, a_{40}$ $a_{46}, a_{47}, a_{52}, a_{54}, a_{55}, a_{58}, a_{59}, a_{62}, a_{63}, a_{64}, a_{66}, a_{67}, a_{68}, a_{70}$
BOU(O)	$a_3, a_5, a_6, a_7, a_9, a_{12}, a_{15}, a_{16}, a_{17}, a_{19}, a_{20}, a_{21}, a_{22}, a_{24}, a_{26}, a_{28}, a_{30}, a_{31}$ $a_{37}, a_{41}, a_{42}, a_{43}, a_{45}, a_{48}, a_{49}, a_{50}, a_{51}, a_{53}, a_{56}, a_{57}, a_{60}, a_{61}, a_{65}, a_{69}$
NEG(O)	$a_{13}, a_{14}, a_{33}, a_{44}, a_{71}, a_{72}$

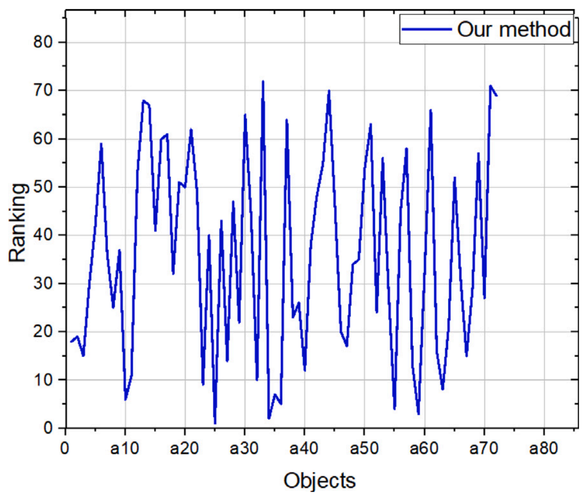

Fig. 2. The outcome of echocardiography ranking.

and preventive measures or timely treatment can be taken. However, treatment for heart disease can be difficult to find early on. The reasons include the possible loss of patient examination data results during storage or migration processes, resulting in incomplete information in diagnosis. Secondly, patient examination results are susceptible to their own emotions, as decision-making may involve some uncertainty and risk.

4.1. An example of lacking information: An echocardiography

This section will use an incomplete dataset from the UCI repository (http://archive.ics.uci.edu/m-l/datasets/Echocardiogram) to test the PT-TWD-PPTDR approach described in this research. The background is as follows: a hospital plans to diagnose patients with heart problems using echocardiography. We selected a dataset that has five conditional features and seventy-two items, where each item has a maximum of two missing evaluation values, while the decision attributes have no missing values, facilitating experimentation and analysis. Therefore, the dataset consists of a total of 72 objects, with each object potentially containing up to two missing evaluation values. We address this problem by using the approach suggested in this work. The results of the classification are shown in Fig. 3, the ranking results are displayed in Fig. 2, and Table 7 is presented below.

The ranking results obtained from applying our method to the dataset are displayed in Fig. 2. Here, we can see that the a_{25} includes the greatest overall ranking, indicating that this item has the highest probability of having heart disease is the 25th. The actual situation of applying this approach to categorize all items into three domains is represented in Fig. 3 and Table 7, along with the number of items included in each sector. As a result, the approach we propose could assist in resolving medical problems in FIIS.

5. The comparison of echocardiogram

The approach can effectively address FIIS decision issues, according to the analysis above. To prove the superiority and efficacy of this

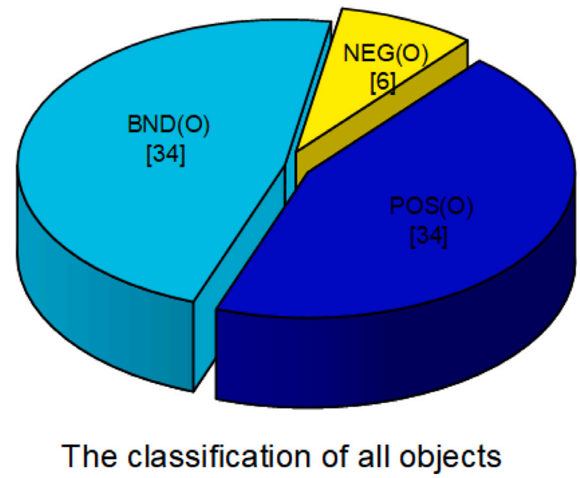

Fig. 3. The echocardiography categorization result.

strategy, we further compare it with different approaches utilizing databases of echocardiography in this section.

5.1. Comparison and analysis of ranking results

The purpose of the upcoming comparative experiments is to evaluate the ranking outcomes produced by various FIIS algorithms. First, as seen in Fig. 4, we compare the ranking results achieved in this paper's Section 4.1 with those obtained by Wang et al. (2023) It is evident that the ranking outcomes produced by the methodology suggested in this research are comparable to those produced by FIIS. The two approaches have different theoretical bases, which results in somewhat different rankings, but the best outcome stays the same. Therefore, it can be said that the approach suggested in this work works well.

