JNCA

Journal of Nonlinear and Convex Analysis

Yokohama Publishers

NACA-ICOTA2019

The 11th International Conference on

Nonlinear Analysis and Convex Analysis (NACA)
Optimization: Techniques and Applications (ICOTA)

Hakodate, Japan, August 26–31, 2019

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Official HP:

http://wgnaca.org/naca-icota2019/

NACA

The International Research Working Group in Nonlinear Analysis and Convex Analysis

Journal of Nonlinear and Convex Analysis
Volume 20, Number 6, 2019, 1077–1087

PARTIALLY CONSISTENT REDUCTION BASED ON DISCERNIBILITY INFORMATION TREE IN INTERVAL-VALUED FUZZY ORDERED INFORMATION SYSTEMS WITH DECISION

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Abstract. Attribute reduction is a hot issue in the field of rough set research in recent years, among which identifiable matrix is one of the most commonly used methods for attribute reduction. However, the elements of identifiable attribute set in identifiable matrix are interlaced and repeated, which brings a lot of inconvenience for reduction. Therefore, a novel method based on discernibility information tree (DIT) proposed by Jiang to overcome the above issues. On this basis, this paper constructs the discernibility information tree ($DIT^{\succeq}$) under the dominance relation and generalizes it to the interval-valued fuzzy ordered information system with decision ($IVFOIS_d$). Furthermore, combining the discernibility information tree of $IVFOIS_d$ with the partially consistent function, a complete partially consistent reduction algorithm based on $DIT^{\succeq}$ is presented. At the same time, some related properties and the complexity of the algorithm are studied. Finally, the effectiveness and accuracy of the $DIT^{\succeq}$-based reduction method are demonstrated by a concrete example.

1. Introduction

Rough set theory is proposed by Polish mathematician Pawlak [10]. It is a theory used to deal with imprecise and incomplete data problems. One of its main contents is attribute reduction [4, 14, 15, 19, 20]. Attribute reduction is to delete irrelevant attributes while keeping the classification ability unchanged, so as to reduce redundant data and simplify rules. On the basis of Pawlak Rough Set, many scholars have given different attribute reduction algorithms according to various reduction ideas [1–3, 6, 9, 12]. Skowron [11] put forward an intuitive and clear discernibility matrix method for attribute reduction, and can obtain all reductions under the information system. Wang [13] proposed a fast and complete heuristic minimum reduction algorithm based on discernibility matrix. Hou [5] proposed an incremental reduction algorithm, which effectively reduces redundant attributes of dynamic decision table by establishing reduction tree. Yao [21] proposed a reduction method based on simplified discernibility matrix, and presented two heuristic reduction construction algorithms from the perspective of attribute importance degrees. Jiang [7] discussed a complete reduction algorithm, which combined the importance degree of attributes with the idea of iteration to find the minimum reduction of decision

2010 *Mathematics Subject Classification.* 03E72, 68T30, 68U35.

Key words and phrases. Discernibility information tree(DIT), identifiable matrix, interval-valued fuzzy ordered information system with decision, partially consistent reduction.

This work is supported by the National Natural Science Foundation of China (No.61772002).

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tables. All non-empty sets in the discernibility matrix corresponding to each information system makes conjunctive, which form the attribute reduction set of the information system. From the conjunction operation, we can see that the repeated elements and the parent set elements of the discernibility matrix do not play any role in the attributes reduction, but these elements occupy a large amount of storage space and also increase the time for us to obtain reduction. Yang [16] proposed a novel storage method to implement compressed storage of repeated sets in the identifiable matrix. However, this approach has some drawbacks, which retains the extra parent set element, so there are still a certain number of redundant elements. In order to eliminate redundant parent set elements in the discernibility matrix, Jiang [8] constructed a virtual tree structure to realize attribute reduction, which is called discernibility information tree (DIT). It can delete attribute sets that are repeated in the reduction process, and also make use of the characteristics of the tree to merge the redundant parent set objects into one tree node. However, DIT is based on equivalence relation, which is too strict and has no fault tolerance. Therefore, this paper establishes the discernibility information tree ($\text{DIT}^{\succeq}$) under the dominance relation, studies the attribute reduction method based on $\text{DIT}^{\succeq}$, and further extends it to the interval-valued fuzzy ordered information system with decision (IVFOIS_d). Furthermore, combined with $\text{DIT}^{\succeq}$ and partially consistent function in IVFOIS_d, a partially consistent reduction algorithm based on the constructed $\text{DIT}^{\succeq}$ will be established.

