Multiview Fuzzy Concept-Cognitive Learning With High-Order Information Fusion of Fuzzy Attributes

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***Abstract*—Concept-cognitive learning (CCL) is an emerging computing paradigm that is widely employed in knowledge discovery. It considers concepts as the basic computing units, emphasizing the representation of knowledge through extent–intent pairs. Some studies explore CCL models on single view data, with a particular focus on fuzzy CCL models, demonstrating notable performance in classification tasks. However, data are always obtained from multiple views in reality, necessitating the crucial task of representing and integrating concepts across multiple views. Hence, this article proposes a novel multi-view fuzzy concept-cognitive learning (MVFCCL) model to address this issue. The process of multiview fuzzy concept cognition is first introduced to learn fuzzy concepts from each view. Specifically, the process provides an intraview fusion method to reconstruct fuzzy attributes by modeling both high-order information and correlation information, thereby enhancing the conceptual representation ability of fuzzy concepts. Then, the multiview fuzzy concept recognition process is established to predict new objects decision attributes by considering their similarity to the multiview fuzzy concept space. Finally, some experiments are conducted to examine the effectiveness of MVFCCL, including comparisons with other methods and ablation experiments for validating the contributions of each step. Experimental results show that the proposed MVFCCL can effectively represent and fuse knowledge from multiview data via fuzzy concepts.**

***Index Terms*—Concept-cognitive learning (CCL), fuzzy formal concept analysis (FCA), granular computing, multiview learning (MVL).**

I. Introduction

A KEY characteristic of human intelligence is the ability to extract knowledge during the process of understanding things. Concepts are an important form of knowledge representation, and there is research on concept learning in many fields, such as cognitive philosophy [1], data mining [2], [3], computer vision [4], and granular computing [5], [6]. Recently, concept-cognitive learning (CCL) [6], [7], [8] has emerged as a burgeoning research direction, offering an effective model for knowledge discovery.

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From a broader perspective, CCL is the science of cognition and learning things via concepts [9]. In current research, CCL can be viewed as a framework for mining knowledge from data. Its defining characteristic is that it uses concepts as the basic knowledge carrier. Formal concept analysis (FCA) [10] played a significant role in the development of CCL. The mathematical definition of concept in FCA is first proposed by Wille [10]. Formally, concepts are composed of extent–intent pairs, where extent refers to a set of objects and intent is the shared attributes possessed by these objects. The unique extent and intent structure of concepts naturally provides a method for mining decision rules from data [5], [11], [12], [13]. Due to the distinctive structure of concepts, as well as their capacity for representing knowledge, FCA garnered the attention of scholars in fields, such as mathematics [14], data science [15], and granular computing [5]. Given that the construction of concept lattices (the primary tool of FCA) is time-consuming and inefficient, the quest for more efficient methods of concept learning became an important research topic. In this context, inspired by the processes of human cognition, concept learning based on cognitive perspective attracted growing interest.

Xu et al. [6] first built a concept learning model from cognitive perspective. Subsequently, the cognitive perspective-based concept learning models have gradually emerged as an important research direction. From the perspective of concept learning methods, several CCL models were successively proposed, such as two-way CCL model [16], [17], [18], [19], incremental CCL model [20], and many more. In terms of the formal representation of concepts, various concepts such as fuzzy concepts [21] and three-way concepts [22] have been investigated to cater to diverse needs. Among them, the fuzzy CCL models received widespread attention due to their ability to handle continuous data directly and achieve better classification performance.

The procedure of fuzzy CCL for the classification task [21], [22], as illustrated in Fig. 1, is comprised of two stages: fuzzy concept cognition and fuzzy concept recognition. In fuzzy concept cognition stage, a fuzzy concept space is constructed to characterize the fuzzy ontology. The fuzzy concept space is composed of fuzzy concept subspaces, with each fuzzy concept subspace associated with a decision attribute. The fuzzy concepts within each fuzzy concept subspace reflect the essential features associated with the specific decision attribute.

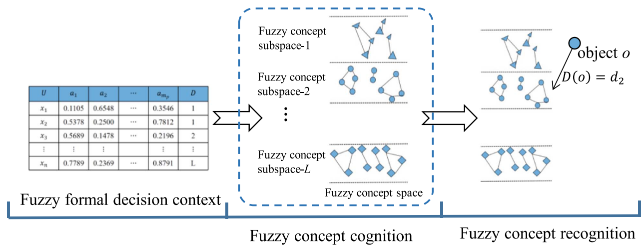


Fig. 1. Procedure of fuzzy CCL.

In fuzzy concept recognition stage, the classification of a new object into a specific fuzzy concept subspace is determined by the maximum similarity between the object and the fuzzy concepts within the fuzzy concept subspace. The process of fuzzy concept recognition is similar to some KNN algorithms, classifying based on the similarity between the object's attributes and the intent of fuzzy concepts [23], [24], [25]. Based on this framework, several fuzzy CCL methods [22], [26], [27], [28] have been proposed. Yuan et al. [22] proposed a fuzzy CCL model for generating progressive fuzzy three-way concepts by simultaneously introducing positive and negative information, and corresponding dynamic updating mechanism was also designed. To enhance the efficiency of CCL, inspired by human memory mechanisms, Guo et al. [26] developed a fuzzy CCL method that can forget unimportant concepts during the process of concept cognition. Considering the different importance of attributes in the process of learning concepts, Zhang et al. [29] investigated a weighted fuzzy CCL model. And CCL models based on multiattention mechanisms [30] and stochastic strategy [31] were successively studied. In addition, various CCL models have been introduced, taking into account different forms of concepts and addressing diverse application scenario. By summarizing existing CCL models, we can derive a criterion for constructing a concept space: the boundaries between concept subspaces have a significant impact on classification performance. In more intuitive terms, larger boundaries result in better classification performance. Existing studies tend to focus on achieving this objective from a model-level perspective, while overlooking the attainment from the inherent nature of fuzzy attributes. From the perspective of FCA, classical attributes dictate that the distance between any two objects under a given attribute is either 0 or 1. However, the fact that fuzzy attributes provide a richer spectrum of measurements is always ignored. Therefore, exploring how to effectively utilize the measurement information to expand the boundaries between fuzzy concept subspaces is a worthwhile research problem, and it is one of the innovations of this article.

Although recent studies indicate that fuzzy concepts can effectively represent the knowledge in fuzzy data, existing fuzzy CCL models are conducted only under single-view data. In reality, data are often recorded from multiple views (presented in various modalities, obtained from multiple of information sources, or else). For example, a scene can be described by an image or a text passage; An image can be extracted into different types of features (scale-invariant features, profile correlations, Zernike moments, etc.); Epileptic electroencephalographic signals can be recorded at different points in time, resulting in multiview data [32]. The emergence of multiview data has driven the development of multi-view learning (MVL) research [33], [34]. Theoretically, it has been proven that MVL has superior generalization performance [35]. Intuitively, MVL is more robust and exhibits stronger practicality in real-life scenarios. Fusion is one of the key challenges in MVL, it aims at integrating information from multiple views to make predictions. Especially with the advent of large language models, the fusion of multiview data has become a hot research direction. Some methods based on certain criteria have been successively proposed in MVL. These criteria aim to enhance the correlation between each view [36], evaluate the confidence of each view [37], [38], improve the predictive performance of all views [39], [40], explore the collaborative mechanism between views [41], [42], etc., [43]. Luo et al. [44] proposed a tensor canonical correlation analysis method for multiview dimension reduction to enhance the correlation between multiple views. By evaluating and fusing the reliability of each view, Han et al. [37] developed trusted multiview classification model. Liang et al. [39] investigated multimodal classification methods, which enhance the classification performance of each modal by fusing the association between features.

Although existing MVL methods effectively fuse data from multiple views, few studies have addressed multiview data from the perspective of formalized knowledge representation and cognition, which is crucial for a deeper understanding of multiview data. On the one hand, formalized knowledge representation aligns with the human process of understanding things. On the other hand, using concepts for knowledge representation demonstrates better adaptability in complex environments, such as dynamic [7], [26] and incomplete environments [31]. CCL provides such a framework to tackle this issue. Furthermore, if we directly apply a CCL model to each view of multiview data, two phenomena are observed: 1) The intent of concepts can effectively represent knowledge within each view. 2) The extent of concepts naturally provides a way to retrieve information across views. This precisely corresponds to the two main fusion methods in MVL: intraview fusion and interview fusion. Therefore, researching MVL methods based on CCL can both expand the application scope of CCL models and provide a new knowledge representation paradigm for MVL.

These facts motivate us to address the research challenges related to the representation, integration, cognition, and recognition of multiview fuzzy concepts. Hence, a multiview fuzzy concept-cognitive learning (MVFCCL) model is proposed in this article. The research contributes in the following ways.

1) A MVFCCL framework is established to learn and integrate fuzzy concepts under multiple views. The framework provides a multiview data fusion mechanism with concepts as the knowledge carrier, and its effectiveness is validated through a series of experiments.

2) The notion of multiview fuzzy concept space is defined, which offers a multiview knowledge representation method that simultaneously characterizes the features associated with a given decision attribute using both attribute and object information. The corresponding multiview concept recognition mechanism is further explored to provide improved predictions for the decision attributes of new objects.
3) A uniform fusion method of fuzzy attributes is proposed to construct fuzzy concept spaces under each view. The rich measures provided by fuzzy attributes are fully explored, and redundancy among attributes is eliminated via correlation. By employing this method, the boundaries between fuzzy concept subspaces under each view are expanded.