In addition, this method is compared with the methods proposed by Zhu et al.'s methond (Zhu, Ma, Zhan, & Yao, 2022), Wang et al.'s methond (Wang et al., 2022), Zhang et al.'s methond (Zhang & Fan, 2012), Liu et al.'s methond (Liu and Zhu and Liu, 2014), J.C.R. Alcantud et al.'s methond (Alcantud, de Andrés Calle, & Torrecillas, 2016), and Liu et al.'s methond (Liu, Zhu, & Liu, 2014). Fig. 5 shows the comparison of rankings between the method proposed in this paper and these other methods. Simultaneously, it can also be observed that our method yields the same optimal results as these approaches. In addition, for the qualitative assessment mentioned above, we will conduct a quantitative evaluation, thus introducing the Spearman rank correlation coefficient (SRCC) (Gauthier, 2001), which displays the correlation between variables. The calculation formulas for the two ranking results are as follows:

$$SRCC = 1 - \frac{6\sum_{i=1}^{n}(y_i - x_i)^2}{n^3 - n} \tag{15}$$

The correlation between two samples is compared using Spearman's rank correlation threshold table and the SRCC value. According to the SRCC critical value table, with an average sample size of 100 and a level of significance of 0.01 in the two-tailed test, a strong correlation between the two identical samples is indicated if the SRCC value obtained by comparing them is more than 0.257.

It is clear that when the threshold table has larger sample sizes, there is less of a requirement to compare the SRCC values from two samples. However, an absolute value of SRCC greater than or equal to 0.6847 is often seen as indicating a strong correlation when the sample size is limited. The SRCC values produced using this method and those acquired using alternative methods are compared in this research. The results of our approach and the other techniques' SRCC values are

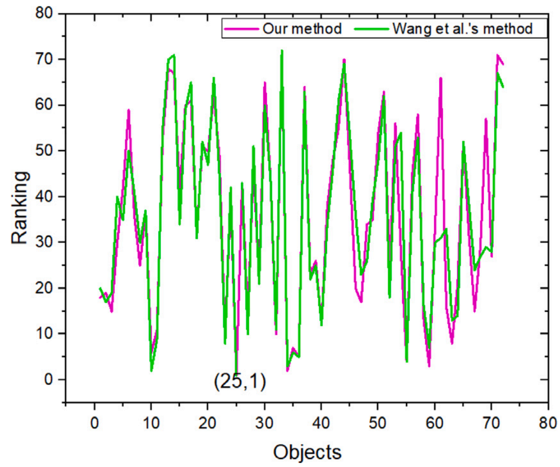

Fig. 4. A comparative analysis of Wang et al.'s technique ranking in FIIS.

all larger than or equal to 0.6847, as shown in Fig. 6 (Wang et al.'s method is denoted as Wang and others (2022), Liu et al.'s method is denoted as Liu et al. (2014)). This suggests that there is a great degree of consistency between the ranking outcomes produced by these approaches and the results gained by our method.

Furthermore, our technique and the other methods had higher SRCC values than Wang et al.'s method (Wang et al., 2023). The findings show that our technique's ranking performance is noticeably better than Wang et al.'s (Wang et al., 2023) method. To sum up, our approach shows some superiority in effectiveness.

5.2. Comparative examination of the classification outcomes

In this section, we will compare our method's categorization results with those of other methods. Other approaches include those presented by Wang et al. (2023), Wang and others (2022), and Zhu et al. (2022). Our method splits all objects into three regions. From Fig. 7, we can clearly see that (the second column is Wang et al. (2023), and the fourth column is Wang and others (2022)), with the exception of Wang et al.'s method (Wang & others, 2022), all objects can be classified into three regions using our method, Wang et al.'s method (Wang et al., 2023), and Zhu et al.'s method (Zhu et al., 2022). This is consistent with the reasonable semantic interpretation of the three regions in Three-Way Decision (TWD) in decision theory and rough sets. TWD takes into account boundary regions, or delayed decision-making, where objects may have an ambiguous categorization because they are at the junction of positive and negative areas. These areas have practical applications that aid in the understanding and classification of data, particularly when dealing with ambiguous and imprecise information. As a result, while handling decision-making challenges, our approach takes practicality into account more, showing both fault tolerance and pragmatism. Next, we examine the misclassification rates acquired by different approaches, as shown in Fig. 8, where our method yields a lower misclassification rate than other methods. It is obvious that the model performs better the lower the misclassification percentage. The following formula may be used to get the misclassification percentage:

$$\text{Misclassification rate} = \frac{n_{O\to \mathrm{NEG}(O)} + n_{\neg O\to \mathrm{POS}(O)}}{|A|} \tag{16}$$

Where $n_{O\to \mathrm{NEG}(O)}$ represents the number of objects belonging to the negative region $\mathrm{NEG}(O)$ of O, $n_{\neg O\to \mathrm{POS}(O)}$ represents the number of objects belonging to the positive region of $\neg O$, and $|A|$ represents the total number of objects.

