Rough Fuzzy Set Based on Logical Disjunct Operation of Variable Precision and Grade in Ordered Information System

Yanting Guo [1], Weihua Xu[1,2]
1. School of mathematics and statistics, Chongqing University of Technology, Chongqing, 400054
E-mail: 1290237984@qq.com
2. The Key Laboratory of Intelligent Perception and Systems for High-Dimensional Information,
Ministry of Education, Nanjing University of Science and Technology, Nanjing, 210094
E-mail: chxuwh@gmail.com

Abstract: Combining the respective advantages of relative and absolute quantitative information, this paper proposes the model of the rough fuzzy set, which is based on logical disjunct operation of variable precision and grade in ordered information system. Moreover, the basic structure and precise description of rough set regions of the model are studied, and some important properties of the model are investigated carefully. Further, the significance of this study is interpreted by combining the analysis of an illustrative case study about the medical diagnosis.

Key Words: Graded Rough Fuzzy Set, Logical Disjunct Operation, Ordered Information System, Variable Precision Rough Fuzzy Set

1 INTRODUCTION

Rough set theory [1] is an effective tool to deal with all kinds of incomplete information. It has been widely applied in many fields, such as machine learning, knowledge discovery, data mining, decision support and analysis, information security, networking, cloud computing and biological information processing. Ordered information systems based on dominance relations [2] as a basic tool of knowledge representation in the rough set theory, can solve many practical problems. The main idea of rough set theory [3-7] is that undefinable sets can be approximated by definable sets, namely the upper and lower approximation set. The lower approximation set is the union of equivalence classes completely included in the undefinable set and the upper approximation set is the union of equivalence classes partially included in the undefinable set.

The classical rough set is a model which not allows any uncertainty, so it lacks fault tolerance capability. In real life, however, a lot of problems to solve are allowed to have certain fault tolerance. Therefore, people proposed various generalized rough set modes [8-11], such as the decision rough set model [12]proposed by Yao, Ziarko's variable precision rough set model [13], parameterized rough set model [14], Herbert and Yao's game rough set model [15], graded rough set model [16]. The variable precision rough set model and graded rough set model reflect absolute quantitative information and relative quantitative information of the approximation space, respectively. The two models are both independent and complementary. Through the combination of variable precision and graded rough set theory [16-21], some higher level extension models are established. These models can provide a more detailed description for the approximation space. For example, by considering the combination of absolute and relative quantitative information in the upper and lower approximation set, two kinds of double-quantitative rough set model can be obtained [18]. From the perspective of logic, a generalized model [20-21] obtained by considering the absolute and relative information. Quantitative composite models with fault tolerance ability can be used for the qualitative analysis of knowledge, which is of great significance and is a very good research direction. Therefore, the rough fuzzy set model based on logical disjunct operation of variable precision and grade are proposed in the ordered information system. This model provides a more accurate approximation of fuzzy concepts. Meanwhile the research significance of the model is analyzed through a specific example about medical diagnosis.

This work is supported by Natural Science Foundation of China (No. 61105041, No. 61472463, No. 61402064), National Natural Science Foundation of CQ CSTC (No. cstc2013jcyjA40051, cstc2015jcyjA40053), and Graduate Innovation Foundation of Chongqing University of Technology (No. YCX2015227, YCX2014236), and Graduate Innovation Foundation of CQ (No.CYS15223).

2 PRELIMINARY

2.1 Variable Precision Rough Set Theory

Let $|C|$ denote the cardinal number of the set C and $c([x]_R, X)$ denote error classification rate of the equivalence class $[x]_R$ with respect to X . $c([x]_R, X)$ can be calculated by $c([x]_R, X) = 1 - |[x]_R \cap X| / |[x]_R|$, where β is a real number between 0 and 1. β is called adjustable error classification level and $1-\beta$ is called precision. Suppose $\overline{R_\beta}X$ and $\underline{R_\beta}X$ denote the upper and lower approximation sets of X when the precision is $1-\beta$, where $\overline{R_\beta}X = \cup\{[x]_R : c([x]_R, X) < 1-\beta\}$ and $\underline{R_\beta}X = \cup\{[x]_R : c([x]_R, X) \le \beta\}$. If $\overline{R_\beta}X \ne \underline{R_\beta}X$, then X is rough when the precision is $1-\beta$; otherwise X is precise. The upper approximation set $\overline{R_\beta}X$ is the union of equivalence classes satisfying $c([x]_R, X) < 1-\beta$, and the lower approximation set $\underline{R_\beta}X$ is the union of equivalence classes satisfying $c([x]_R, X) \le \beta$.