In the following, we will give some preparatory knowledge related to $\text{IVFOIS}_{\succeq}$, and review partially consistent reduction in Section 2. At the same time, the basic content of DIT will be also introduced in Section 2. In Section 3, a $\text{DIT}^{\succeq}$ algorithm in a decision table based on interval-valued fuzzy ordered relation and a complete partially consistent reduction algorithm based on $\text{DIT}^{\succeq}$ are presented. The related theorems are introduced, and the complexity of time and space is analyzed. In Section 4, according to the theoretical knowledge of the previous section, a concrete example is given to analysis. The feasibility and validity of the theory in this paper are proved by comparing with the calculation results of the original method. In Section 5, the current research progress of $\text{DIT}^{\succeq}$ is summarized, and the direction of the next research content is provided.

2. Preliminary

In this section, we will introduce the basic concepts for the rest of the article.

2.1. Interval-valued fuzzy ordered information systems with decision ($IVFOIS_d$).

Giving an interval-valued fuzzy information system (IVFIS), it is denoted by $IS = (U, AT, V, F)$, where U is a non-empty finite universe, AT is a finite nonempty set of attributes, $V = \cup_{a \in AT} V_a$ represents the domain of attributes. Since it is an interval-valued fuzzy information system, V is a set composed of a number of interval numbers, and the range of the interval number is between 0 and 1. $F = \{f : U \to V\}$ is a set of relation sets, representing the mapping sets from objects to attribute values, where $f(x, a) \in V_a$ represents the attribute value of object x under attribute a. At the same time, it can also be expressed as $f(x, a) = [a^L(x), a^R(x)]$, in which $a^L : U \to [0, 1]$ and $a^R : U \to [0, 1]$. It satisfies

$a^L(x) \leq a^R(x)$ for each $x \in U$. Especially, when $a^L(x) = a^R(x)$, $f(x, a)$ degenerates into a real number.

In the IVFIS, universe U is classified according to the binary relation between attributes and objects. The comparisons of interval-valued fuzzy numbers are as follows

$$\begin{aligned} f(x_i, a) \leq f(x_j, a) &\Leftrightarrow a^L(x_i) \leq a^L(x_j), a^R(x_i) \leq a^R(x_j) \\ f(x_i, a) \geq f(x_j, a) &\Leftrightarrow a^L(x_i) \geq a^L(x_j), a^R(x_i) \geq a^R(x_j) \end{aligned} \tag{2.1}$$

By sorting the attribute values of all objects under one attribute according to the above size relations, an increasing or decreasing sequence can be obtained, which can be called a criterion.

Definition 2.1. Let $IS = (U, AT, V, F)$ be an IVFIS. If all attributes are criteria, it is called an interval-valued fuzzy ordered information system [17, 18]. In general, we denote it by $IS^{\succeq}$.

An interval-valued fuzzy ordered information system with decision is a quadruple $IS_d^{\succeq} = (U, C \cup D, V, F)$, where C is a condition attribute set and D represents a decision attribute set. Meanwhile, it is satisfied a conclusion: $C \cap D = \emptyset$. The attribute values of condition attributes are interval numbers, ranging from 0 to 1.

2.2. **Partially consistent reduct in an $IVFOIS_d$.** Partially consistent reduction is based on inconsistent decision information systems. Suppose that $I = (U, C \cup D, V, F)$ is a decision information system. If $R_C \subseteq R_D$, then I is called an consistent information system, otherwise it is inconsistent. In the following, all information systems are inconsistent. For convenience, it will not be explained one by one.

Definition 2.2. Let $IS_d^{\succeq} = (U, C \cup D, V, F)$ be an $IVFOIS_d$ and decision classes $U/R_D^{\succeq} = \{D_1, D_2, \ldots, D_s\}$. $\forall\, CT \subseteq C$, partially consistent function is defined by

$$\delta_{CT}^{\succeq}(x) = \{D_i |\ [x]_{CT}^{\succeq} \subseteq D_i, x \in U\} \tag{2.2}$$

Definition 2.3. Let $IS_d^{\succeq} = (U, C \cup D, V, F)$ be an $IVFOIS_d$ and $CT \subseteq C$. If $\delta_{CT}^{\succeq}(x) = \delta_C^{\succeq}(x)$ for every $x \in U$, then CT is called as partially consistent coordination set with respect to $R_{CT}^{\succeq}$. In addition, if for any $A \subseteq CT$, it does not satisfy that $\delta_{CT}^{\succeq}(x) = \delta_A^{\succeq}(x)$. Then CT is called as partially consistent reduction in an $IVFOIS_d$.