The rest of this article is organized as follows. Section II provides a summary of fuzzy CCL models. Section III demonstrates the proposed MVFCCL model. The experimental results and analysis are conducted in Section IV. Finally, Section V concludes this article.

II. Preliminaries

In this section, some notions related to fuzzy concepts are first reviewed, and several fuzzy CCL models are then summarized.

A. Fuzzy Concepts

Some basic definitions are given in this section, and the specific information is available at [45], [46], [47]. In fuzzy FCA, data are represented via the notion of fuzzy formal context (FFC) or fuzzy formal decision context (FFDC). A FFDC is a quintuple $(U, A, \widetilde{R}, D, J)$, where

1) $U = \{o_1, o_2, \ldots, o_M\}$ is a set of objects.
2) $A = \{a_1, a_2, \ldots, a_N\}$ is a set of fuzzy attributes.
3) $\widetilde{R} = \{< (o_m, a_n), \widetilde{R}(o_m, a_n) > |(o_m, a_n) \in U \times A\}$ is a fuzzy binary relation on $U \times A$, $\widetilde{R} : U \times A \to [0, 1]$, and $\widetilde{R}(o_m, a_n)$ represents the degree to which object o_m possesses attribute a_n.
4) $D = \{d_1, d_2, \ldots, d_L\}$ is a set of decision attributes, and $J : U \times D \to \{0, 1\}$ is a crisp binary relation on $U \times D$. $J(o_m, d_l) = 1$ indicates object o_m possesses decision attribute d_l.
5) Each object has and only has one decision attribute in this article. And the division on U induced by J is denoted as $O/J = \{U_1, U_2, \ldots, U_L\}$, where the elements in $U_l (l = 1, 2, \ldots, L)$ are the objects that possess the decision attribute d_l.

A FFC is a triplet $(U, A, \widetilde{R})$. The power set of U and the fuzzy power set of A are denoted as 2^U and $\mathcal{F}^A$, respectively. In order to discover knowledge in the form of fuzzy concepts in FFCs, a pair of cognitive operators, $\widetilde{\mathcal{H}} : 2^U \to \mathcal{F}^A$ and $\mathcal{G} : \mathcal{F}^A \to 2^U$ can be defined.

Definition 1: Let $(U, A, \widetilde{R})$ be a FFC, for $E \in 2^U$ and $\widetilde{B} \in \mathcal{F}^A$, a pair of cognitive operators, denoted as $\widetilde{\mathcal{H}}$ and $\mathcal{G}$, is defined as follows:

$$\widetilde{\mathcal{H}}(E)(a) = \bigwedge_{o \in U} \widetilde{R}(o, a), a \in A$$

$$\mathcal{G}(\widetilde{B}) = \{o \in U | \widetilde{B}(a) \leq \widetilde{R}(o, a), \forall a \in A\}. \quad (1)$$

It should be pointed out that there are multiple methods for defining cognitive operators, and Definition 1 presents one commonly used approach. In a FFC, fuzzy concepts can be learned according to the cognitive operators $\widetilde{\mathcal{H}}$ and $\mathcal{G}$.

Definition 2: Let $(U, A, \widetilde{R})$ be a FFC. A pair $(E, \widetilde{B})$ with $E \in 2^U$ and $\widetilde{B} \in \mathcal{F}^A$ is referred to as a fuzzy concept if $\widetilde{\mathcal{H}}(E) = \widetilde{B}$ and $\mathcal{G}(\widetilde{B}) = E$. E and $\widetilde{B}$ are, respectively, called the extent and intent of $(E, \widetilde{B})$.

In simple terms, $\widetilde{B}$ can be interpreted as a fuzzy attribute set shared by the objects in E, and E is an object set possesses all the fuzzy attributes in $\widetilde{B}$. Therefore, fuzzy concepts can be regarded as a carrier for representing knowledge in fuzzy data. The set of all fuzzy concepts is denoted as $C_{\mathcal{G}\widetilde{\mathcal{H}}}$.

Although fuzzy concepts serve as an effective means of expressing knowledge, obtaining all fuzzy concepts in FFCs is a challenging task. Therefore, fuzzy granular concepts have received extensive attention in fuzzy CCL, as they are relatively easier to learn. Fuzzy granular concepts are learned based on individual objects as clues. For $o \in U$, a fuzzy granular concept $(\mathcal{G}\widetilde{\mathcal{H}}(\{o\}), \widetilde{\mathcal{H}}(\{o\})$[simply designated as $(\mathcal{G}\widetilde{\mathcal{H}}(o), \widetilde{\mathcal{H}}(o)$] can be learned, yielding the set of all fuzzy granular concepts

$$GC_{\mathcal{G}\widetilde{\mathcal{H}}} = \{(\mathcal{G}\widetilde{\mathcal{H}}(o), \widetilde{\mathcal{H}}(o) | o \in U\}. \quad (2)$$

$GC_{\mathcal{G}\widetilde{\mathcal{H}}}$ can be referred to as the "knowledge base" of $C_{\mathcal{G}\widetilde{\mathcal{H}}}$, as each fuzzy concept in $C_{\mathcal{G}\widetilde{\mathcal{H}}}$ can be learned from the fuzzy granular concepts in $GC_{\mathcal{G}\widetilde{\mathcal{H}}}$ through $\widetilde{\mathcal{H}}$ and $\mathcal{G}$. Furthermore, all fuzzy concepts in $C_{\mathcal{G}\widetilde{\mathcal{H}}}$ constitute a complete lattice, and there exists a specialization–generalization relationship between fuzzy concepts [45], [46], [47].

B. Fuzzy Concept Spaces

Current fuzzy CCL models for constructing fuzzy concept spaces are summarized in this section.

Let $F = (U, A, \widetilde{R}, D, J)$ be a FFDC. For $U_l \in U/J$, a pair of local cognitive operators $\widetilde{\mathcal{H}}_l : 2^{U_l} \to \mathcal{F}^A$ and $\mathcal{G}_l : \mathcal{F}^A \to 2^{U_l}$ is defined as follows:

$$\widetilde{\mathcal{H}}_l(E)(a) = \bigwedge_{o \in U_l} \widetilde{R}(o, a), a \in A$$

$$\mathcal{G}_l(\widetilde{B}) = \{o \in U_l | \widetilde{B}(a) \leq \widetilde{R}(o, a), \forall a \in A\} \quad (3)$$

where $E \subseteq U_l$, $\widetilde{B} \in \mathcal{F}^A$. Then, the fuzzy granular concept subspace with respect to U_l is represented by $GC^l_{\mathcal{G}\widetilde{\mathcal{H}}} = \{(\mathcal{G}_l\widetilde{\mathcal{H}}_l(o), \widetilde{\mathcal{H}}_l(o) | o \in U_l\}$, and a fuzzy granular concept space is the collection $\{GC^l_{\mathcal{G}\widetilde{\mathcal{H}}} | l = 1, 2, \ldots, L\}$. It should be point out that the cognitive operators in Definition 1 is defined based on the entire object set, while the local cognitive operators are defined based on a subset of the object set.

A fuzzy granular concept subspace describes the features of objects with the same decision attribute, from the perspectives of extent and intent. However, direct application of fuzzy granular concept spaces for concept recognition is fraught with certain challenges, such as: 1) An excess of elements within $GC^l_{\mathcal{G}\widetilde{\mathcal{H}}}$, leading to inefficient retrieval of fuzzy concepts. 2) Poor classification performance, due to the poor conceptual representation ability of fuzzy granular concepts.

Therefore, several fuzzy CCL models have been proposed to overcome the aforementioned problems. For any $l \in \{1, 2, \ldots, L\}$, these models can be formalized as a surjective

$$\mathcal{L}_{\mathcal{G}\widetilde{\mathcal{H}}} : GC^l_{\mathcal{G}\widetilde{\mathcal{H}}} \to \Omega^l \tag{4}$$

where $\Omega^l \subseteq \{(E, \widetilde{B}) | E \subseteq U_l, \widetilde{B} \in \mathcal{F}^A\}$. Each element in Ω^l can be regarded as "fuzzy ontology concepts" of $GC^l_{\mathcal{G}\widetilde{\mathcal{H}}}$, since it reflects the essential features of fuzzy concept subspace associated with decision attribute d_l. The fuzzy ontology concept space is denoted as $\{\Omega^1, \Omega^2, \ldots, \Omega^L\}$, and we refer to it as fuzzy concept space without causing confusion. Some fuzzy CCL models for generating fuzzy ontology concepts have been proposed, and we review three typical fuzzy ontology concepts here: progressive fuzzy concepts, big fuzzy concepts, and weighted fuzzy concepts.

Progressive fuzzy concepts are generated through the fusion of similar concepts. For any progressive fuzzy concepts $(E, \widetilde{B}) \in \Omega^l$, there exists $\{(E_1, \widetilde{B}_1), (E_2, \widetilde{B}_2), \ldots, (E_k, \widetilde{B}_k)\} \subseteq GC^l_{\mathcal{G}\widetilde{\mathcal{H}}}$, such that

$$\begin{aligned} E &= E_1 \cup E_2 \cup \cdots \cup E_k \\ \widetilde{B} &= \frac{1}{2^{k-1}}(\widetilde{B}_1 + \widetilde{B}_2 + 2^1\widetilde{B}_3 + 2^2\widetilde{B}_4 + \cdots + 2^{k-2}\widetilde{B}_k) \end{aligned} \tag{5}$$

where $\{(E_1, \widetilde{B}_1), (E_2, \widetilde{B}_2), \ldots, (E_k, \widetilde{B}_k)\}$ is a conceptual cluster of $GC^l_{\mathcal{G}\widetilde{\mathcal{H}}}$. This method is adopted by [9], [21], [22], and [28] to construct fuzzy concept spaces.