It should be noted that the range of parameter β is generally $[0, 0.5)$ in the variable precision rough set model [11]. Of course, the range can also be is limited to $[0.5,1]$, which can be regarded as promotional rough set theory [2]. Without loss of generality, the range of the parameter β is limited to $[0,1]$ in this paper. It is evident that the former $\beta \in (0, 0.5]$ and the latter $\beta \in (0.5,1]$ are symmetrical. Therefore, in general, we only need to study the former case $\beta \in (0, 0.5]$ and the latter case can be obtained similarly.

2.2 Graded Rough Set Theory

Let $\overline{R_k}X$ and $\underline{R_k}X$ denote the upper and lower approximation sets of X when the grade is any natural number k. $\overline{R_k}X$ and $\underline{R_k}X$ can be defined as $\overline{R_k}X = \cup\{[x]_R : |[x]_R \cap X| > k\}$ and $\underline{R_k}X = \cup\{[x]_R : |[x]_R| - |[x]_R \cap X| \le k\}$. If $\overline{R_k}X \ne \underline{R_k}X$, then X is rough when the grade is k; otherwise X is precise. The upper approximation $\overline{R_k}X$ is the union of some equivalence classes, and all these equivalence classes must satisfy $|[x]_R \cap X| > k$, and the lower approximation $\underline{R_k}X$ is the union of some equivalence classes, and all these equivalence classes must satisfy $|[x]_R| - |[x]_R \cap X| \le k$.

2.3 The Rough Set of Ordered Information Systems

Let $I = (U, A, V, F)$ be an information system, where the universe of discourse $U = \{x_1, x_2, \cdots, x_n\}$ is a nonempty finite set of objects, A is a nonempty finite set of attributes, $V = \bigcup_{a\in A} V_a$, and $F = \{f \mid U \to V_a, a \in A\}$ is the relationship set from U to A, and V_a is the finite domain of a.

For given information systems, if there is a partial ordered relation on V_a, then the attribute a is called a criterion. $\forall x, y \in U$, $x \ge_a y$ represents that x with respect to the criterion a is superior to y, and $x \ge_a y \Leftrightarrow f(x,a) \ge_a f(y,a)$. For $\forall B \subseteq A$, $x \ge_B y$ represents that x with respect to the criterion set B is superior to y and $x \ge_B y \Leftrightarrow$ it is true that $f(x,a) \ge_a f(y,a)$ for any $a \in B$. If each attribute is a criterion in I, then I is called a ordered information system which can be denoted by $I^{\ge} = (U, A, V, F)$. Sometimes for convenience, it can also be denoted by $I^{\ge} = (U, A)$.

Let $I^{\ge} = (U, A, V, F)$ be an ordered information system. For $\forall B \subseteq A$, the dominance relation $R_B^{\ge}$ of the criterion set B can be defined as $R_B^{\ge} = \{(x, y) \in U \times U : \forall a \in B, f(x,a) \le f(y,a)\}$. The dominance class of x with respect to $R_B^{\ge}$ is defined as $[x]_B^{\ge} = \{y \in U : (x, y) \in R_B^{\ge}\} = \{y \in U \mid \forall a \in B, f(x,a) \le f(y,a)\}$. In the ordered information system, for $\forall X \subseteq U$, the upper and lower approximation sets of X with respect to $R_B^{\ge}$ are defined as $\overline{R_A^{\ge}}(X) = \cup\{[x]_A^{\ge} \mid [x]_A^{\ge} \cap X \ne \varnothing\}$ and $\underline{R_A^{\ge}}(X) = \cup\{[x]_A^{\ge} \mid [x]_A^{\ge} \subseteq X\}$. And the positive, negative and boundary region of X with respect to $R_B^{\ge}$ are defined as $posR_A^{\ge}(X) = \underline{R_A^{\ge}}(X)$, $negR_A^{\ge}(X) = \sim \overline{R_A^{\ge}}(X)$ and $bndR_A^{\ge}(X) = \overline{R_A^{\ge}}(X) - \underline{R_A^{\ge}}(X)$. If $\overline{R_A^{\ge}}X \ne \underline{R_A^{\ge}}X$, then X with respect to $R_B^{\ge}$ is rough; if $\overline{R_A^{\ge}}X = \underline{R_A^{\ge}}X$, then X with respect to $R_B^{\ge}$ is precise or describable[2].

3 ROUGH FUZZY SET THEORY

In this section, a new rough set model is proposed, namely the rough fuzzy set model based on logical disjunct operation of variable precision and grade in the ordered information system.