Definition 2.4. Let $IS_d^{\succeq} = (U, C \cup D, V, F)$ be an $IVFOIS_d$ and denote

$$\mathcal{D}_{\succeq C}^{\delta} = \{(x, y) |\ \delta_C(x) \supset \delta_C(y)\} \tag{2.3}$$

$$\mathcal{D}_{\succeq C}^{\delta}(x, y) = \begin{cases} \{a_j \in C |\ f(x, a_j) > f(y, a_j)\} & (x, y) \in \mathcal{D}_{\succeq C}^{\delta} \\ \emptyset & (x, y) \notin \mathcal{D}_{\succeq C}^{\delta} \end{cases} \tag{2.4}$$

And $\mathcal{D}_{\succeq C}^{\delta}(x, y)$ is called a partially consistent identifiable attribute set in an $IVFOIS_d$. Based on these above, partially consistent identifiable matrix can be constructed by

$$Dis_{\succeq C}^{\delta} = (\mathcal{D}_{\succeq C}^{\delta}(x, y) |\ x, y \in U)_{|U| \times |U|} \tag{2.5}$$

Definition 2.5. Let $IS_d^{\succeq} = (U, C \cup D, V, F)$ be an $IVFOIS_d$ and the minimal disjunctive normal form $M_{\succeq min}^{\delta}$ of identifiable formula ($M_{\succeq AT}^{\delta} = \wedge\{\vee\{\ a_j \mid a_j \in \mathcal{D}_{\succeq C}^{\delta}(x, y), (x, y) \in U\}\}$) is defined by

$$M_{\succeq min}^{\delta} = \vee_{t=1}^{p}(\wedge_{j=1}^{q_j} a_j) \tag{2.6}$$

Meanwhile, $B_s = \{a_j | j = 1, 2, \ldots, q_j\}$, then $\{B_s | s = 1, 2, \ldots, p\}$ are the sets formed by all partially consistent reductions.

2.3. **Discernibility information tree (DIT).** DIT is a virtual ordered tree [19], in which the order of nodes is arranged from left to right according to the order of condition attributes, and the arrangement can not be reversed or changed. The following is a detailed introduction to the characteristics of DIT.

(1) In every node of DIT, the number of branch nodes it can have must not exceed the number of condition attributes.
(2) There are many branch nodes extending from the root of the DIT. The branch node of each node is called a child node. Meanwhile, for a child node, the node is called a parent node. a node without a child node is called a leaf node. In the branch nodes of same node, the repeated nodes are placed in the nodes with the same name, and no new branches are generated.

3. The method of the partially consistent reduction based on DIT$^{\succeq}$ in an $IVFOIS_d$

In the section, based on the partially consistent identifiable matrix, we will construct the discernibility information tree under dominance relation (DIT$^{\succeq}$) in an $IVFOIS_d$. In addition, we wil discuss the attribute reduction based on DIT$^{\succeq}$ and study the related properties.

Algorithm 1: The algorithm of DIT$^{\succeq}$ based on the partially consistent identifiable matrix in an $IVFOIS_d$

Input : an inconsistent $IS_d^{\succeq} = (U, C \cup D, V, F)$.
Output : a discernibility information tree (DIT$^{\succeq}$) in $IVFOIS_d$.

begin
  step 1:
  create the root node TN of the DIT$^{\succeq}$;
  $TN \leftarrow \emptyset$;
  step 2:
  according to $\mathcal{D}_{\succeq C}^{\delta} = \{(x, y) | \delta_C(x) \supset \delta_C(y)\}$, it obtains the partially consistent identifiable attribute set $\mathcal{D}_{\succeq C}^{\delta}(x, y)$ for any $x, y \in U$. Furthermore, partially consistent identifiable matrix can be obtained. Suppose that $\forall CT \subseteq C$ satisfies $CT = \mathcal{D}_{\succeq C}^{\delta}(x, y)$ for any $x, y \in U$. Then, the sequence of elements in CT is arranged in sequence from left to right based on the order of conditional attributes in the information system.
  Meanwhile, $TN \leftarrow$ root node trail based on CT;

```
    step 3:
    if CT == ∅ then
        select the other of the identifiable set and insert it into the DIT⪰.
    else
        choose the leftmost attribute called a in CT;
        if exist a attribute a of child node CN in root node TN then
            if CN is a leaf node then
                Choose the strategy of non-expanding trail and do not construct
                branches for other attributes in CT;
            end
            if  a is the last attribute in CT then
                By using subtree deletion strategy, all existing child notes (subtrees)
                of node CN on DIT⪰ are removed;
            end
            else
                TN = CN;
            end
        else
            A new child node N is created for the node TN. At the same time, the
            attribute name of the node N is set initially to a, and then connected to
            the node with the same attribute name as the node. Finally, a same name
            node chain is formed according to the above method;
            TN = N;
        end
        CT ⇐ CT − {a};
    end
    return  : DIT⪰;
end
```

In algorithm 1, the construction process of DIT$^{\succeq}$ is given. In this process, two strategies are adopted, namely, non-expanding trail strategy and deleting subtree strategy. The specific method is to select the shortest trail in a node's sub-node, without generating other extended trails of the node. For example, both attribute sets $\{a, c\}$ and $\{a, c, d, f\}$ take node a as the parent node, but only choose to construct shorter trail $\langle a, c\rangle$ on DIT$^{\succeq}$. At the same time, the same partially identifiable attribute set is mapped to the same trail.