5.3. Comparison among decision theories

Our method and Wang et al.'s method (Wang & others, 2022) combine prospect theory in decision theory, while the other methods, Wang et al.'s method (Wang et al., 2023) and Zhu et al.'s method (Zhu et al., 2022), integrate regret theory. Both are significant theories in decision-making. In the field of decision theory, numerous studies have explored the superiority of prospect theory over regret theory and provided evidence supporting prospect theory.

Prospect theory focuses on the relative changes in gains and losses rather than their absolute values, making it more sensitive to gains and losses. It emphasizes individuals' sensitivity to potential losses, known as the loss aversion effect. This sensitivity makes prospect theory more realistic in explaining decision-making behaviors since, in many cases, individuals tend to focus more on losses than gains. Kahneman (1979), Kahneman and Tversky (1984), Tversky and Kahneman (1992) conducted a series of experiments to investigate individuals' cognitive processes regarding uncertainty and verified the effectiveness of prospect theory in explaining decision-making behaviors. Their research found that individuals are more concerned about potential losses, exhibiting loss aversion effect, thus providing higher accuracy and explanatory power in describing and predicting decision-making behaviors.

On the other hand, regret theory primarily focuses on comparing decision outcomes with possible alternative solutions, overlooking individuals' actual feelings toward decision outcomes. It assumes that individuals make decisions by comparing the degree of regret, but in reality, individuals may be influenced by emotions, attitudes, and external factors during decision-making. Regret theory does not emphasize individuals' aversion to losses and risk-averse behaviors. It mainly focuses on the degree of regret regarding choices already made, overlooking individuals' emotional experiences during the decision-making process, i.e., sensitivity to gains and losses. Therefore, regret theory lacks in addressing the emotional dimension of individuals' decision-making behaviors compared to prospect theory. Regret theory does not consider individuals' cognitive processes regarding uncertainty as prospect theory does. It assumes that individuals can accurately assess the degree of regret for different decision choices, but when facing uncertainty, individuals often make irrational decisions, which is a limitation of regret theory.

Hence, our method, combined with prospect theory, exhibits higher accuracy and explanatory power in explaining and predicting individuals' decision-making behaviors.

5.4. Discussion

In this study, we constructed a PPTDR model in the context of FIIS, introducing an objective method to obtain weights, which was then combined with PT to consider the psychological states of decision-makers. This method derived the calculation methods for weight functions and value functions of decision objects, ultimately developing an algorithm to determine the expected values of all objects for sorting and classification. Nevertheless, some approaches exclusively deal with decision-making in fully integrated information systems, neglecting incomplete information or the emotional impact of decision-makers' choices. As a result, we thoroughly compared our approach to nine different approaches. We evaluated and synthesized our approach with the other nine techniques based on the table in a number of different ways:

(1) Our approach can efficiently resolve making choices issues in full information systems and FIIS. As shown in Table 8, only our method and those proposed by Liu et al. (2016), Zhan et al. (2021), and Yang and Li (2020) can handle problems in FIIS, whereas other methods are limited to complete information systems. This capability is crucial in the era of big data where missing data is common due to losses during data acquisition and storage.