Let $I^{\ge} = (U, A, V, F)$ be an ordered information system. For $\forall \widetilde{X} \in F(U)$, $R^{\ge}$ is the dominance relation of $I^{\ge}$. In the ordered information system $I^{\ge}$, generalized models of classical variable precision rough sets and graded rough sets are proposed through the promotion from classic sets to fuzzy sets. Definitions of variable precision rough fuzzy set, the graded rough fuzzy set, the rough fuzzy set based on the dominance relation and the rough fuzzy set based on logical disjunct operation of variable precision and grade are proposed.

Definition 3.1 The resolution ratio $c([x]_R^{\ge}, \widetilde{X})$ of the dominance class $[x]_R^{\ge}$ with respect to the fuzzy set $\widetilde{X}$ is defined as $c([x]_R^{\ge}, \widetilde{X}) = 1 - \sum_{y\in[x]_R} \widetilde{X}(y) \Big/ |[x]_R^{\ge}|$, where the real number $\beta \in [0,1]$ is called adjustable resolution level and $1-\beta$ is called precision. For any object x of U, the precision is $1-\beta$, the upper and lower approximation sets of $\widetilde{X}$ with respect to the dominance relation $R^{\ge}$ are defined as follows:

$$(\overline{R_\beta^{\ge}}\widetilde{X})(x) = \bigvee_{y\in[x]_R^{\ge}} \{\widetilde{X}(y) : c([x]_R^{\ge}, \widetilde{X}) < 1-\beta\},$$

$$(\underline{R_\beta^{\ge}}\widetilde{X})(x) = \bigwedge_{y\in[x]_R^{\ge}} \{\widetilde{X}(y) : c([x]_R^{\ge}, \widetilde{X}) \le \beta\}.$$

If $\overline{R_\beta^{\ge}}\widetilde{X} \ne \underline{R_\beta^{\ge}}\widetilde{X}$, then $\widetilde{X}$ with respect to $R^{\ge}$ is rough when the precision is $1-\beta$; otherwise $\widetilde{X}$ with respect to $R^{\ge}$ is precise. It is true that $\overline{R_\beta^{\ge}}\widetilde{X}$ and $\underline{R_\beta^{\ge}}\widetilde{X}$ are both fuzzy sets on the universe of discourse U. The membership degree of x with respect to $\overline{R_{\beta}^{\geq}}\widetilde{X}$ is the supremum of x with respect to dominance classes whose the resolution ratio is less than $1-\beta$; the membership degree of x with respect to $\overline{R_{\beta}^{\geq}}\widetilde{X}$ is the infimum of x with respect to dominance classes whose the resolution ratio is not more than β.

Definition 3.2 Let $N_{0.5}=\{0.5n \mid n\in N\}$, $k\in N_{0.5}$, where N is the natural numbers set. For $\forall x\in U$, when the grade is k, the upper and lower approximation set of $\widetilde{X}$ with respect to $R^{\geq}$ are defined as follows:

$$(\overline{R_{k}^{\geq}}\widetilde{X})(x)=\vee_{y\in[x]_R^{\geq}}\{\widetilde{X}(y):\sum_{y\in[x]_R^{\geq}}\widetilde{X}(y)>k\},$$

$$(\underline{R_{k}^{\geq}}\widetilde{X})(x)=\wedge_{y\in[x]_R^{\geq}}\{\widetilde{X}(y):\sum_{y\in[x]_R^{\geq}}1-\widetilde{X}(y)\leq k\}.$$

If $\overline{R_{k}^{\geq}}\widetilde{X}\neq\underline{R_{k}^{\geq}}\widetilde{X}$, then $\widetilde{X}$ with respect to $R^{\geq}$ is rough when the grade is k; otherwise $\widetilde{X}$ is precise. It is evident that $\overline{R_{k}^{\geq}}\widetilde{X}$ and $\underline{R_{k}^{\geq}}\widetilde{X}$ both are fuzzy sets on U. The membership degree of x with respect to $\overline{R_{k}^{\geq}}\widetilde{X}$ is the supremum of x with respect to dominance classes, and all these dominance classes must satisfy $\sum_{y\in[x]_R^{\geq}}\widetilde{X}(y)>k$; the membership degree of x with respect to $\underline{R_{k}^{\geq}}\widetilde{X}$ is the infimum of x with respect to dominance classes, and all these dominance classes must satisfy $\sum_{y\in[x]_R^{\geq}}1-\widetilde{X}(y)\leq k$.