Therefore, the DIT$^{\succeq}$ based on identifiable matrix constructed in an $IVFOIS_d$. Compression storage of partially consistent identifiable matrices is realized. Thus, reduce the time and space complexity of construction DIT$^{\succeq}$.

Theorem 3.1. *$DIT^{\succeq}$ based on partially consistent identifiable matrix in an $IVFOIS_d$ contains all trails needed for attribute reduction of information system.*

Proof. Let the set DS be identifiable attribute sets composed by all trails of DIT$^{\succeq}$. According to the establishment steps of DIT$^{\succeq}$, we can see that $DS \subseteq Dis^{\delta}_{\succeq C}$. As to $\forall(x, y) \in U \times U$, it satisfies the conclusion that $Dis^{\delta}_{\succeq C}(x, y) \in Dis^{\delta}_{\succeq C}$. There are $\exists\ (x_0, y_0)$ and $Dis^{\delta}_{\succeq C}(x_0, y_0) \in DS$ such that $Dis^{\delta}_{\succeq C}(x_0, y_0) \subseteq Dis^{\delta}_{\succeq C}(x, y)$. From partially consistent identifiable matrix, we know that $(Dis^{\delta}_{\succeq C}(x_0, y_0)) \wedge (Dis^{\delta}_{\succeq C}(x, y)) =$

$Dis^{\delta}_{\succeq C}(x,y)$. Therefore, DIT$^{\succeq}$ based on partially consistent identifiable matrix in $IVFOIS_d$ contains all trails needed for attribute reduction of information system. □

Theorem 3.2. *In the DIT$^{\succeq}$, the union of partially consistent identifiable attribute sets corresponding to all trails which have only one node composes the $Core^{\succeq}_{D}(C)$ of the decision table.*

Proof. If there is a node named a on the DIT$^{\succeq}$ and there is a path only containing the node a, then in the partially consistent identifiable matrix, there exist the attribute set $\{a\}$ which only contains one element named a. Furthermore, the attribute set consisting of a single element in the partially consistent identifiable matrix is necessary attribute. If $\{a\}$ $(a \in C)$ is a single element set in the partially consistent identifiable matrix, then it is necessary. All necessary attributes constitute the relative core $Core^{\succeq}_{D}(C)$ of the information system. Finally, the above theory is proved. □

Theorem 3.3. *Let CT be a condition attributes set which is composed of all child notes of the root node in the DIT$^{\succeq}$. Then $\delta^{\succeq}_{CT}(x) = \delta^{\succeq}_{C}(x)$ in an $IVFOIS_d$.*

Proof. Based on the theorem 3.1, DIT$^{\succeq}$ based on partially consistent identifiable matrix in an $IVFOIS_d$ contains all attribute trails needed for attribute reduction of information system. And DS represents identifiable attribute sets composed by all trails of DIT$^{\succeq}$ based on the partially consistent identifiable matrix. So, we have $\mathcal{D}^{\delta}_{\succeq C}(x,y) \cap CT \neq \emptyset$ for any $(x,y) \in \mathcal{D}^{\delta}_{\succeq C}$. According to the correlation theorem of partially consistent sets, we know $\delta^{\succeq}_{CT}(x) = \delta^{\succeq}_{C}(x)$.

Given an $IS^{\succeq}_{d} = (U, C \cup D, V, F)$, the cardinality of the universe is $|U|$ and the cardinality of conditional attribute is $|C|$. Then, in the partially consistent identifiable matrix, the number of non-empty condition attribute subsets that can be obtained is at most $|U|^2$. Assuming that the number of nonempty subset of conditional attributes is P and $P \ll |U|^2$ generally. The DIT$^{\succeq}$ can have at most P different trails, with at most $|C|$ nodes per trail. Therefore, the total number of nodes on DIT$^{\succeq}$ will not exceed $|C| * P$. In addition, there are many trails with the same parent nodes that can be used as shared prefixes, thus reducing the space occupation of repetitive elements, making the actual number of nodes of DIT$^{\succeq}$ much smaller than $|C| * P$. To sum up, the spatial complexity of the DIT$^{\succeq}$ is $O(|C||U|^2)$.