Big fuzzy concepts are the fuzzy granular concepts in $GC^l_{\mathcal{G}\widetilde{\mathcal{H}}}$ with larger extents. They have better effectiveness in dynamic concept cognition and concept recognition. For any $(E, \widetilde{B}) \in \Omega^l$, the following conditions are satisfied:

1) $(E, \widetilde{B}) \in GC^l_{\mathcal{G}\widetilde{\mathcal{H}}}$ and there exists $(E_1, \widetilde{B}_1) \in GC^l_{\mathcal{G}\widetilde{\mathcal{H}}}$, such that $E_1 \subseteq E$.
2) If there exists $(E_2, \widetilde{B}_2) \in GC^l_{\mathcal{G}\widetilde{\mathcal{H}}}$ and $E \subseteq E_2$, then $E = E_2$.

This strategy is utilized by [26] and [27].

Weighted fuzzy concepts refer to fuzzy concepts within $GC^l_{\mathcal{G}\widetilde{\mathcal{H}}}$ that possess better conceptual representation ability. Some CCL models for generating weighted concepts, such as based on information entropy and attention mechanism, have been proposed. This process can be described as

$$\Omega^l = \{(E, \widetilde{B}) \in GC^l_{\mathcal{G}\widetilde{\mathcal{H}}} | w((E, \widetilde{B})) > \delta\} \tag{6}$$

where $w((E, \widetilde{B}))$ reflects conceptual representation ability of $(E, \widetilde{B})$, and δ is a threshold. This approach is adopted by [29] and [30].

Based on various fuzzy CCL models, a fuzzy concept space $\{\Omega^1, \Omega^2, \ldots, \Omega^L\}$ is generated. $\Omega^l (l = 1, 2, \ldots, L)$ is referred to as a fuzzy concept subspace associated with decision attribute d_l. In summary, the fuzzy CCL models can map fuzzy granular concepts to fuzzy ontology concepts (progressive fuzzy concepts, weighted fuzzy concepts, big fuzzy concepts, etc.) with stronger representation capabilities. This article focuses on the fusion of multiple fuzzy concept subspaces, rather than proposing a new fuzzy CCL model, and existing fuzzy CCL models can be embedded in the proposed fusion framework. For convenience, the process of obtain a fuzzy concept space from a FFDC F is simply denoted as

$$\mathcal{L}_{\mathcal{G}\widetilde{\mathcal{H}}}[F] = \{\Omega^1, \Omega^2, \ldots, \Omega^L\} \tag{7}$$

where $\mathcal{G}$ and $\widetilde{\mathcal{H}}$ are a pair of cognitive operator, and $\mathcal{L}$ is a specific fuzzy CCL method for generating fuzzy ontology concepts. In this article, inspired by [26] and [27], big fuzzy concepts are adopted as fuzzy ontology concepts in MVFCCL, since it does not introduce additional parameter. More specifically, the fuzzy granular concepts in $GC^l_{\mathcal{G}\widetilde{\mathcal{H}}}$ satisfy conditions 1) and 2) of big fuzzy concepts are selected as the elements in Ω^l. The meanings of the main symbols used in this article are listed in Table I.

TABLE I
MEANINGS OF THE MAIN SYMBOLS

Symbol	Meaning
U	The set of objects
A	The set of fuzzy attributes
$\widetilde{R}$	The fuzzy binary relation on $U \times A$
D	The set of decision attributes
J	The crisp binary relation on $U \times D$
$d^l \in D$	A decision attribute
U^l	The set of objects with decision attribute d^l
$\mathcal{G}$, $\widetilde{\mathcal{H}}$	A pair of cognitive operators
$C_{\mathcal{G}\widetilde{\mathcal{H}}}$	The set of all fuzzy concepts
$GC_{\mathcal{G}\widetilde{\mathcal{H}}}$	The set of all fuzzy granular concepts
$\mathcal{G}^l$, $\widetilde{\mathcal{H}}^l$	A pair of local cognitive operators associated with d^l
$GC^l_{\mathcal{G}\widetilde{\mathcal{H}}}$	The fuzzy granular concept subspace associated with d^l
Ω^l	The fuzzy concept subspace associated with d^l
$\mathcal{L}_{\mathcal{G}\widetilde{\mathcal{H}}}$	A fuzzy concept-cognitive learning model that can map $GC^l_{\mathcal{G}\widetilde{\mathcal{H}}}$ to Ω^l
A_v	The set of fuzzy attributes under the vth view
$\widetilde{\mathcal{R}}_v$	The fuzzy binary relation under the vth view
Ω^l_v	The fuzzy concept subspace with respect to d_l under the vth view
$\Omega_v^{\sim}$	The fuzzy concept space under the vth view
Ω	The multiview fuzzy concept space
$\mathcal{B}^P$	The fuzzy attribute extension process with parameter P
$\mathcal{C}^\delta$	The correlation fusion process with parameter δ

III. MULTI-VIEW FUZZY CONCEPT-COGNITIVE LEARNING

In this section, the definitions of multiview FFDC and multiview fuzzy concept space are first defined, and the procedures of multiview fuzzy concept cognition and multiview fuzzy concept recognition (MVCR) are then presented in detail.

A. Multiview Fuzzy Concept Spaces

In practice, data are often recorded from multiple views, making the fusion of data from different views an important topic. Suppose an entity is characterized by V views. From the perspective of fuzzy CCL, the data obtained from the vth$(v = 1, 2, \ldots, V)$ view can be formalized into a FFDC $F_v = (U, A_v, \widetilde{R}_v, D, J)$, and a multiview FFDC with V views

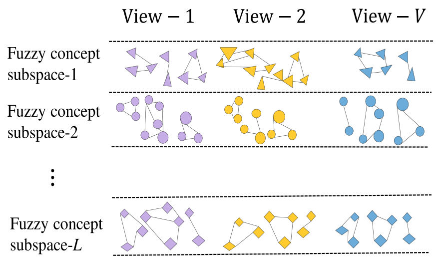


Fig. 2. Multiview fuzzy concept space.

is represented by

$$\mathrm{MVF} = \{F_v | v = 1, 2, \ldots, V\}. \tag{8}$$

For $F_v \in$ MVF, a fuzzy concept space $\Omega_v^{\sim}$ with respect to vth view is generated through the fuzzy CCL model $\mathcal{L}_{\mathcal{G}\widetilde{\mathcal{H}}}$, i.e.,

$$\Omega_v^{\sim} = \mathcal{L}_{\mathcal{G}\widetilde{\mathcal{H}}}[F_v] = \{\Omega_v^1, \Omega_v^2, \ldots, \Omega_v^L\}. \tag{9}$$

A multiview fuzzy concept space Ω is then obtained, which is denoted as

$$\Omega = \{\Omega_v^l | v = 1, 2, \ldots, V, l = 1, 2, \ldots, L\} \tag{10}$$

where Ω_v^l represents the fuzzy concept subspace with respect to decision attribute d_l under the vth view. Fig. 2 shows a multiview fuzzy concept space. Compared to Fig. 1, the number of fuzzy concept subspaces increases from L to $L * V$.

This section establishes a MVFCCL model to effectively represent knowledge from each view in a conceptual form and fuse concepts under different views. The proposed MVFCCL is divided into two steps: multiview fuzzy concept cognition and MVCR.

1) Multiview fuzzy concept cognition aims at learning a multiview fuzzy concept space, which can effectively represent knowledge across multiple views via fuzzy concepts. It can be divided into two successive steps: fuzzy attribute extension (FAE) (see Section III-B) and correlation fusion (CF) (see Section III-C).
2) MVCR focuses on integrating the multiview fuzzy concept space to make a prediction. The process is shown in Section III-D.

The total procedure of MVFCCL is shown in Fig. 3. Liang et al. [39] proposed a intramodal fusion method by modeling both high-order information and correlation information for multimodal classification. Inspired by [39], the proposed multiview fuzzy concept cognition can be regarded as an intraview fusion method to improve the conceptual representation ability of fuzzy concepts.

B. Fuzzy Attribute Extension

We concentrate on a single FFDC $F_v = (U, A_v, \widetilde{R}_v, D, J) \in MVF$ in this section and Section III-C, where $U = \{o_1, o_2, \ldots, o_M\}$ and $A_v = \{a_1, a_2, \ldots, a_{N_v}\}$.

FAE aims at exploring the rich measurement information provided by fuzzy attributes. In this process, a parameter P is introduced, and it represents the extension rate of fuzzy attributes.

TABLE II
EXAMPLE OF FFDC

Objects	a_1	a_2	a_3	a_4	d_1	d_2
o_1	0.91	0.89	0.12	0.90	1	0
o_2	0.88	0.85	0.11	0.70	1	0
o_3	0.90	0.88	0.20	0.50	1	0
o_4	0.15	0.23	0.79	0.30	0	1
o_5	0.19	0.21	0.88	0.10	0	1

The specific approach involves converting N_v-dimension fuzzy attributes into N_vP-dimension fuzzy attributes by applying a power operation to each fuzzy attribute. A FFDC $F_v' = (U, A_v', \widetilde{R}_v', D, J)$ is then obtained. The procedure is describe as follows:

$$F_v' = \mathcal{B}^P[F_v] = \left(U, A_v', \widetilde{R}_v', D, J\right). \tag{11}$$

The connections between F_v and F_v' are detailed in Definition 3.