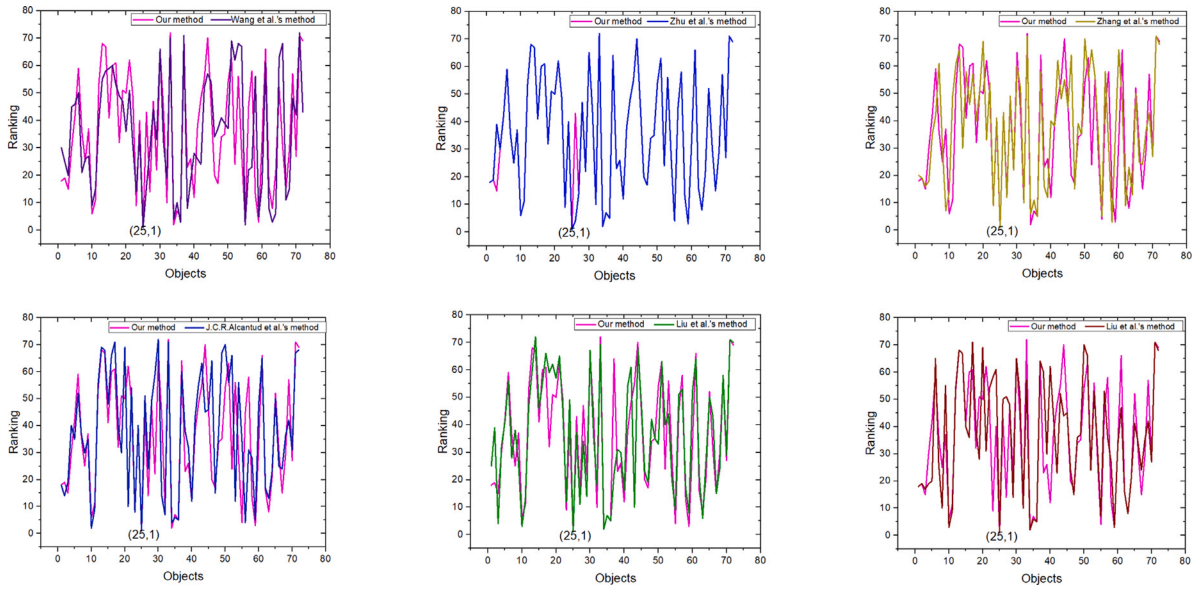

Fig. 5. Analysis of the rankings of various approaches in FIIS.

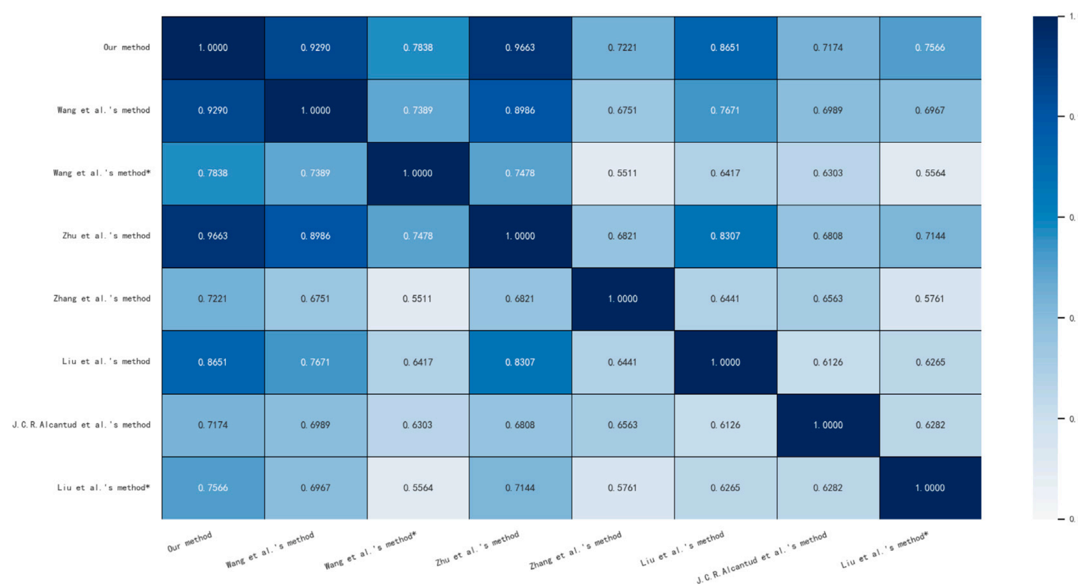

Fig. 6. SRCCs of the ranking outcomes acquired in FIIS using various techniques.

(2) From Table 8, it can be observed that only four methods can be used for decision-making in FIIS. Yang et al.'s (Yang & Li, 2020) and Liu et al.'s (Liu et al., 2016) methods classify objects without ranking them. Zhan et al.'s (Zhan et al., 2021) method can sort and classify objects but relies on known fuzzy decision attribute values, which are often unavailable in real-world scenarios, limiting its applicability. Our method calculates the expected value based on evaluation values of all objects, enabling both sorting and classification. Thus, our method not only classifies objects but also ranks them, providing a complete ranking that aids decision-makers in making informed decisions.

(3) Our method calculates the target state set based on the decision attributes of the dataset. The conditional probability values are derived using the classic conditional probability formula to calculate the weight function. This process minimizes the influence of subjective preferences on decision results, as the conditional probability is objectively obtained. Additionally, our method fully utilizes the dataset's information, enhancing its effectiveness.

(4) Only our approach, Bell's approach (Bell, 1982), and Wang et al.'s approach (Wang et al., 2020) combine PT and RT to take decision-makers' psychological states into account. This consideration aligns better with actual decision-making situations, making these methods more effective in practice. Our method also calculates the maximum deviation weight of all objects based on classical conditional probability, dividing objects into two states and three behaviors in TWD, which effectively addresses behavioral decision problems in FIIS.