In the process of constructing DIT$^{\succeq}$, the maximum number of insertion trails into the DIT$^{\succeq}$ is $|U|^2$. And in the process of building the trail, compare and insert $|C|$ nodes at most and delete P_i nodes $(i = 1, 2 \ldots, |U|^2)$. The complexity of the DIT$^{\succeq}$ is $|C||U|^2 + (P_1 + P_2 + \cdots + P_{|U|^2})$. It is known that there are at most $|C||U|^2$ nodes in a DIT$^{\succeq}$, and the value of $(P_1 + P_2 + \cdots + P_{|U|^2})$ is at most $|C||U|^2$. Hence, the time complexity of the DIT$^{\succeq}$ is also $O(|C||U|^2)$.

Next, a partially consistent reduction algorithm (Algorithm 2) in an $IVFOIS_d$ is given. According to the trail of the DIT$^{\succeq}$, the iteration process in Algorithm 2 selects the necessary condition attributes of the DIT$^{\succeq}$ from right to left, and deletes the other trails that contains the necessary attributes.

Algorithm 2: The partially consistent reduction algorithm based on $\text{DIT}^{\succeq}$ in an $IVFOIS_d$

Input : an $\text{DIT}^{\succeq}$ in $IVFOIS_d$.
Output : a complete partially consistent reduction CT

begin
  step 1:
  $CT \leftarrow \emptyset$;
  step 2:
  get the trail of only one node in the $\text{DIT}^{\succeq}$ based on the partially consistent identifiable matrix, and place the corresponding attributes of these nodes in a set S ;
  step 3:
  if $S \neq \emptyset$ **then**
    for any $a \in S$, remove all trails containing attribute a in the $\text{DIT}^{\succeq}$ obtained from the partially consistent identifiable matrix;
    $\text{NDIT}^{\succeq} \leftarrow$ new $\text{DIT}^{\succeq}$;
    $CT \leftarrow S$;
  end
  step 4:
  while *the $DIT^{\succeq}$ obtained from the partially consistent identifiable matrix contains not only root nodes;* **do**
    select the right child node (denoted by b) of the $\text{NDIT}^{\succeq}$ and delete all trails containing the node b;
    $\text{DIT}^{\succeq} \leftarrow$ new changed $\text{NDIT}^{\succeq}$;
    $CT = CT \cup \{b\}$;
  end
  return : CT;
end

In this paper, two algorithms are used to obtain partially consistent reduction. This method is a complete reduction algorithm. That is to satisfy the following two conditions (suppose $CT \subseteq C$ is a partially consistent reduction)

- $R^{\succeq}_{CT} = R^{\succeq}_{C}$;
- $\forall a \in R^{\succeq}_{C}$, $R^{\succeq}_{CT-\{a\}} \neq R^{\succeq}_{C}$.

Thus, for partially consistent identifiable matrices, the above two conditions can be transformed into the following two points, and the completeness of partially consistent reduction can also be preserved.

- $\forall (x,y) \in \mathcal{D}^{\delta}_{\succeq C}(x,y)$, then $\mathcal{D}^{\delta}_{\succeq C}(x,y) \cap CT \neq \emptyset$;
- $\forall a \in CT$, $\exists \mathcal{D}^{\delta}_{\succeq C}(x_i,y_j)$ such that $\mathcal{D}^{\delta}_{\succeq C}(x_i,y_j) \cap (CT - \{a\}) = \emptyset$.

According to the Theorem 3.1, we know that $\text{DIT}^{\succeq}$ based on partially consistent identifiable matrix in an $IVFOIS_d$ contains all attribute trails needed for attribute reduction of information system. It satisfies that the following when DS is expressed as a set of all trails in a $\text{DIT}^{\succeq}$.

- $\forall T \in DS$, $T \cap CT \neq \emptyset$;
- $\forall a \in CT$, $\exists T \in DS$ such that $T \cap (CT - \{a\}) = \emptyset$.

To prove that Algorithm 2 is complete, we only need to prove that the attribute set CT obtained by Algorithm 2 satisfies both of the above conditions. In Algorithm 2, $T \cap CT \neq \emptyset$ for any $T \in DS$. Combined Theorem 3.2, the attribute set in the DIT$^{\succeq}$ consisting of a single element in the partially consistent identifiable matrix is necessary attribute of the condition attribute set in the decision table. All necessary attributes constitute the relative core $Core_D^{\succeq}(C)$ of the information system. The second step of Algorithm 2 is used to get the core $Core_D^{\succeq}(C)$ of partially consistent reduction in $IVFOIS_d$. Then delete all trails which contain core elements in the DIT$^{\succeq}$. Let $P = CT - Core_D^{\succeq}(C)$. Assuming that b is the rightmost element in P, the attribute corresponding to the rightmost child of the root node in the current DIT$^{\succeq}$ must be b and a subtree whose root is this node must not contain any attributes in $P - \{a\}$. Therefore, $\forall a \in CT$, $\exists T \in DS$ such that $T \cap (CT - \{a\}) = \emptyset$. Similarly, other elements in CT also meet above conditions. To sum up, the partially consistent reduction obtained by Algorithm 2 is a complete reduction.