Definition 3.3 For $\forall\widetilde{X}\in F(U)$, $x\in U$, in the ordered information system, the upper and lower approximation, the positive, negative and boundary region of $\widetilde{X}$ with respect to $R^{\geq}$ are defined as follows:

$$(\overline{R_{A}^{\geq}}\widetilde{X})(x)=\vee_{y\in[x]_R^{\geq}}\{\widetilde{X}(y):\sum_{y\in[x]_R^{\geq}}\widetilde{X}(y)>0\},$$

$$(\underline{R_{A}^{\geq}}\widetilde{X})(x)=\wedge\{\widetilde{X}(y):y\in[x]_R^{\geq}\},$$

$$posR_A^{\geq}(\widetilde{X})=\underline{R_{A}^{\geq}}(\widetilde{X}),$$

$$negR_A^{\geq}(\widetilde{X})=\sim\overline{R_{A}^{\geq}}(\widetilde{X}),\ bnR_A^{\geq}(\widetilde{X})=\overline{R_{A}^{\geq}}(\widetilde{X})-\underline{R_{A}^{\geq}}(\widetilde{X}).$$

If $\overline{R_{A}^{\geq}}\widetilde{X}\neq\underline{R_{A}^{\geq}}\widetilde{X}$, then $\widetilde{X}$ with respect to $R_A^{\geq}$ is rough; otherwise $\widetilde{X}$ is precise or definable.

Definition 3.4 Let $I^{\geq}=(U,A,V,F)$ be an ordered information system and $R^{\geq}$ is the dominance relation of $I^{\geq}$. In the system $I^{\geq}$, $\forall\widetilde{X}\in F(U)$, the upper and lower approximation sets of $\widetilde{X}$ with respect to $R^{\geq}$ based on logical disjunct operation of precision $1-\beta$ and grade k can be defined as follows:

$(\overline{R_{\beta\vee k}^{\geq}}\widetilde{X})(x)=\vee_{y\in[x]_R^{\geq}}\{\widetilde{X}(y):c([x]_R^{\geq},\widetilde{X})<1-\beta$ or $\sum_{y\in[x]_R^{\geq}}\widetilde{X}(y)>k\}$, $(\underline{R_{\beta\vee k}^{\geq}}\widetilde{X})(x)=\wedge_{y\in[x]_R^{\geq}}\{\widetilde{X}(y):$ $c([x]_R^{\geq},\widetilde{X})\leq\beta$ or $\sum_{y\in[x]_R^{\geq}}1-\widetilde{X}(y)\leq k\}$.

According to the positive, negative, and boundary definition of classical rough set, the definition of this model in other regions can be similarly defined. Details are as follows:

$$posR_{\beta\vee k}^{\geq}\widetilde{X}=\overline{R_{\beta\vee k}^{\geq}}\widetilde{X}\cap\underline{R_{\beta\vee k}^{\geq}}\widetilde{X},$$

$$negR_{\beta\vee k}^{\geq}\widetilde{X}=\sim(\overline{R_{\beta\vee k}^{\geq}}\widetilde{X}\cup\underline{R_{\beta\vee k}^{\geq}}\widetilde{X}),$$

$$UbnR_{\beta\vee k}^{\geq}\widetilde{X}=\overline{R_{\beta\vee k}^{\geq}}\widetilde{X}-\underline{R_{\beta\vee k}^{\geq}}\widetilde{X},$$

$$LbnR_{\beta\vee k}^{\geq}\widetilde{X}=\underline{R_{\beta\vee k}^{\geq}}\widetilde{X}-\overline{R_{\beta\vee k}^{\geq}}\widetilde{X},$$

$$bnR_{\beta\vee k}^{\geq}\widetilde{X}=UbnR_{\beta\vee k}^{\geq}\widetilde{X}\cup LbnR_{\beta\vee k}^{\geq}\widetilde{X},$$

where $posR_{\beta\vee k}^{\geq}\widetilde{X}$, $negR_{\beta\vee k}^{\geq}\widetilde{X}$, $UbnR_{\beta\vee k}^{\geq}\widetilde{X}$, $LbnR_{\beta\vee k}^{\geq}\widetilde{X}$ and $bnR_{\beta\vee k}^{\geq}\widetilde{X}$ are called the positive, negative, upper boundary, lower boundary and boundary region of $\widetilde{X}$ with respect to $R_{\beta\vee k}^{\geq}$. If $\overline{R_{\beta\vee k}^{\geq}}\widetilde{X}\neq\underline{R_{\beta\vee k}^{\geq}}\widetilde{X}$, then $\widetilde{X}$ with respect to $R_{\beta\vee k}^{\geq}$ is rough, otherwise $\widetilde{X}$ is precise or definable.

The rough fuzzy set model based on logical disjunct operation of variable precision and grade is stated as above. By exploring the content, the following properties can be obtained.