Definition 3: Suppose F_v is a FFDC, F_v' is referred to as the FFDC after FAE with extension rate P if F_v' satisfies the following properties:

1) $A_v' = \{c_1, c_2, \ldots, c_{N_vP}\}$.
2)

$$\widetilde{R}_v' = \{\widetilde{R}_v'(o_m, c_n) | m = 1, 2, \ldots, M, \\ n = 1, 2, \ldots, N_vP\}.$$

3) For any $o_m \in U$, $c_{n_1} \in A_v'$ and $p \in \{1, 2, \ldots, P\}$, there exists $a_{n_2} \in A_v$, such that

$$\widetilde{R}_v'(o_m, c_{n_1}) = \frac{(\widetilde{R}_v(o_m, a_{n_2}))^p}{T^p} \tag{12}$$

where $(\widetilde{R}_v(o_m, a_{n_2}))^p$ is the pth power of $\widetilde{R}_v(o_m, a_{n_2})$. $T \in (0, 1]$ is used to control the decrease ratio of fuzzy values and ensure the fuzzification of attribute values.

It should be pointed out that FAE does not change the degree of possession relationship between any two objects under any attribute. More specifically, for any fuzzy attribute a, if object o_1 has a higher degree of possession of a than object o_2, assuming $c_1, c_2, \ldots, c_P$ are the attributes obtained by extending a, then o_1 has a higher degree of possession than o_2 under $c_1, c_2, \ldots, c_P$. To facilitate understanding, we provide an example that illustrates the process of FAE in Example 1.

Example 1: Table II shows a FFDC with five objects and four fuzzy attributes. After the process of FAE with extension ratio 3, the extended fuzzy attributes are listed in Table III . T is set to 0.92.

In this example, c_1, c_2, c_3 are generated by a_1, c_4, c_5, c_6 are generated by a_2, c_7, c_8, c_9 are generated by a_3, and c_{10}, c_{11}, c_{12} are generated by a_4. On the one hand, the measurement provided by fuzzy attribute is fully explored. On the other hand, the association between some generated attributes and decision attributes is enhanced, i.e., the boundaries between objects with different decision attributes are expanded.

In summary, the advantages for conducting FAE are outlined as follows:

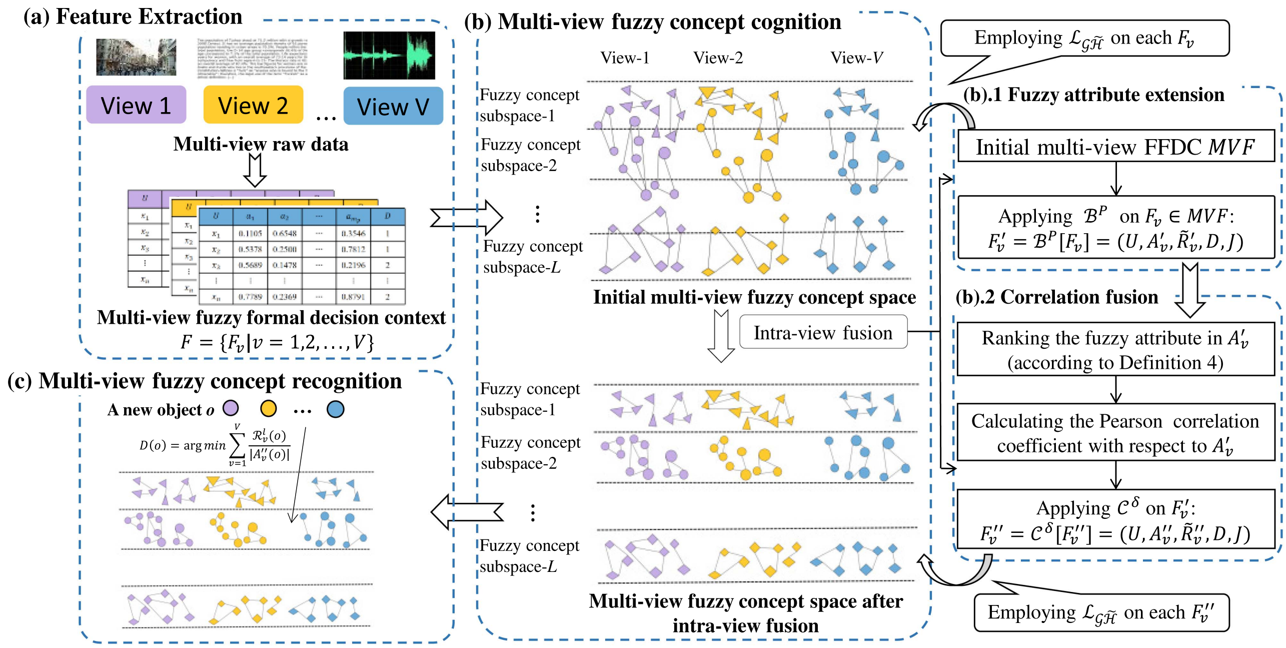


Fig. 3. Overall procedure of MVFCCL.

TABLE III
EXTENDED FUZZY ATTRIBUTES

Objects	c_1	c_2	c_3	c_4	c_5	c_6
o_1	0.99	0.98	0.97	0.97	0.94	0.90
o_2	0.96	0.91	0.88	0.92	0.85	0.79
o_3	0.98	0.96	0.94	0.96	0.91	0.88
o_4	0.16	0.03	0	0.25	0.06	0.02
o_5	0.21	0.04	0	0.23	0.05	0.01
Objects	c_7	c_8	c_9	c_{10}	c_{11}	c_{12}
o_1	0.13	0.02	0	0.98	0.96	0.94
o_2	0.12	0.01	0	0.76	0.58	0.44
o_3	0.22	0.05	0.01	0.54	0.30	0.16
o_4	0.86	0.74	0.63	0.33	0.11	0.03
o_5	0.96	0.91	0.88	0.11	0.01	0

1) It fully explores the rich measurements provided by fuzzy attributes. In classical formal decision context, the distance among objects with respect to a particular attribute is either 0 or 1. However, in FFDCs, there exists various distances between objects. By performing power operation on each fuzzy attribute, the measurement advantages offered by fuzzy attributes can be exploited.
2) It enhances the discriminative power of certain fuzzy attributes. For classification task, the discriminative ability of a fuzzy attribute is manifested by its association with decision attributes. FAE can generate some fuzzy attributes with stronger associations to decision attributes. In a more intuitive sense, it makes objects with a higher degree of a fuzzy attribute more pronounced in their possession, while those with a lower degree become more pronounced in their nonpossession.

Notice that more fuzzy attributes are generated through FAE, how to select better fuzzy attributes will be discussed in the next section.

C. Correlation Fusion

CF aims to select the fuzzy attributes with better discriminative ability based on the correlations among fuzzy attributes. In fact, correlation-based MVL constitute a significant research domain, such as MVL methods based on canonical correlation analysis. From the perspective of CCL, correlation to some extent reflects the redundancy among attributes. The objective of this step is to eliminate redundancy among attributes while preserving the diversity of correlations.

Continued from Section III-B, the fuzzy attributes in A_v' are first ranked based on their discernibility. The ranking criterion is the distances between fuzzy concepts in different fuzzy concept subspaces under fuzzy attributes. Thus, a fuzzy CCL model $\mathcal{L}_{\mathcal{G}\widetilde{\mathcal{H}}}$ need to be applied on F_v' to construct a fuzzy concept space, i.e.,

$$\Omega_v^{\sim} = \mathcal{L}_{\mathcal{G}\widetilde{\mathcal{H}}}[F_v'] = \{\Omega_v^1, \Omega_v^2, \dots, \Omega_v^L\}. \tag{13}$$

For any $(E_1, \widetilde{B}_1), (E_2, \widetilde{B}_2) \in \cup\Omega_v^{\sim}$, if $(E_1, \widetilde{B}_1)$ and $(E_2, \widetilde{B}_2)$ are in the same fuzzy concept subspace, they are denoted as $d(E_1, E_2) = 1$; otherwise, $d(E_1, E_2) = 0$. Inspired by [9], the discernibility of a fuzzy attribute is given in Definition 4.

Definition 4: Let $F_v = (U, A_v', \widetilde{R}_v', D, J)$ be a FFDC, and $\Omega_v^{\sim} = \mathcal{L}_{\mathcal{G}\widetilde{\mathcal{H}}}[F_v'] = \{\Omega_v^1, \Omega_v^2, \dots, \Omega_v^L\}$. For any $c_m \in A_v'$, the discernibility of c_m on $(E_1, \widetilde{B}_1)$ and $(E_2, \widetilde{B}_2)$ is defined as follows:

$$dis_{c_m}((E_1, \widetilde{B}_1), (E_2, \widetilde{B}_2)) = \begin{cases} 0, & d(E_1, E_2) = 1. \\ |\widetilde{B}_1(c_m) - \widetilde{B}_2(c_m)|, & d(E_1, E_2) = 0. \end{cases} \tag{14}$$

And the discernibility of c_m is denoted as

$$\text{dis}_{c_m} = \frac{1}{2} \sum_{(E_1,\widetilde{B}_1)\in\cup\Omega_v^{\sim}} \sum_{(E_2,\widetilde{B}_2)\in\cup\Omega_v^{\sim}} dis_{c_m}((E_1,\widetilde{B}_1),(E_2,\widetilde{B}_2)). \tag{15}$$

According to Definition 4, the large the value of dis_{c_m}, the stronger the discernibility of c_m. Subsequently, we can arrange the fuzzy attributes in A'_v in descending order based on their discernibility, ranging from high to low. Following this section, the sorted fuzzy attributes set is represented as $A'_v = \{e_1, e_2, \ldots, e_{N_v P}\}$.