By analyzing these differences, we summarize the advantages of our method. Our method effectively solves decision problems in FIIS by accommodating missing data in information systems. It not only

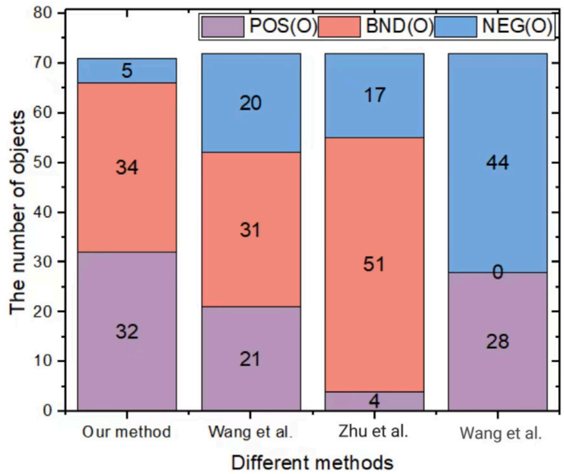

Fig. 7. Analysis of how various FIISs techniques are classified in comparison.

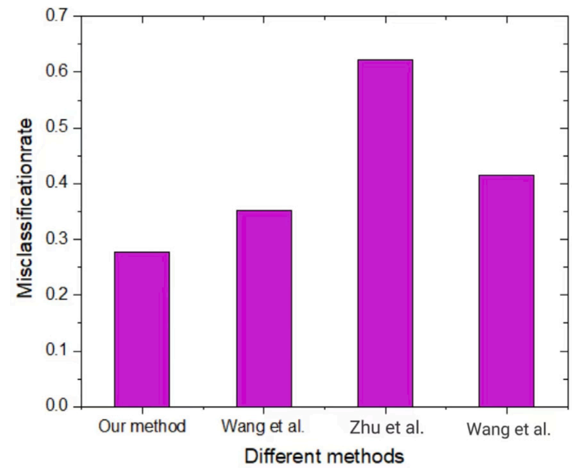

Fig. 8. Rate of misclassification for various techniques in FIIS.

classifies but also ranks all objects, providing a comprehensive decision-making tool. By incorporating the psychological attitudes of decision-makers, it addresses behavioral decision problems in FIIS. Additionally, it derives maximum deviation weights and categorizes objects into states and behaviors based on conditional probability, ensuring robust decision-making.

6. Analytical experimentation

In this part, we conduct a study using the echocardiography case mentioned above to analyze the sensitivity of the model through parameter analysis. This paper involves five parameters: α, ϕ, θ, μ, and ω. We will analyze α and μ in three aspects.

6.1. The sensitivity analysis of the parameter α

We investigate the effect of changing parameter α on the choice outcomes while its value is set at 0.1. The variations in the sorting and classification outcomes are shown in Figs. 9 and 10, respectively.

It is evident from Fig. 9 that the general trend remains constant and that changes in parameter α have minimal impact on the sorting outcomes of all items. Furthermore, the ideal result obtained for the 25th object is likewise consistent. As α grows, the number of items in the positive and border regions steadily rises, whereas the number of objects in the negative region gradually falls, as shown in Fig. 10. This trend reflects the impact of the α parameter on the stringency of classification. That is, a lower α value leads to stricter classification, but may cause the model to excessively exclude uncertain objects, thereby increasing classification error; conversely, a higher α value relaxes the classification criteria, which may reduce classification accuracy in some scenarios. Therefore, selecting an appropriate α value is crucial for balancing classification accuracy and generalization ability.

6.2. The sensitivity analysis of the parameter μ

Parameters α, ϕ, θ, and ω have fixed values of 0.1, 0.61, 0.69, and 0.88, respectively. We observed that all items maintained their ranking while the value of parameter μ was varied. As a result, as seen in Fig. 111, we examined how item categorization changed as the parameter μ changed. We limited our analysis to the impact of parameter μ on overall object categorization. Findings indicate that while the number of items in the positive and negative zones decreases, the number of items in the border zone increases as the value of parameter μ rises. In the experiment, changes in the parameter μ directly affect the distribution of objects in the boundary region and the positive and negative regions. The parameter μ reflects the model's sensitivity to loss. When μ is large, the model tends to make conservative decisions, increasing the proportion of the boundary region and thus enhancing fault tolerance in high-risk scenarios; however, an excessively high μ value can reduce the model's ability to identify objects in the positive region. Therefore, the selection of μ should be based on the risk preference in the decision-making scenario to avoid potential suppression of profitable decisions.

6.3. The sensitivity analysis of parameters μ and α

We note that when the values of α and μ vary concurrently, the categorization of all objects changes when the values of parameters ϕ, θ, and ω are fixed at 0.61, 0.69, and 0.88, respectively. The number of items in the positive zone progressively decreases as these two parameters are increased, whereas the number of objects in the negative region and boundary region gradually grows, as seen in Fig. 12. The experimental results show that the joint variation of α and μ has a significant impact on the classification results. Particularly, when α and μ are increased, the number of objects in the negative and boundary regions both increases. This phenomenon indicates that the interaction between α and μ may cause the model to become risk-averse under high tolerance, especially in situations with high data uncertainty, which expands the boundary region to enhance robustness but may also reduce the model's ability to identify objects in the positive region. Therefore, the α and μ parameters need to be jointly tuned in different application scenarios to achieve a balance between risk management and decision accuracy.