Theorem 3.1 Let $I^{\geq}=(U,A,V,F)$ be an ordered information system, $\widetilde{X}$ is any fuzzy set of U and $R^{\geq}$ is the dominance relation of $I^{\geq}$. The following conclusion is true, namely

(1) $\overline{R_{\beta\vee k}^{\geq}}\widetilde{X}=\overline{R_{\beta}^{\geq}}\widetilde{X}\cup\overline{R_{k}^{\geq}}\widetilde{X}$,

(2) $\underline{R_{\beta\vee k}^{\geq}}\widetilde{X}=\underline{R_{\beta}^{\geq}}\widetilde{X}\cup\underline{R_{k}^{\geq}}\widetilde{X}$,

where $\beta\in[0,1]$ and $k\in N_{0.5}$.

Proof (1) For $\forall x\in U$, it is true that $(\overline{R_{\beta\vee k}^{\geq}}\widetilde{X})(x)=\vee_{y\in[x]_R^{\geq}}\{\widetilde{X}(y):c([x]_R^{\geq},\widetilde{X})<1-\beta$ or $\sum_{y\in[x]_R^{\geq}}\widetilde{X}(y)>k\}$. It is evident that $(\overline{R_{\beta\vee k}^{\geq}}\widetilde{X})(x)=(\vee_{y\in[x]_R^{\geq}}\{\widetilde{X}(y):c([x]_R^{\geq},\widetilde{X})<1-\beta\})\vee(\vee_{y\in[x]_R^{\geq}}\{\widetilde{X}(y):\sum_{y\in[x]_R^{\geq}}\widetilde{X}(y)>k\})$. According to the definition 3.1 and 3.2, there are $(\overline{R_{\beta}^{\geq}}\widetilde{X})(x)=\vee_{y\in[x]_R^{\geq}}\{\widetilde{X}(y):c([x]_R^{\geq},\widetilde{X})<1-\beta\}$, $\overline{R_{k}^{\geq}}\widetilde{X}(x)=\vee(\vee_{y\in[x]_R^{\geq}}\{\widetilde{X}(y):\sum_{y\in[x]_R^{\geq}}\widetilde{X}(y)>k\})$. Therefore, it is true that $\overline{R_{\beta\vee k}^{\geq}}\widetilde{X}=\overline{R_{\beta}^{\geq}}\widetilde{X}\cup\overline{R_{k}^{\geq}}\widetilde{X}$.

(2) The method analogous to that used above. □

Theorem 3.2 Let $I^{\geq}=(U,A,V,F)$ be an ordered information system, $\widetilde{X}$ is any fuzzy set of U and $R^{\geq}$ is the dominance relation of $I^{\geq}$. It is true that

(1) $\overline{R^{\geq}_{\beta\vee k}}\widetilde{X}=posR^{\geq}_{\beta\vee k}\widetilde{X}\cup UbnR^{\geq}_{\beta\vee k}\widetilde{X}$,

(2) $\underline{R^{\geq}_{\beta\vee k}}\widetilde{X}=posR^{\geq}_{\beta\vee k}\widetilde{X}\cup LbnR^{\geq}_{\beta\vee k}\widetilde{X}$,

where $\beta\in[0,1]$ and $k\in N_{0.5}$.

Proof According to the definition 3.4 it follows. □

Theorem 3.3 Let $I^{\geq}=(U,A,V,F)$ be an ordered information system, $\widetilde{X}$ is any fuzzy set of U and $R^{\geq}$ is the dominance relation of $I^{\geq}$. It is true that

$$(1)\ (\overline{R^{\geq}_{\beta\vee k}}\widetilde{X})(x)=\vee_{y\in[x]_R^{\geq}}\{\widetilde{X}(y):\sum_{y\in[x]_R^{\geq}}\widetilde{X}(y)>\min(k,\beta|[x]_R^{\geq}|)\},$$

$$(2)\ (\underline{R^{\geq}_{\beta\vee k}}\widetilde{X})(x)=\wedge_{y\in[x]_R^{\geq}}\{\widetilde{X}(y):\sum_{y\in[x]_R^{\geq}}\widetilde{X}(y)\geq\min(|[x]_R^{\geq}|-k,|[x]_R^{\geq}|-\beta|[x]_R^{\geq}|)\},$$

where $\beta\in[0,1]$ and $k\in N_{0.5}$.