Then, our aim is to select the top-c_v fuzzy attributes $A''_v = \{e_1, e_2, \ldots, e_{c_v}\}$ from A'_v. The Pearson correlation coefficient is used to measure correlation. The Pearson correlation coefficient matrix [48] with respect to A'_v is denoted as $\text{corr}_{A'_v}$, for any $e_{m_1}, e_{m_2} \in A'_v$

$$\text{corr}_{A'_v}(e_{m_1}, e_{m_2}) = \frac{\mathbb{E}((e_{m_1} - \mu_{e_{m_1}})(e_{m_2} - \mu_{e_{n_2}}))}{\sigma_{e_{m_1}}\sigma_{e_{m_2}}} \tag{16}$$

where $\mu_{e_{m_1}}$ and $\sigma_{e_{m_1}}$ are the mean and the standard deviation of e_{m_1}, respectively.

A parameter $\delta(-1 \le \delta \le 1)$ is used in the CF process. If the correlation between two fuzzy attributes is greater than δ, it is considered that there exists a correlation between them.

The selected fuzzy attribute set A''_v satisfies the following definition.

Definition 5: Suppose $A'_v = \{e_1, e_2, \ldots, e_{N_v P}\}$ is the ranked fuzzy attribute set, $corr_{A'_v}$ is the corresponding Pearson correlation coefficient matrix, and δ is a parameter ($\delta \in [-1, 1]$). The selected fuzzy attribute set $A''_v = \{e_1, e_2, \ldots, e_{c_v}\}$ satisfies the following conditions:

1)
$$\bigcup_{m_1=1,2,\ldots,c_v} \{e_{m_2} \in A'_v | \text{corr}_{A'_v}(e_{m_1}, e_{m_2}) > \delta\} = A'_v.$$

2) For any $k < c_v$
$$\bigcup_{m_1=1,2,\ldots,k} \{e_{m_2} \in A'_v | \text{corr}_{A'_v}(e_{m_1}, e_{m_2}) > \delta\} \subset A'_v.$$

Definition 5 illustrates the relationship between A'_v and A''_v. In order to select the fuzzy attribute subset A''_v from A'_v, a search method based on the Pearson correlation coefficient matrix is proposed, corresponding to lines 9 to 20 of Algorithm 1. In line 12 of Algorithm 1, $\text{corr}_{A'_v}(1:k, n)$ denotes the vector composed of the elements from the first to the kth row of the nth column in $\text{corr}_{A'_v}$.

Through CF, a FFDC $F''_v = (U, A''_v, \widetilde{R}''_v, D, J)$ is obtained. The process is denoted as $\mathcal{C}^\delta$, i.e.,

$$F''_v = \mathcal{C}^\delta[F'_v] = (U, A''_v, \widetilde{R}''_v, D, J). \tag{17}$$

CF expands the boundaries between fuzzy concept subspaces within the same view. Since we directly rank the fuzzy attributes based on their contribution to expanding the boundaries, it ensures the selected fuzzy attributes have better discernibility. Besides, Definition 5 ensures the elimination of redundancy among fuzzy attributes while preserving the diversity of correlation.

At this point, the intraview fusion has been completed. The intraview fusion framework can be represented by

$$F''_v = \mathcal{C}^\delta \circ \mathcal{B}^P[F_v] = (U, A''_v, \widetilde{R}''_v, D, J). \tag{18}$$

By applying the fuzzy CCL model $\mathcal{L}_{\mathcal{G}\widetilde{\mathcal{H}}}$ on F''_v, a fuzzy concept space after intraview fusion is obtained. The overall process is described as

$$\Omega_v^{\sim} = \mathcal{L}_{\mathcal{G}\widetilde{\mathcal{H}}}[\mathcal{C}^\delta \circ \mathcal{B}^P[F_v]] = \{\Omega_v^1, \Omega_v^2, \ldots, \Omega_v^L\}. \tag{19}$$

By employing the intraview fusion and the fuzzy CCL model on each F_v in MVF, a multiview fuzzy concept space Ω after intraview fusion is then generated

$$\Omega = \{\Omega_v^l | v = 1, 2, \ldots, V, l = 1, 2, \ldots, L\}. \tag{20}$$

The multiview fuzzy concept cognition process is completed at this point, and the entire process of multiview fuzzy concept cognition is discussed in detail in Algorithm 1, which corresponds to "(b).1" in Fig. 3.

In FAE step (lines 1–3), the time complexity is $O(|U||A_v|P)$. In CF step (lines 4–25), the time complexity of applying $\mathcal{L}_{\mathcal{G}\widetilde{\mathcal{H}}}$ on F'_v is $O(|U|^2|A_v|P)$(line 5), ranking the fuzzy attributes (lines 6–7) is $O(|U|^2|A_v|P)$, calculating the Pearson correlation coefficient matrix (line 8) is $O(|U||A_v|^2P^2)$, and selecting the top-c_v attribute (lines 9–20) is $O(|A_v|^2P^2)$. Thus, the total time complexity of multiview fuzzy concept cognition is $O(\sum_{v=1}^V(|U|^2|A_v|P + |U||A_v|^2P^2))$.

D. Multiview Fuzzy Concept Recognition

Multiview concept fuzzy recognition focuses on integrating the multiview fuzzy concept space for classification tasks. Let o be a new object, and $A_v(o)$ be the fuzzy attribute values of o under the vth view. The objective of this section is identifying the decision attribute of o based on the multiview fuzzy concept space Ω.

First, the same fuzzy attribute transformation should be applied on o, i.e., $A_v(o)$ is transformed into $A''_v(o)$. More specifically, for each view, perform a power operation on each attribute value of o, then select the same top-c_v attributes. The distance between $A''_v(o)$ and a fuzzy concept $(X, \widetilde{B}) \in \Omega_v^l$ is defined by

$$\text{DIST}(A''_v(o), \widetilde{B}) = ||A''_v(o) - \widetilde{B}||^2 \tag{21}$$

where $||\cdot||^2$ represents 2-norm. Then, the definition of fuzzy concept recognition degree is given to determine the decision attribute.

Definition 6: For any $\Omega_v^l \in \Omega_v^{\sim}$, the fuzzy concept recognition degree of Ω_v^l on o is defined as follows:

$$\mathcal{R}_v^l(o) = \min\{\text{DIST}(A''_v(o), \widetilde{B}) | (X, \widetilde{B}) \in \Omega_v^l\}. \tag{22}$$

The smaller the value of $\mathcal{R}_v^l(o)$, the higher the degree of association between o and decision attribute d_l. Intuitively, based on the principles of CCL, the decision attribute of object o, denoted as $D(o)$, can be determined using the following formula:

$$D(o) = \arg\min_{d_l} \sum_{v=1}^V \mathcal{R}_v^l(o). \tag{23}$$

Algorithm 1: Multi-View Fuzzy Concept Cognition.

Input: A multi-view FFDC $MVF = \{F_v | v = 1, 2, \ldots, V\}$, parameter P and δ, and a fuzzy CCL model $\mathcal{L}_{\mathcal{G}\widetilde{\mathcal{H}}}$;
Output: A multi-view fuzzy concept space Ω;

for $v \in \{1, 2, \ldots, V\}$ **do**
  Let $F'_v = \mathcal{B}^P[F_v]$, where $\mathcal{B}^P[F_v] = (U, A'_v, \widetilde{R}'_v, D, J)$;
end for
for $v \in \{1, 2, \ldots, V\}$ **do**
  Apply the fuzzy CCL model $\mathcal{L}_{\mathcal{G}\widetilde{\mathcal{H}}}$ on F'_v;
  Compute the discernibility of each fuzzy attribute in A'_v according to Definition 4;
  Get the ranked fuzzy attribute set $A'_v = \{e_1, e_2, \ldots, e_{N_v P}\}$;
  Calculate $corr_{A'_v}$;
  Define $minper = (-1, -1, \ldots, -1)$ and $temp$ are two $1 \times N_v P$ vector;
  for $k \in \{2, 3, \ldots, N_v P\}$ **do**
    for $n \in \{1, 2, \ldots, N_v P\}$ **do**
      $temp(n) \leftarrow \max\{corr_{A'_v}(1:k, n) | n = 1, 2, \ldots, N_v P\}$;
    end for
    $minper(k) \leftarrow \min temp$;
    if $\max minper > \delta$ **then**
      Break;
    end if
  end for
  $c_v \leftarrow k$;
  $A'' \leftarrow \{e_1, e_2, \ldots, e_{c_v}\}$;
  Let $F''_v = \mathcal{C}^\delta[F'_v]$, where $\mathcal{C}^\delta[F'_v] = (U, A''_v, \widetilde{R}''_v, D, J)$;
end for
for $v \in \{1, 2, \ldots, V\}$ **do**
  Let $\Omega^{\sim}_v = \mathcal{L}_{\mathcal{G}\widetilde{\mathcal{H}}}[F''_v]$, where $\mathcal{L}_{\mathcal{G}\widetilde{\mathcal{H}}}[F''_v] = \{\Omega^1_v, \Omega^2_v, \ldots, \Omega^L_v\}$;
end for
$\Omega \leftarrow \{\Omega^l_v | v = 1, 2, \ldots, V, l = 1, 2, \ldots, L\}$.

However, through the process of intraview fusion, the number of attributes under each view may change. In order to balance the impact of different numbers of attributes under different views on the classification performance, the decision attribute is derived by the following formula:

$$D(o) = \arg\min_{d_l} \sum_{v=1}^{V} \frac{\mathcal{R}^l_v(o)}{||A''_v(o)||} \tag{24}$$

where $||A''_v(o)||$ is the number of fuzzy attribute in $A''_v(o)$.