6.4. Parameter tuning and analysis

In incomplete information systems, model performance is significantly influenced by parameter settings, making meticulous parameter tuning a crucial step in enhancing decision accuracy and robustness. Firstly, it is necessary to clarify the roles and expected ranges of key parameters in the model (such as α, μ, ϕ, θ, and ω), and set reasonable initial values for each parameter based on relevant literature and domain knowledge. For example, the value range of the α parameter can be set to [0, 1], while the value range of μ is adjusted according to the intensity of loss aversion. Secondly, cross-validation techniques (Kohavi, 1995) can be used to effectively evaluate the model's performance under different parameter settings. Specifically, the datasets can be divided into k subsets (usually 5 or 10), with $k-1$ subsets used for training the model each time, and the remaining subset used for validation. By repeating this process k times, ensuring that

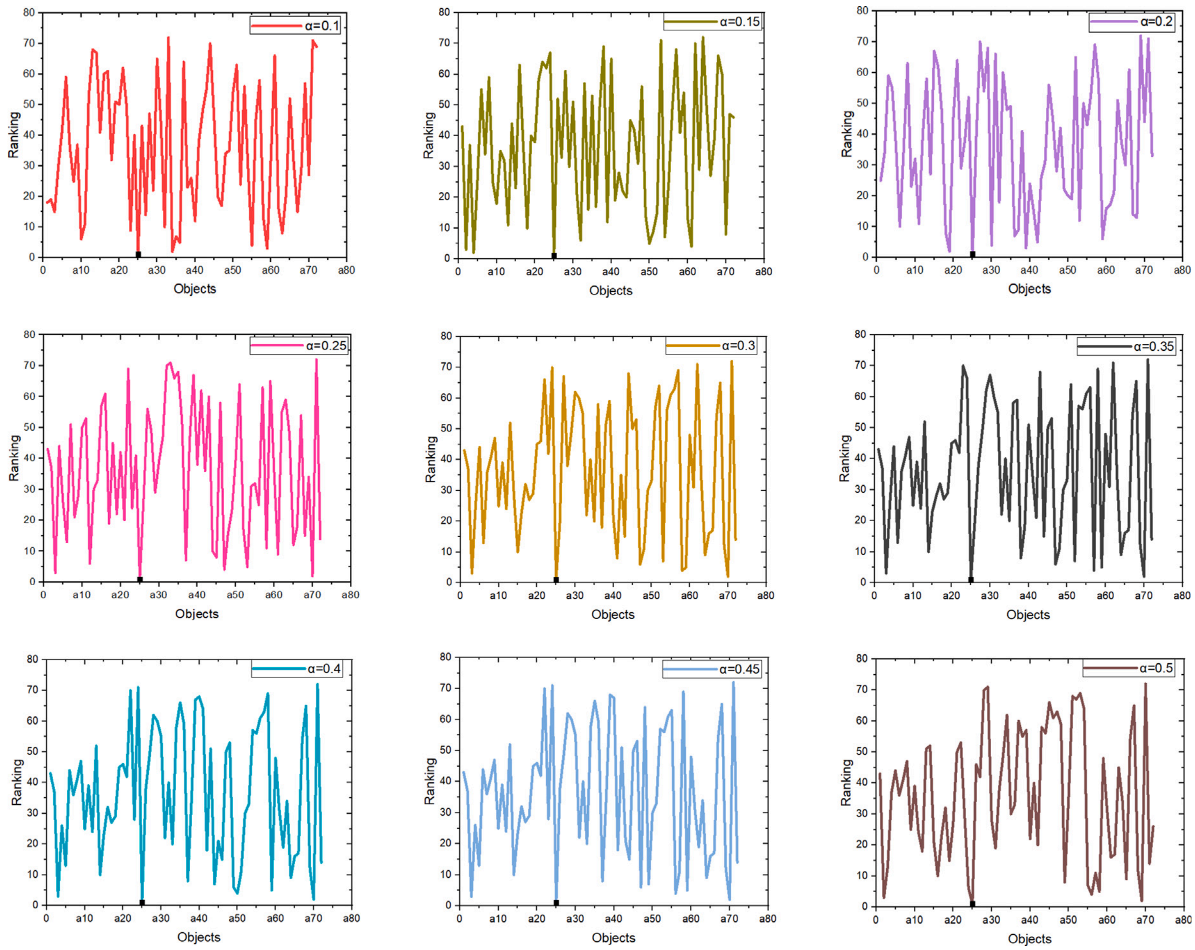

Fig. 9. Ordering of all items when α changes in value.

Table 8
Comparing several approaches.