Proof (1) According to the definition 3.4, it is true that $(\overline{R^{\geq}_{\beta\vee k}}\widetilde{X})(x)=\vee_{y\in[x]_R^{\geq}}\{\widetilde{X}(y):c([x]_R^{\geq},\widetilde{X})<1-\beta$ or $\sum_{y\in[x]_R^{\geq}}\widetilde{X}(y)>k\}$ for any object x of U. By considering the condition $c([x]_R^{\geq},\widetilde{X})=1-\sum_{y\in[x]_R}\widetilde{X}(y)\Big/|[x]_R^{\geq}|$, there is $(\overline{R^{\geq}_{\beta\vee k}}\widetilde{X})(x)=\vee_{y\in[x]_R^{\geq}}\{\widetilde{X}(y):\sum_{y\in[x]_R^{\geq}}\widetilde{X}(y)>\beta|[x]_R^{\geq}|$ or $\sum_{y\in[x]_R^{\geq}}\widetilde{X}(y)>k\}$. Therefore, there are $(\overline{R^{\geq}_{\beta\vee k}}\widetilde{X})(x)=\vee_{y\in[x]_R^{\geq}}\{\widetilde{X}(y):\sum_{y\in[x]_R^{\geq}}\widetilde{X}(y)>\beta|[x]_R^{\geq}|$ or $\sum_{y\in[x]_R^{\geq}}\widetilde{X}(y)>k\}$.

(2) The property can be proved in similar (1). □

Theorem 3.4 Let $I^{\geq}=(U,A,V,F)$ be an ordered information system, $\widetilde{X}$ is any fuzzy set of U and $R^{\geq}$ is the dominance relation of $I^{\geq}$. There are the following assertions set up.

(1) When $0<\beta<0.5$ and $k\neq 0$, there are

$$(posR^{\geq}_{\beta\vee k}\widetilde{X})(x)=(\wedge_{y\in[x]_R^{\geq}}\{\widetilde{X}(y):|[x]_R^{\geq}|\geq k/\beta,\sum_{y\in[x]_R^{\geq}}\widetilde{X}(y)\geq(1-\beta)|[x]_R^{\geq}|\})\vee(\wedge_{y\in[x]_R^{\geq}}\{X(y):k/(1-\beta)<|[x]_R^{\geq}|<k/\beta,\sum_{y\in[x]_R^{\geq}}X(y)\geq|[x]_R^{\geq}|-k\})\vee(\wedge_{y\in[x]_R^{\geq}}\{\widetilde{X}(y):|[x]_R^{\geq}|\leq k/(1-\beta),\sum_{y\in[x]_R^{\geq}}\widetilde{X}(y)>\beta|[x]_R^{\geq}|\});$$

$$(negR^{\geq}_{\beta\vee k}\widetilde{X})(x)=1-\{(\vee_{y\in[x]_R^{\geq}}\{X(y):|[x]_R^{\geq}|\geq k/\beta,\sum_{y\in[x]_R^{\geq}}X(y)>k\})\vee(\vee_{y\in[x]_R^{\geq}}\{X(y):|[x]_R^{\geq}|<k/\beta,\sum_{y\in[x]_R^{\geq}}X(y)>\beta|[x]_R^{\geq}|\})\vee(\wedge_{y\in[x]_R^{\geq}}\{X(y):|[x]_R^{\geq}|\leq k/(1-\beta),|[x]_R^{\geq}|-k\leq\beta|[x]_R^{\geq}|\})\};$$

$$(UbnR^{\geq}_{\beta\vee k}\widetilde{X})(x)=(\vee_{y\in[x]_R^{\geq}}\{\widetilde{X}(y):|[x]_R^{\geq}|\geq k/\beta,k<\sum_{y\in[x]_R^{\geq}}\widetilde{X}(y)<(1-\beta)|[x]_R^{\geq}|\})\vee(\{(\vee_{y\in[x]_R^{\geq}}\widetilde{X}(y))\wedge(1-\wedge_{y\in[x]_R^{\geq}}\widetilde{X}(y)):|[x]_R^{\geq}|\geq k/\beta,\sum_{y\in[x]_R^{\geq}}\widetilde{X}(y)\geq(1-\beta)|[x]_R^{\geq}|\})\vee(\{(\vee_{y\in[x]_R^{\geq}}\widetilde{X}(y))\wedge(1-\wedge_{y\in[x]_R^{\geq}}\widetilde{X}(y)):k/(1-\beta)<|[x]_R^{\geq}|<k/\beta,\sum_{y\in[x]_R^{\geq}}\widetilde{X}(y)\geq|[x]_R^{\geq}|-k\})\vee$$

$$(\vee_{y\in[x]_R^{\geq}}\{\widetilde{X}(y):k/(1-\beta)<|[x]_R^{\geq}|<k/\beta,\beta|[x]_R^{\geq}|<\sum_{y\in[x]_R^{\geq}}\widetilde{X}(y)<|[x]_R^{\geq}|-k\})\vee(\{(\vee_{y\in[x]_R^{\geq}}\widetilde{X}(y))\wedge(1-\wedge_{y\in[x]_R^{\geq}}\widetilde{X}(y)):|[x]_R^{\geq}|\leq k/(1-\beta),\sum_{y\in[x]_R^{\geq}}\widetilde{X}(y)>\beta|[x]_R^{\geq}|\});$$