The algorithm of MVCR is shown in Algorithm 2, and the time complexity of Algorithm 2 is $O(\sum_{v=1}^{V} |U||A_v|P)$.

IV. Experiments

In this section, the superiority of MVFCCL is evaluated through several experiments. The purpose of the experiments

Algorithm 2: Multi-View Fuzzy Concept Recognition.

Input: A multi-view fuzzy concept space $\Omega = \{\Omega^l_v | v = 1, 2, \ldots, V, l = 1, 2, \ldots, L\}$, and a new object o with fuzzy attribute $\{A_v(o) | v = 1, 2, \ldots, V\}$;
Output: The decision attribute $D(o)$;

for $v \in \{1, 2, \ldots, V\}$ **do**
  Transform $A_v(o)$ into $A''_v(o)$;
  for $l \in \{1, 2, \ldots, L\}$ **do**
    $\mathcal{R}^l_v(o) \leftarrow \min\{DIST(A''_v(o), \widetilde{B}) | (X, \widetilde{B}) \in \Omega^l_v\}$;
  end for
end for
$D(o) \leftarrow \arg\min_{d_l} \sum_{v=1}^{V} \frac{\mathcal{R}^l_v(o)}{||A''_v(o)||}$.

TABLE IV
Detailed Information of Selected Datasets

ID	Dataset	Training	Testing	Views	Class
D1	PS	700	300	$(1024, 100)$	20
D2	TV	1558	500	$(1024, 100)$	10
D3	Wiki	2173	693	$(1024, 100)$	10
D4	NWO	3360	2315	$(64, 225, 144, 73, 128)$	5
D5	CH-CM	3360	2315	$(64, 225)$	5
D6	CH-CORR	3360	2315	$(64, 144)$	5
D7	CH-WT	3360	2315	$(64, 128)$	5
D8	CORR-EDH	3360	2315	$(144, 73)$	5
D9	MF	1800	200	$(216, 76, 64, 6, 240, 47)$	10
D10	Pix-Fac	1800	200	$(240, 216)$	10
D11	Pix-Fou	1800	200	$(240, 76)$	10
D12	Pix-Zer	1800	200	$(240, 47)$	10

is to examine MVFCCL is effective in representing and integrating fuzzy concepts across multiple views, as evidenced by classification accuracy.

A. Experiment Settings

The proposed MVFCCL aims to comprehensively integrate data from various views. Therefore, several multimodal and multiview datasets are employed in the experiments, including: Pascal Sentences [49] (PS), TVGraz [50] (TV), Wikipedia [51] (Wiki), Nus-wide-object (NWO) [52], and Multiple features[1] (MF). PS, TV and Wiki are three multimodal datasets consists of image/text pairs. For these three datasets, we use the same data provided by [53]. Images are described by using 1024-dimension SIFT features, and texts are represented by 100-dimension LDA features. NWO is a one object image dataset categorized into 31 classes. 5675 images with respect to five classes(bear, birds, cars, dog, and fish) are used in the experiments. Five types of features are extracted from the images, including CH, CORR, EDH, WT, and CM. By combining some pairs of features, four additional multiview datasets are obtained, including: CH-CM, CH-CORR, CH-WT, and CORR-EDH. MF consists of features of handwritten numerals (0–9). Six types of features: Fac, Fou, Kar, Mor, Pix, and Zer are extracted. Similar to [39], by combining Pix with Fac, Fou, and Zer, three additional datasets are generated.

TABLE V
BASIC CLASSIFICATION ACCURACY OF THE FUZZY CCL MODELS UNDER EACH VIEW

View	Baseline	ILMPFTC	MFCCL	DMPWFC
PS(image)	0.2233	0.2233	0.2233	0.0867
PS(text)	0.5367	0.5133	0.5367	0.2467
TV(image)	0.4700	0.4700	0.4700	0.0960
TV(text)	0.6620	0.6240	0.6620	0.1540
Wiki(image)	0.2092	0.2121	0.2092	0.1414
Wiki(text)	0.7937	0.7864	0.7937	0.3838
NWO(CH)	0.4376	0.4376	0.4376	0.2393
NWO(CORR)	0.4389	0.4471	0.4389	0.2894
NWO(EDH)	0.4605	0.4665	0.4657	0.4194
NWO(WT)	0.3728	0.3650	0.3728	0.2704
NWO(CM)	0.4406	0.4363	0.4406	0.2760
MF(Fou)	0.9605	0.9620	0.9605	0.7050
MF(Fac)	0.8010	0.8000	0.8010	0.4265
MF(Kar)	0.9625	0.9660	0.9625	0.6090
MF(Mor)	0.3210	0.5920	0.6585	0.6515
MF(Pix)	0.9750	0.9750	0.9750	0.6745
MF(Zer)	0.7810	0.7945	0.7870	0.5515

The detailed informations are listed in Table IV. For D1–D8, the authors have provided the division of training and testing data, and we use the same division for the corresponding datasets. For D9–D12, the experiments are conducted with 10-fold cross-validations independently, and mean classification accuracy values are recorded. Besides, all attribute values are scaled to be between 0.001 and 0.999, according to the following formula:

$$\widetilde{R}(o,a) = (0.999 - 0.001)\frac{a(o) - \min(a)}{\max(a) - \min(a)} + 0.001$$

where $a(o)$ denotes the attribute value of object o under attribute a, and $\min(a)$ and $\max(a)$ represent the minimum and maximum values of all objects under a. T in (12) is set to 0.999 to ensure the fuzzification of attribute values.

The experiments are conducted on a personal computer with OS: Microsoft WIN10; Processor: Intel(R) Core(TM) i7-13700H CPU @2400 Mhz; Memory: 32 GB; Programming language: MATLAB R2020b.

B. Comparison With Other Methods

In order to demonstrate the effectiveness of MVFCCL, this section examines its fusion performance in comparison to other methods. Given that MVFCCL is a fusion framework based on CCL models, it is crucial to consider comparative methods from both the perspectives of CCL models and fusion methods.

For CCL model, three typical fuzzy CCL models, ILMPFTC [22], MFCCL [26], and DMPWFC [29] are employed in the experiments. Progressive fuzzy three-way concepts are generated through ILMPFTC, DMPWFC employs fuzzy entropy to select weighted fuzzy concepts, and MFCCL generates big fuzzy three-way concepts in concept recognition. ILMPFTC, DMPWFC, and MFCCL all require a parameter δ, and δ takes values in $\{0, 0.1, \ldots, 1\}$ in our experiments. To provide readers with a preliminary understanding of the classification capabilities of these fuzzy CCL methods, the basic classification

TABLE VI
TIME COMPLEXITY COMPARISON OF MVFCCL WITH OTHER METHODS

Method	Time complexity
MVFCCL	$O(\sum_{v=1}^{V}(\lvert U\rvert^2\lvert A_v\rvert P + \lvert U\rvert\lvert A_v\rvert^2 P^2))$
Baseline-B	$O(\lvert U\rvert^2 \sum_{v=1}^{V} \lvert A_v\rvert)$
ILMPFTC-B	$O(\lvert U\rvert^2 \sum_{v=1}^{V} \lvert A_v\rvert)$
DMPWFC-B	$O(\lvert U\rvert^2 \sum_{v=1}^{V} \lvert A_v\rvert)$
MFCCL-B	$O(\lvert U\rvert^2 \sum_{v=1}^{V} \lvert A_v\rvert)$
AFKNN-B	$O(L^2\lvert U\rvert \sum_{v=1}^{V} \lvert A_v\rvert^2)$
FENN-B	$O(\lvert U\rvert \sum_{v=1}^{V} \lvert A_v\rvert)$
IFKNN-B	$O(\lvert U\rvert \sum_{v=1}^{V} \lvert A_v\rvert)$
Baseline-C	$O(\lvert U\rvert^2 \sum_{v=1}^{V}(\lvert A_v\rvert))$
ILMPFTC-C	$O(\lvert U\rvert^2 \sum_{v=1}^{V}(\lvert A_v\rvert))$
DMPWFC-C	$O(\lvert U\rvert^2 \sum_{v=1}^{V}(\lvert A_v\rvert))$
MFCCL-C	$O(\lvert U\rvert^2 \sum_{v=1}^{V}(\lvert A_v\rvert))$
AFKNN-C	$O(L^2\lvert U\rvert \sum_{v=1}^{V} \lvert A_v\rvert^2)$
FENN-C	$O(\lvert U\rvert \sum_{v=1}^{V} \lvert A_v\rvert)$
IFKNN-C	$O(\lvert U\rvert \sum_{v=1}^{V} \lvert A_v\rvert)$
FISH-MML	$O(\sum_{v=1}^{V}(\lvert U\rvert^2\lvert A_v\rvert + \lvert U\rvert\lvert A_v\rvert^2 + \lvert A_v\rvert^3))$

accuracy of the CCL methods under each view is listed in Table V. In Table V, Baseline refers to the CCL model that generates big fuzzy concepts, which have been discussed in Section II. It is also the basic fuzzy CCL model adopted in the proposed MVFCCL.

For fusion method, we adopt two widely used methods in multimodal learning and MVL, which have been summarized in [39]: feature concatenation and best fusion. Feature concatenation involves concatenating the features from all views together once they are extracted. Specifically, the data under all views are concatenated into a new dataset to train a classifier. The experimental results employing this fusion method are denoted by "-C", such as ILMPFTC-C. Best fusion uses the view with the best performance to make a prediction. The procedure entails independently training a classifier based on data from different views. Subsequently, the view with the highest classification accuracy performance is selected. This fusion method is denoted by "-B", such as ILMPFTC-B.