4. Example analysis

Many reality situations in life are not always a certain value rather often a rough range, so interval-value information system is often more suitable for practical problems. In order to avoid too big difference in the range of interval values, the data is pre-processed. Next, the inconsistent decision information system is given in Table 1. There are ten objects $U = \{x_1, x_2, x_3, x_4, x_5, x_6, x_7, x_8, x_9, x_{10}\}$, a condition attribute set $C = \{a_1, a_2, a_3, a_4, a_5\}$ and a decision attribute set $D = \{d_1, d_2, d_3\}$.

Table 1: an interval-valued fuzzy ordered decision table

U	a_1	a_2	a_3	a_4	a_5	d_1	d_2	d_3
x_1	[0.3,0.6]	[0.2,0.5]	[0.5,0.7]	[0.3,0.6]	[0.4,0.7]	1	2	1
x_2	[0.2,0.5]	[0.1,0.2]	[0.4,0.7]	[0.5,0.7]	[0.3,0.6]	2	3	1
x_3	[0.2,0.5]	[0.1,0.2]	[0.4,0.7]	[0.3,0.6]	[0.3,0.6]	3	4	2
x_4	[0.1,0.2]	[0.1,0.2]	[0.3,0.6]	[0.5,0.7]	[0.3,0.6]	2	3	1
x_5	[0.5,0.7]	[0.4,0.7]	[0.7,0.9]	[0.4,0.7]	[0.6,0.8]	3	4	2
x_6	[0.2,0.7]	[0.4,0.7]	[0.4,0.8]	[0.4,0.7]	[0.6,0.8]	1	2	1
x_7	[0.3,0.6]	[0.3,0.6]	[0.5,0.7]	[0.3,0.6]	[0.5,0.7]	2	3	1
x_8	[0.4,0.7]	[0.4,0.7]	[0.6,0.8]	[0.5,0.7]	[0.6,0.8]	3	4	2
x_9	[0.4,0.7]	[0.5,0.7]	[0.6,0.8]	[0.2,0.7]	[0.7,0.9]	0	2	1
x_{10}	[0.5,0.7]	[0.5,0.7]	[0.7,0.9]	[0.2,0.5]	[0.7,0.9]	0	2	1

By computing, we have dominance classes in $IVFOIS_d$.

$[x_1]_C^{\succeq} = \{x_1, x_5, x_7, x_8\}$, $[x_2]_C^{\succeq} = \{x_2, x_8\}$,
$[x_3]_C^{\succeq} = \{x_1, x_2, x_3, x_5, x_6, x_7, x_8\}$, $[x_4]_C^{\succeq} = \{x_2, x_4, x_8\}$,
$[x_5]_C^{\succeq} = \{x_5\}$, $[x_6]_C^{\succeq} = \{x_5, x_6, x_8\}$,
$[x_7]_C^{\succeq} = \{x_5, x_7, x_8\}$, $[x_8]_C^{\succeq} = \{x_8\}$,
$[x_9]_C^{\succeq} = \{x_9\}$, $[x_{10}]_C^{\succeq} = \{x_{10}\}$.

$D_1 = [x_3]_D^{\succeq} = [x_5]_D^{\succeq} = [x_8]_D^{\succeq} = \{x_3, x_5, x_8\}$,
$D_2 = [x_2]_D^{\succeq} = [x_4]_D^{\succeq} = [x_7]_D^{\succeq} = \{x_2, x_3, x_4, x_5, x_7, x_8\}$,
$D_3 = [x_1]_D^{\succeq} = [x_6]_D^{\succeq} = \{x_1, x_2, x_3, x_4, x_5, x_6, x_7, x_8\}$,
$D_4 = [x_9]_D^{\succeq} = [x_{10}]_D^{\succeq} = \{x_1, x_2, x_3, x_4, x_5, x_6, x_7, x_8, x_9, x_{10}\}$.