$$(LbnR^{\geq}_{\beta\vee k}\widetilde{X})(x)=(\wedge_{y\in[x]_R^{\geq}}\{\widetilde{X}(y):|[x]_R^{\geq}|\leq k/1-\beta,|[x]_R^{\geq}|-k\leq\sum_{y\in[x]_R^{\geq}}\widetilde{X}(y)\leq\beta|[x]_R^{\geq}|\})\vee(\{(\wedge_{y\in[x]_R^{\geq}}\widetilde{X}(y))\wedge(1-\vee_{y\in[x]_R^{\geq}}\widetilde{X}(y)):|[x]_R^{\geq}|\geq k/\beta,\sum_{y\in[x]_R^{\geq}}\widetilde{X}(y)\geq(1-\beta)|[x]_R^{\geq}|\})\vee(\{(\wedge_{y\in[x]_R^{\geq}}\widetilde{X}(y))\wedge(1-\vee_{y\in[x]_R^{\geq}}\widetilde{X}(y)):k/(1-\beta)<[x]_R^{\geq}|<k/\beta,\sum_{y\in[x]_R^{\geq}}\widetilde{X}(y)\geq|[x]_R^{\geq}|-k)\vee(\{(\wedge_{y\in[x]_R^{\geq}}\widetilde{X}(y))\wedge(1-\vee_{y\in[x]_R^{\geq}}\widetilde{X}(y)):|[x]_R^{\geq}|\leq k/(1-\beta),\sum_{y\in[x]_R^{\geq}}\widetilde{X}(y)>\beta|[x]_R^{\geq}|\}).$$

(2) When $0.5\leq\beta<1$ and $k\neq 0$, there are

$$(posR^{\geq}_{\beta\vee k}\widetilde{X})(x)=(\wedge_{y\in[x]_R^{\geq}}\{\widetilde{X}(y):|[x]_R^{\geq}|\geq k/(1-\beta),\sum_{y\in[x]_R^{\geq}}\widetilde{X}(y)\geq(1-\beta)|[x]_R^{\geq}|\})\vee(\wedge_{y\in[x]_R^{\geq}}\{\widetilde{X}(y):k/\beta<|[x]_R^{\geq}|<k/1-\beta,\sum_{y\in[x]_R^{\geq}}\widetilde{X}(y)>k\})\vee(\wedge_{y\in[x]_R^{\geq}}\{\widetilde{X}(y):|[x]_R^{\geq}|\leq k/\beta,\sum_{y\in[x]_R^{\geq}}\widetilde{X}(y)>\beta|[x]_R^{\geq}|\});$$

$$(negR^{\geq}_{\beta\vee k}\widetilde{X})(x)=1-\{(\vee_{y\in[x]_R^{\geq}}\{\widetilde{X}(y):|[x]_R^{\geq}|>k/\beta,\sum_{y\in[x]_R^{\geq}}\widetilde{X}(y)>k\})\vee(\wedge_{y\in[x]_R^{\geq}}\{\widetilde{X}(y):k/\beta<|[x]_R^{\geq}|<k/(1-\beta),(1-\beta)|[x]_R^{\geq}|\leq\sum_{y\in[x]_R^{\geq}}\widetilde{X}(y)\leq k\})\vee(\vee_{y\in[x]_R^{\geq}}\{\widetilde{X}(y):|[x]_R^{\geq}|\leq k/\beta,\sum_{y\in[x]_R^{\geq}}\widetilde{X}(y)>\beta|[x]_R^{\geq}|\})\vee$$

$$(\bigwedge_{y\in[x]_R^{\geq}}\{\widetilde{X}(y): |[x]_R^{\geq}|\leq k/\beta, |[x]_R^{\geq}|-k\leq\sum_{y\in[x]_R^{\geq}}\widetilde{X}(y)\leq\beta|[x]_R^{\geq}|\}\})\,;$$