Besides, three multimodal/MVL method AFKNN-C [39], AFKNN-B [39], and FISH-MML [40], along with two fuzzy classification methods, IFKNN [54] and FENN [55], are selected as comparative methods.

In summary, the comparing methods can be divided into following four groups.

1) Fuzzy CCL model adopting best fusion method, including ILMPFTC-B, DMPWFC-B, MFCCL-B, Baseline-B.
2) Fuzzy CCL model adopting feature concatenation method, including ILMPFTC-C, DMPWFC-C, MFCCL-C, Baseline-C.
3) Fuzzy classification method, including IFKNN-B, IFKNN-C, FENN-B, and FENN-C, and neighbor parameter *K* is set to 3 in our experiments;
4) MVL model, including AFKNN-C, AFKNN-B, and FISH-MML. Due to the unsuitability of AFKNN-B, AFKNN-C, and FISH-MML for fuzzy data, these three

[1][Online]. Available: http://archive.ics.uci.edu/dataset/72/multiple+features

TABLE VII
CLASSIFICATION ACCURACY COMPARISON WITH OTHER METHODS BASED ON BEST FUSION

Dataset	MVFCCL	Baseline-B	ILMPFTC-B	DMPWFC-B	MFCCL-B	AFKNN-B	FENN-B	IFKNN-B
D1	0.5600	0.5367	0.5133	0.2467	0.5367	**0.5767**	0.5467	0.5367
D2	**0.7460**	0.6620	0.6240	0.1540	0.6620	0.7420	0.6220	0.6640
D3	0.8052	0.7937	0.7864	0.3838	0.7937	**0.8312**	0.8052	0.8009
D4	**0.6333**	0.4605	0.4665	0.4194	0.4657	0.4773	0.4963	0.4834
D5	**0.5050**	0.4389	0.4471	0.2894	0.4389	0.4350	0.4713	0.4752
D6	**0.5171**	0.4605	0.4665	0.4194	0.4657	0.4773	0.4963	0.4834
D7	**0.5443**	0.4406	0.4376	0.2704	0.4406	0.4104	0.4687	0.4683
D8	**0.5369**	0.4605	0.4665	0.4194	0.4657	0.4773	0.4963	0.4834
D9	**0.9865**	0.9750	0.9755	0.7050	0.9750	0.9655	0.9740	0.9760
D10	**0.9835**	0.9765	0.9765	0.6560	0.9765	0.9575	0.9760	0.9770
D11	**0.9905**	0.9750	0.9750	0.6380	0.9750	0.9600	0.9730	0.9750
D12	0.9735	0.9750	0.9750	0.6390	0.9750	0.9600	0.9755	**0.9760**
Ave. value	**0.7318**	0.6796	0.6758	0.4367	0.6809	0.6892	0.6918	0.6916
Ave. rank	**2.2083**	9.7950	9.4583	14.5833	9.5000	9.3333	7.0000	6.5817

TABLE VIII
CLASSIFICATION ACCURACY COMPARISON WITH OTHER METHODS BASED ON FEATURE CONCATENATION

Dataset	MVFCCL	Baseline-C	ILMPFTC-C	DMPWFC-C	MFCCL-C	AFKNN-C	FENN-C	IFKNN-C	FISH-MML
D1	0.5600	0.2667	0.2667	0.1200	0.2700	0.2333	0.1900	0.2100	**0.5967**
D2	**0.7460**	0.5020	0.5020	0.1160	0.5020	0.5780	0.4240	0.4540	0.6840
D3	**0.8052**	0.3117	0.3117	0.1544	0.3146	0.6104	0.3117	0.3088	0.7965
D4	**0.6333**	0.5486	0.5486	0.2911	0.5486	0.5158	0.5525	0.5616	0.5780
D5	**0.5050**	0.4531	0.4531	0.2829	0.4531	0.4436	0.4864	0.4955	0.4730
D6	**0.5171**	0.4708	0.4708	0.3425	0.4708	0.4946	0.4946	0.4998	0.4812
D7	**0.5443**	0.5201	0.5222	0.2242	0.5201	0.4479	0.5430	0.5438	0.5417
D8	0.5369	0.4877	0.4903	0.3317	0.4877	0.5300	0.4998	0.5080	**0.5378**
D9	**0.9865**	0.9790	0.9790	0.7085	0.9790	0.8910	0.9780	0.9805	0.9535
D10	**0.9835**	0.9785	0.9785	0.6495	0.9785	0.8385	0.9760	0.9780	0.9550
D11	**0.9905**	0.9780	0.9780	0.6485	0.9780	0.9515	0.9760	0.9765	0.9770
D12	0.9735	0.9740	0.9740	0.6395	0.9740	0.9295	**0.9770**	0.9765	0.8725
Ave. value	**0.7318**	0.6225	0.6229	0.3757	0.6230	0.6220	0.6174	0.6244	0.7039
Ave. rank	**2.2083**	7.7083	7.4583	15.7500	7.4167	10.2083	7.1667	5.8333	6.0833

TABLE IX
FRIEDMAN ANOVA TABLE

Source	SS	df	MS	Chi-sq	Prob>Chi-sq
Columns	1919.38	15	127.958	85.4	7.0940×10^{-12}
Error	2126.12	165	12.886	–	–
Total	4045.5	191	–	–	–

TABLE X
SELECTED STEPS FOR ABLATION EXPERIMENTS

Selected steps	FAE	CF	MVCR
Baseline	×	×	×
Step1	✓	×	×
Step2	×	✓	×
Step3	×	×	✓
Step4	✓	×	✓
Step5	×	✓	✓
MVFCCL	✓	✓	✓

methods do not undergo fuzzification of the data. These three methods are based on KNN algorithm, and K is set to 3. Besides, as FISH-MML yields complex eigenvalues, we utilize the real part of the eigenvalues in the experiments.

A detailed analysis of the time complexity of MVFCCL and its comparison methods is presented in Table VI. P denotes the parameter utilized in MVFCCL, and L is the parameter utilized in AFKNN-C and AFKNN-B. $|U|$ represents the number of objects, $|A_v|$ is the number of attributes under view v, and V is the number of views.

The comparisons of MVFCCL with best fusion-based methods and feature concatenation-based methods are shown in Tables VII and VIII, respectively. The best classification accuracies under each dataset in Table VII and VIII are highlighted in bold for each respective table, and the best classification results in both Table VII and VIII are indicated by underlines. It can be seen that MVFCCL outperforms 15 comparative methods in terms of classification performance on 8 out of 12 datasets. It also achieves the highest average classification accuracy (0.7318) and the best average ranking (2.2083). The comparisons between MVFCCL and the classification performances of the four groups of methods are visually presented in Figs. 4 and 5.

The Friedman test [56] at a significance level of $\alpha = 0.05$ is employed to assess the significance of the differences in classification performance. Table IX is the corresponding Friedman ANOVA table. It can be seen that the resulting p-value is 7.0940×10^{-12}, indicating a significant difference in classification accuracy. Then, a Nemenyi post-hoc test [57] is conducted to test the difference between MVFCCL and each of other compared methods. The corresponding critical distance $\mathrm{CD} = q_\alpha \sqrt{\frac{k(k+1)}{6N}} = 6.6589 (k = 15, N = 12, q_{0.05} = 3.391)$, and the CD diagram is shown in Fig. 6. The average rank of each algorithm is plotted on the axis, and any two algorithms that have significant differences in classification performance are not connected by a line.

TABLE XI
CLASSIFICATION ACCURACY OF SELECTED STEPS ON 12 DATASETS

Steps	D1	D2	D3	D4	D5	D6	D7	D8	D9	D10	D11	D12	Ave. acc.
Baseline	0.4467	0.6740	0.6595	0.6039	0.4942	0.4868	0.5352	0.4946	0.9865	0.9790	0.9835	0.9765	0.6934
Step1	0.4460	0.6740	0.6595	0.6039	0.4942	0.4903	0.5352	0.4985	0.9880	0.9800	0.9850	0.9780	0.6944
Step2	0.4533	0.6740	0.6609	0.6043	0.4942	0.4877	0.5400	0.4976	**0.9910**	0.9815	0.9865	**0.9825**	0.6961
Step3	0.5567	0.7360	0.8038	0.6225	0.5037	0.5024	0.5352	0.5153	0.7745	0.9790	0.9845	0.9630	0.7064
Step4	0.5567	0.7360	0.8038	0.6225	0.5045	0.5067	0.5356	0.5201	0.8495	0.9795	0.9875	0.9700	0.7144
Step5	**0.5600**	**0.7460**	**0.8052**	**0.6333**	**0.5050**	0.5089	**0.5443**	0.5261	0.9690	0.9820	0.9875	0.9680	0.7279
MVFCCL	**0.5600**	**0.7460**	**0.8052**	**0.6333**	**0.5050**	**0.5171**	**0.5443**	**0.5369**	0.9865	**0.9835**	**0.9905**	0.9735	**0.7318**

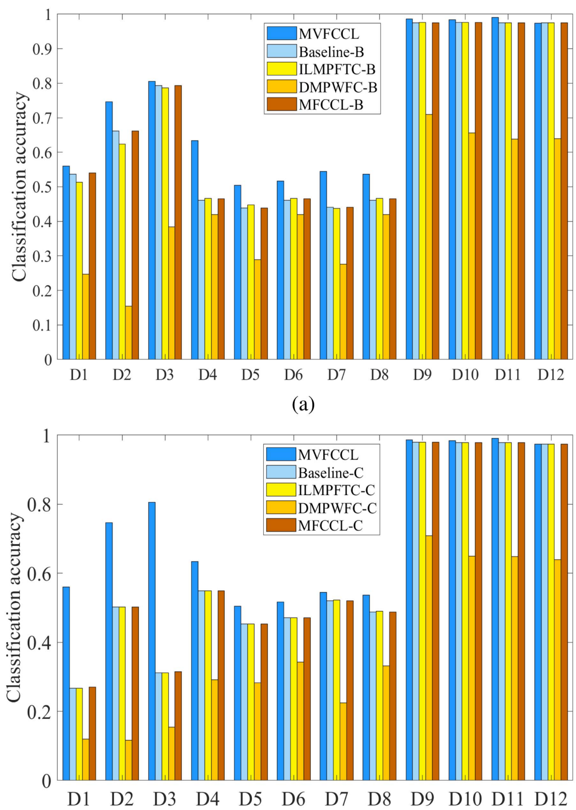


Fig. 4. Comparisons of classification accuracy with fuzzy CCL models. (a) Comparisons with fuzzy CCL models based on best fusion. (b) Comparisons with fuzzy CCL models based on feature concatenation fusion.