Due to $R_C \nsubseteq R_D$, the information system is inconsistent. Then we have

$\delta_C^{\succeq}(x_1) = \delta_C^{\succeq}(x_3) = \delta_C^{\succeq}(x_6) = \{D_3, D_4\}$,
$\delta_C^{\succeq}(x_2) = \delta_C^{\succeq}(x_4) = \delta_C^{\succeq}(x_7) = \{D_2, D_3, D_4\}$,
$\delta_C^{\succeq}(x_5) = \delta_C^{\succeq}(x_8) = \{D_1, D_2, D_3, D_4\}$,
$\delta_C^{\succeq}(x_9) = \delta_C^{\succeq}(x_{10}) = \{D_4\}$.

Then, based on the definition of partially consistent identifiable matrix, the identifiable matrix under dominance relation is obtained in Table 2.

Table 2: an partially consistent identifiable matrix in $IVFOIS_d$

U	x_1	x_2	x_3	x_4	x_5	x_6	x_7	x_8	x_9	x_{10}
x_1	$\varnothing$	$\varnothing$	$\varnothing$	$\varnothing$	$\varnothing$	$\varnothing$	$\varnothing$	$\varnothing$	$\varnothing$	$\{a_4\}$
x_2	$\{a_4\}$	$\varnothing$	$\{a_4\}$	$\varnothing$	$\varnothing$	$\{a_4\}$	$\varnothing$	$\varnothing$	$\{a_4\}$	$\{a_4\}$
x_3	$\varnothing$	$\varnothing$	$\varnothing$	$\varnothing$	$\varnothing$	$\varnothing$	$\varnothing$	$\varnothing$	$\varnothing$	$\{a_4\}$
x_4	$\{a_4\}$	$\varnothing$	$\{a_4\}$	$\varnothing$	$\varnothing$	$\{a_4\}$	$\varnothing$	$\varnothing$	$\{a_4\}$	$\{a_4\}$
x_5	C	$\{a_1, a_2, a_3, a_5\}$	C	$\{a_1, a_2, a_3, a_5\}$	$\varnothing$	$\{a_1, a_3\}$	C	$\varnothing$	$\{a_1, a_3, a_4\}$	$\{a_4\}$
x_6	$\varnothing$	$\varnothing$	$\varnothing$	$\varnothing$	$\varnothing$	$\varnothing$	$\varnothing$	$\varnothing$	$\{a_4\}$	$\{a_4\}$
x_7	$\{a_2, a_5\}$	$\varnothing$	$\{a_1, a_2, a_3, a_5\}$	$\varnothing$	$\varnothing$	$\varnothing$	$\varnothing$	$\varnothing$	$\varnothing$	$\{a_4\}$
x_8	C	$\{a_1, a_2, a_3, a_5\}$	C	$\{a_1, a_2, a_3, a_5\}$	$\varnothing$	$\{a_1, a_3, a_4\}$	C	$\varnothing$	$\{a_4\}$	$\{a_4\}$
x_9	$\varnothing$	$\varnothing$	$\varnothing$	$\varnothing$	$\varnothing$	$\varnothing$	$\varnothing$	$\varnothing$	$\varnothing$	$\varnothing$
x_{10}	$\varnothing$	$\varnothing$	$\varnothing$	$\varnothing$	$\varnothing$	$\varnothing$	$\varnothing$	$\varnothing$	$\varnothing$	$\varnothing$

Based on the original method of disjunctive and conjunctive formulas, the identifiable attribute set in the identifiable matrix is calculated, and the following conclusions are obtained

$$
\begin{aligned}
&(a_1 \vee a_2 \vee a_3 \vee a_4 \vee a_5) \wedge (a_1 \vee a_2 \vee a_3 \vee a_5) \wedge (a_1 \vee a_3 \vee a_4) \\
&\wedge (a_1 \vee a_3) \wedge (a_2 \vee a_5) \wedge (a_4) \\
=&(a_2 \vee a_5) \wedge (a_1 \vee a_3) \wedge (a_4) \\
=&(a_1 \wedge a_2 \vee a_4) \vee (a_1 \wedge a_4 \vee a_5) \vee (a_2 \wedge a_3 \vee a_4) \vee (a_3 \wedge a_4 \vee a_5).
\end{aligned} \tag{4.1}
$$