Methods	Incomplete information	Decision characteristics	Objective weights/Value functions	Loss/Utility decisions	Behavior	Ranking	Classification
Our method	√	√	√	√	√	√	√
Wang et al. (2020) et al.'s method	×	×	×	√	√	√	√
Wang and others (2022) et al.'s method	×	×	√	×	√	√	√
Zhu et al. (2022) et al.'s method	√	√	×	√	√	√	√
Zhang and Fan (2012) et al.'s method	√	×	√	×	×	√	×
Liu and Zhu and Liu (2014) et al.'s method	×	√	×	×	×	√	×
Alcantud et al. (2016) et al.'s method	×	√	×	√	×	√	×
(Hwang & Tilley, 1981) approach	×	×	×	√	×	√	×
(Ghorabaee, Zavadskas, Olfat, & Turskis, 2015) approach	×	×	×	×	×	√	×
(Harsanyi, 1955) approach	×	×	×	×	×	√	×

each subset serves as a validation set, the average performance metrics (such as accuracy, recall, and F1 score) across all validation processes can be calculated to select the optimal parameter combination.

Based on the results of cross-validation, the grid search method (Hutter, Kotthoff, & Vanschoren, 2019) can systematically explore the parameter space. A predefined grid of parameter combinations (for example, setting multiple values for α such as 0.1, 0.2, 0.3, etc., and different loss aversion levels for μ) can be used for cross-validation to help discover the best parameter combination, enabling the model to exhibit optimal decision-making capabilities in incomplete information systems. Additionally, as a complement to grid search, random search (Bergstra & Bengio, 2012) can experiment with randomly selected parameter combinations in the parameter space. Compared to grid search, random search is more efficient in large-scale parameter spaces, especially suitable for situations with a large number of parameters.

Finally, after tuning, we need to conduct in-depth analysis and visualization of the results from different parameter combinations to identify which parameters have the greatest impact on model performance (Sokolova & Lapalme, 2009). Methods such as learning curves or parameter importance plots can be used to visually showcase the contribution of each parameter to model decisions, providing valuable insights for subsequent model improvements. At the same time, after the model is deployed, we should continuously monitor its performance in real-world applications, collect feedback data, and adjust relevant parameters or introduce new features as needed to continuously enhance the model's decision-making capabilities.

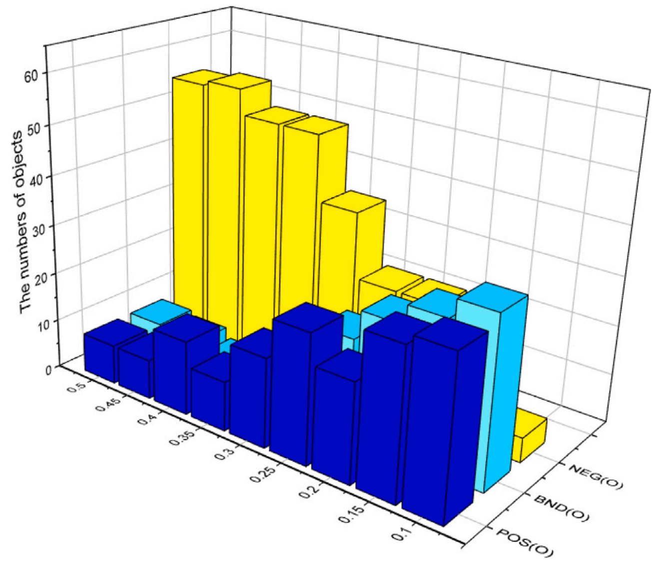

Fig. 10. Entire object classification when α parameter values vary.

Through these systematic parameter tuning methods, we can significantly enhance the model's ability to cope with ambiguity and uncertainty in incomplete information systems, thereby ensuring its stability and practicality in complex application scenarios.

7. Conclusion and future works

In the information age, data frequently tends to be lost or ignored during storage and utilization, leading to a growing interest in behavioral decision-making among academics. Consequently, managing decision difficulties in incomplete information systems (FIIS) has become critically important. In this study, we have examined the PT-TWD-PPTDR approach within FIIS to address these challenges, resulting in several significant advancements. Firstly, we present a PT-TWD-PPTDR approach to resolve decision problems involving missing information, successfully handling partial datasets that arise from the probability of data loss or omission during data collection or storage. Secondly, we employ a PPTDR to manage binary relationships between objects more efficiently. Previous TWD techniques in FIIS often required additional relationships formed through the expansion of equivalency relations, such as tolerance and similarity relations, which improved the handling of missing values. Lastly, TWD typically incorporates two states and three behaviors, with conditional probability and loss function being two critical considerations. We recognize that decision-makers are frequently influenced by varying psychological states and tend to make different decisions, as the loss function in current approaches is either subjectively provided by decision-makers or computed as relative loss functions. Thus, decision psychology must be integrated into the process. This study introduces prospect theory to determine the predicted values of each item for ranking and classification, emphasizing the acquisition of weight and value functions.