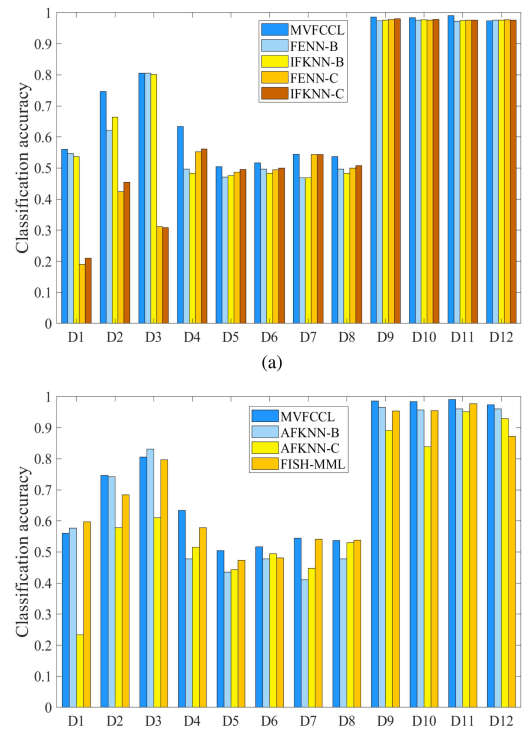


Fig. 5. Comparisons of classification accuracy with fuzzy classification and MVL methods. (a) Comparisons with fuzzy classification methods. (b) Comparisons with MVL methods.

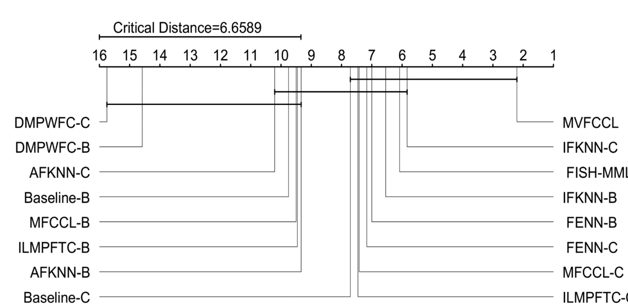


Fig. 6. Nemenyi test between MVFCCL and comparison methods.

C. Ablation Experiments

In this section, we investigate the contributions of each step in MVFCCL.

MVFCCL consists of multiview fuzzy concept cognition and MVCR. For multiview fuzzy concept cognition, it involves two steps: FAE and CF. For MVCR, its main characteristic is to divide the concept recognition degree under each view by $\|A''_v(o)\|$. To validate the significance of each step, we conduct experiments using some selected steps and record the experimental results. The selected steps are listed in Tables X and XI represent the classification accuracy with respect to the steps. In Table X, ✓ and ×, respectively, indicate whether the corresponding step in that column is selected or not. The classification accuracy of the steps that achieve the best performance under each dataset is marked in bold in Table XI. It can be seen that MVFCCL achieved the best performance in 10 out of 12 datasets, indicating that each step in MVFCCL is effective. Furthermore, compared to Step1, Step2, and Step3, Step4 and Step5 have better average classification accuracy, demonstrating that the classification performance is better when more steps are selected. The average classification accuracy for each step is visually presented in Fig. 7, and the contribution of each step

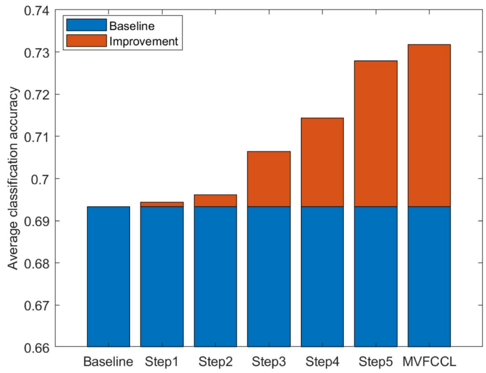
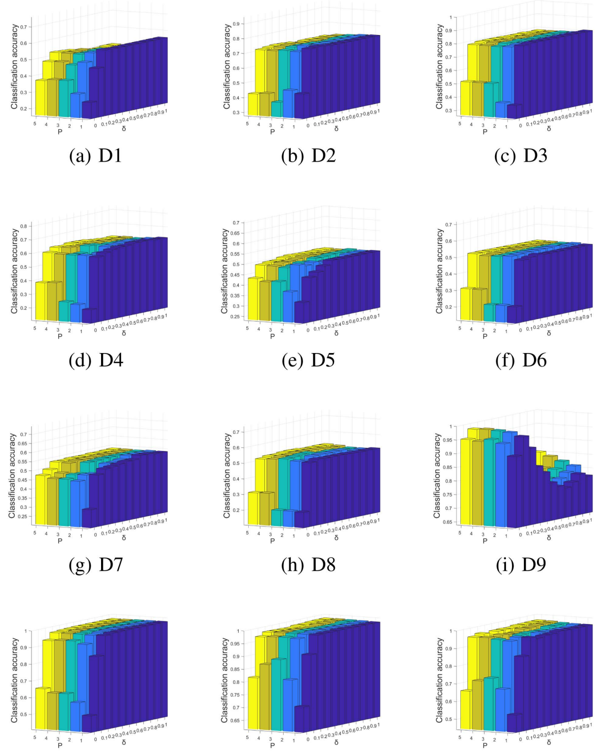


Fig. 7. Average classification accuracy of steps in MVFCCL.



(a) D1 (b) D2 (c) D3
(d) D4 (e) D5 (f) D6
(g) D7 (h) D8 (i) D9
(j) D10 (k) D11 (l) D12

Fig. 8. Classification accuracy under different parameter on 12 datasets. (a) D1. (b) D2. (c) D3. (d) D4. (e) D5. (f) D6. (g) D7. (h) D8. (i) D9. (j) D10. (k) D11. (l) D12.

to the improvement of classification performance is indicated in red.

D. Parameter Analysis

MVFCCL employs two parameters P and δ. P is used to control the FAE ratio, which takes values in $\{1, 2, 3, 4, 5\}$. δ is introduced in CF, and it is adjusted from 0 to 1 with a step of 0.1. In order to assess the sensitivity of MVFCCL to these two parameters, we conducted a study to examine the changes in accuracy performance under different parameter settings. The experimental results are shown in Fig. 8. It can be observed that: 1) When δ is fixed, the classification accuracy steadily improves with an increase in P, particularly when δ is small. It corroborates the notion that FAE enhances conceptual representation ability of fuzzy concepts. 2) In most cases, when $\delta > 0.1$, the classification performance tends to be more stable. It should be noted that as δ increases, more fuzzy attributes are selected. This implies that CF will reduce the number of selected fuzzy attributes. These two findings further validate the efficacy of the intraview fusion step.

V. Conclusion

This article has established a fuzzy CCL model called MVFCCL that can handle multiview fuzzy data. In MVFCCL, concepts have been used as the fundamental carriers to represent and fuse knowledge from multiview fuzzy data, which distinguishes it from existing MVL methods. To enhance the representation capability of concepts, an FAE method tailored for fuzzy CCL has also been investigated. Both high-order information and correlation information of fuzzy attribute have been fully explored in the method. Furthermore, a series of definitions and theories regarding MVFCCL have been provided, such as multiview fuzzy formal decision context and multiview fuzzy concept space. Finally, corresponding algorithms have been developed, and extensive experiments have been conducted to validate the effectiveness of MVFCCL.

In summary, on the one hand, this study has extended the CCL model, which can only handle single-view data, to handle multiview data, and has provided a new concept-based knowledge representation paradigm for MVL. On the other hand, as we all know, concepts are important forms of representation for information granules, which are the basic computing units in Granular computing. Hence, this study has promoted the application of granular computing theory in the field of multiview and multimodal fusion.

One notable characteristic of concepts is the simultaneous use of extent and intent to convey knowledge. The proposed MVFCCL effectively utilizes the intent information of concepts for concept learning. However, the extent information of concepts is not considered. In fact, the extent information of concepts provides valuable retrieval cues across views. Therefore, it is worth investigating how to fully leverage the extent information for cross-view retrieval and achieve collaborative learning among views. In addition, MVFCCL focuses on the intraview fusion, but different views may have varying representation capabilities. Consequently, it is a worthwhile research question to consider the importance of each view and design corresponding weighted methods.

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