So $\{a_1, a_2, a_4\}$, $\{a_1, a_4, a_5\}$,$\{a_2, a_3, a_4\}$, $\{a_3, a_4, a_5\}$ are partially consistent reductions. Next, the method of discernibility information tree is applied to find partially consistent reduction. Firstly, the root node is created, then the attribute order of the identifiable attribute set are arranged in the order of condition attributes. The first trail $\langle a_1, a_2, a_3, a_4, a_5\rangle$ to create is based on identifiable attribute set $\{a_1, a_2, a_3, a_4, a_5\}$ and insert this trail into the $\text{DIT}^{\succeq}$. For the second identifiable attribute set C, no new trail is constructed because the trail $\langle a_1, a_2, a_3, a_4, a_5\rangle$ corresponding to C already exists in the $\text{DIT}^{\succeq}$. Similarly, map all the same identifiable attribute sets into the same trail. Next, create corresponding trails $\langle a_1, a_2, a_3, a_5\rangle$ for third identifiable attribute set $\{a_1, a_2, a_3, a_5\}$. This trail has the same prefix $\langle a_1, a_2, a_3\rangle$ as the trail $\langle a_1, a_2, a_3, a_4, a_5\rangle$. So, only create a branch node a_5 after node a_3. Then, the trail $\langle a_1, a_3\rangle$ corresponding to the identifiable attribute set $\{a_1, a_3\}$ is constructed. For identifiable attribute sets $\{a_1, a_3, a_4\}$, nodes a_4 can be regarded as useless because they have the same node $\{a_1, a_3\}$ as the trail $\langle a_1, a_3\rangle$. Therefore, two identifiable attribute sets are mapped onto the trail $\langle a_1, a_3\rangle$. And then get the trail $\langle a_2, a_5\rangle$ and trail $\langle a_4\rangle$. Finally, the $\text{DIT}^{\succeq}$ in an $IVFOIS_d$ is obtained as shown in Figure 1.

Based on the $\text{DIT}^{\succeq}$ and Algorithm 2, partially consistent reduction of inconsistent information systems is carried out. Find the trail $\langle a_4\rangle$ that contains only one node

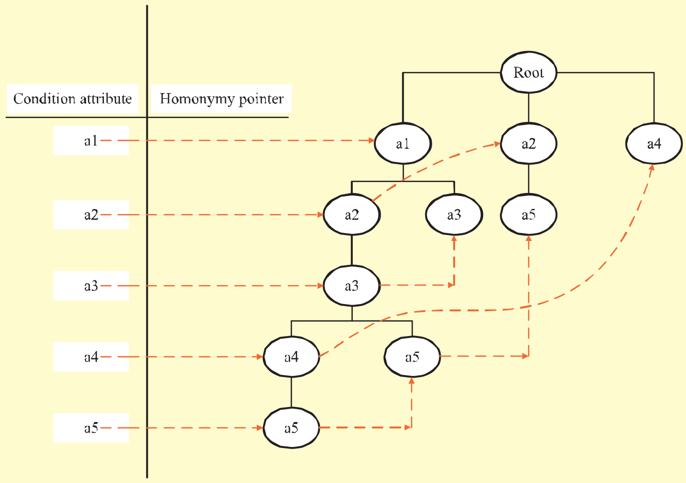

FIGURE 1

a_4 in the DIT$^{\succeq}$, and delete the other trail $\langle a_1, a_2, a_3, a_4, a_5\rangle$, $\langle a_4\rangle$ that contains the node a_4. And denote $CT = \{a_4\}$. In the moment, select the most right child node a_2 of the root node in the changed DIT$^{\succeq}$ and $CT = CT \cup \{a_2\} = \{a_2, a_4\}$. Then delete all trails in the tree containing node a_2, that is, deleting $\langle a_1, a_2, a_3, a_5\rangle$ and $\langle a_2, a_5\rangle$. Now, there is only trail $\langle a_1, a_3\rangle$. Keep the child nodes a_1 of the root node and $CT = CT \cup \{a_1\} = \{a_1, a_2, a_4\}$. Finally, a partially consistent reduction $CT = \{a_1, a_2, a_4\}$ is obtained. Compared with the original algorithm, the correctness and effectiveness of the method are proved.

5. CONCLUSIONS

In this paper, under the background of interval-valued fuzzy ordered decision information system, a data model based on DIT$^{\succeq}$ is proposed, which can compress and store partially consistent identifiable matrices. That is, DIT$^{\succeq}$ based on partially consistent identifiable matrices. Comparing with previous reductions by conjunctive and disjunctive formulas, this paper reduces the space-time complexity of partially consistent reductions in inconsistent $IVFOIS_d$ by DIT$^{\succeq}$. However, the order in which the set of attributes is inserted into the DIT$^{\succeq}$ is the original order of condition attributes. Regardless of the impact of attribute importance on the establishment of DIT$^{\succeq}$ based on partially consistent identifiable matrices, this is a need for improvement. At the same time, attribute reduction through DIT$^{\succeq}$ can only get one reduction result, not all. Thus, the next work is to optimize DIT$^{\succeq}$ combined with attribute importance, and try to get all the reduction results of information system.

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Manuscript received August 31, 2018
revised October 15, 2018

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