Several important and practical issues can be explored in future research based on the framework established in this paper. Our approach integrates prospect theory with TWD, accounting for the psychological states of decision-makers and their effects on outcomes. Future research could delve into additional behavioral decision-making theories, such as regret theory. Another avenue for enhancing decision resilience involves combining TWD with cumulative prospect theory or third-generation prospect theory. Expanding the method's applicability to encompass both single and group decision settings would enrich its relevance in group decision information systems. Additionally, adapting the dominance relation-based TWD technique to dynamic information systems presents a prospective research direction, considering the dynamic nature of contemporary data and information. Moreover, extending the technique to manage missing values across various information systems, including fuzzy, multi-scale, intuitionistic fuzzy, and interval-valued systems, would enhance its adaptability and utility in a multitude of real-world contexts.

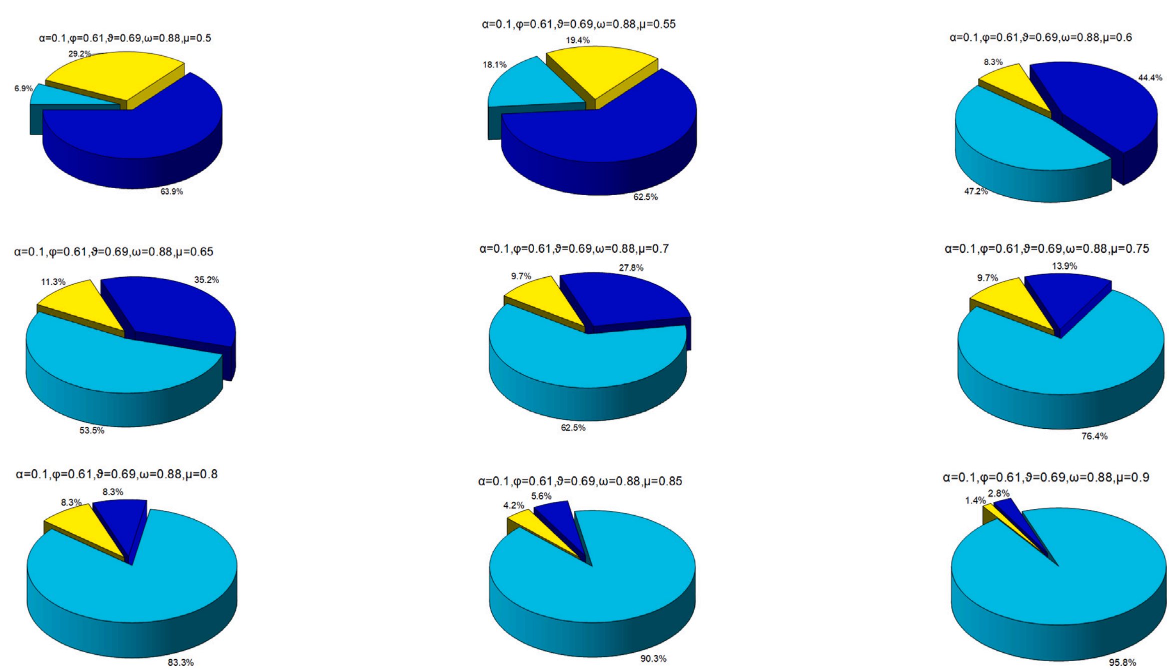

Fig. 11. Categorization of every item when the parameter μ values fluctuate.

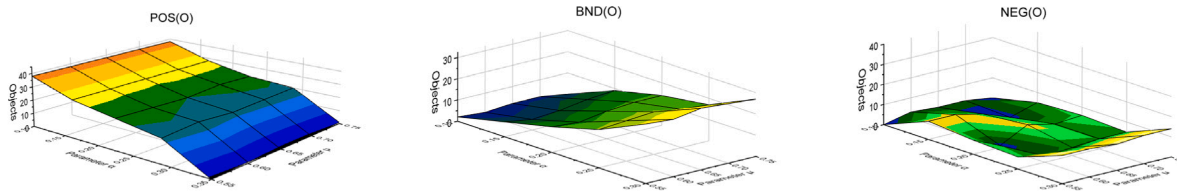

Fig. 12. Categorization of every item when α and μ change in value.

CRediT authorship contribution statement

Xiaoyan Zhang: Conceptualization, Investigation, Methodology, Validation, Writing – review & editing. **Shijia Yu:** Data curation, Methodology, Software, Visualization, Writing – original draft, Writing – review & editing.

Declaration of competing interest

We wish to confirm that there are no known conflicts of interest associated with this publication and there has been no significant financial support for this work that could have influenced its outcome.

We confirm that the manuscript has been read and approved by all named authors and that there are no other persons who satisfied the criteria for authorship but are not listed. We further confirm that the order of authors listed in the manuscript has been approved by all of us.

We confirm that we have given due consideration to the protection of intellectual property associated with this work and that there are no impediments to publication, including the timing of publication, with respect to intellectual property. In so doing we confirm that we have followed the regulations of our institutions concerning intellectual property.

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Data availability

No data was used for the research described in the article.

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