$$(UbnR^{\geq}_{\beta\vee k}\widetilde{X})(x)=(\bigvee_{y\in[x]_R^{\geq}}\{\widetilde{X}(y): |[x]_R^{\geq}|\geq k/(1-\beta), k<\sum_{y\in[x]_R^{\geq}}\widetilde{X}(y)<(1-\beta)|[x]_R^{\geq}|\})\vee(\{(\bigvee_{y\in[x]_R^{\geq}}\widetilde{X}(y))\wedge(1-\bigwedge_{y\in[x]_R^{\geq}}\widetilde{X}(y)): |[x]_R^{\geq}|\geq k/(1-\beta), \sum_{y\in[x]_R^{\geq}}\widetilde{X}(y)\geq(1-\beta)|[x]_R^{\geq}|\})\vee(\{(\bigvee_{y\in[x]_R^{\geq}}\widetilde{X}(y))\wedge(1-\bigwedge_{y\in[x]_R^{\geq}}\widetilde{X}(y)): k/\beta<|[x]_R^{\geq}|<k/(1-\beta), \sum_{y\in[x]_R^{\geq}}\widetilde{X}(y)>k\})\vee(\{(\bigvee_{y\in[x]_R^{\geq}}\widetilde{X}(y))\wedge(1-\bigwedge_{y\in[x]_R^{\geq}}\widetilde{X}(y)): |[x]_R^{\geq}|\leq k/\beta, \sum_{y\in[x]_R^{\geq}}\widetilde{X}(y)>\beta|[x]_R^{\geq}|\})\,;$$

$$(LbnR^{\geq}_{\beta\vee k}\widetilde{X})(x)=(\bigwedge_{y\in[x]_R^{\geq}}\{\widetilde{X}(y): k/\beta<|[x]_R^{\geq}|<k/(1-\beta), (1-\beta)|[x]_R^{\geq}|\leq\sum_{y\in[x]_R^{\geq}}\widetilde{X}(y)\leq k\})\vee(\{(\bigwedge_{y\in[x]_R^{\geq}}\widetilde{X}(y))\wedge(1-\bigvee_{y\in[x]_R^{\geq}}\widetilde{X}(y)): |[x]_R^{\geq}|\geq k/(1-\beta), \sum_{y\in[x]_R^{\geq}}\widetilde{X}(y)\geq(1-\beta)|[x]_R^{\geq}|\})\vee(\{(\bigwedge_{y\in[x]_R^{\geq}}\widetilde{X}(y))\wedge(1-\bigvee_{y\in[x]_R^{\geq}}\widetilde{X}(y)): k/\beta<|[x]_R^{\geq}|<k/(1-\beta), \sum_{y\in[x]_R^{\geq}}\widetilde{X}(y)>k\})\vee(\{(\bigwedge_{y\in[x]_R^{\geq}}\widetilde{X}(y))\wedge(1-\bigvee_{y\in[x]_R^{\geq}}\widetilde{X}(y)): |[x]_R^{\geq}|\leq k/\beta, \sum_{y\in[x]_R^{\geq}}\widetilde{X}(y)>\beta|[x]_R^{\geq}|\})\vee(\bigwedge_{y\in[x]_R^{\geq}}\{\widetilde{X}(y): |[x]_R^{\geq}|\leq k/\beta, |[x]_R^{\geq}|-k\leq\sum_{y\in[x]_R^{\geq}}\widetilde{X}(y)\leq\beta|[x]_R^{\geq}|\})\,.$$

Proof Because the first case $0<\beta<0.5$ and the second case $0.5\leq\beta<1$ are symmetry, so the following only provide the reasoning process of the case (1).

(1) By preconditions $0<\beta<0.5$ and $k\neq0$, it is true that $\beta<1-\beta$ and $k/(1-\beta)<k/\beta$. In order to obtain concise results, the comparison between $\beta|[x]_R^{\geq}|$ and $|[x]_R^{\geq}|-k$ is made. When $|[x]_R^{\geq}|\geq k/\beta$, according to the theorem 3.3, there are $(\overline{R^{\geq}_{\beta\vee k}}\widetilde{X})(x)=\bigvee_{y\in[x]_R^{\geq}}\{\widetilde{X}(y): \sum_{y\in[x]_R^{\geq}}\widetilde{X}(y)>k\}$, $(\underline{R^{\geq}_{\beta\vee k}}\widetilde{X})(x)=\bigwedge_{y\in[x]_R^{\geq}}\{\widetilde{X}(y):\geq|[x]_R^{\geq}|-\beta|[x]_R^{\geq}|\}$. Then by $posR^{\geq}_{\beta\vee k}\widetilde{X}=\overline{R^{\geq}_{\beta\vee k}}\widetilde{X}\cap\underline{R^{\geq}_{\beta\vee k}}\widetilde{X}$, it is obvious that $(posR^{\geq}_{\beta\vee k}\widetilde{X})(x)=\bigwedge_{y\in[x]_R^{\geq}}\{\widetilde{X}(y): |[x]_R^{\geq}|\geq k/\beta, \sum_{y\in[x]_R^{\geq}}\widetilde{X}(y)\geq(1-\beta)|[x]_R^{\geq}|\}$. When $k/(1-\beta)<|[x]_R^{\geq}|<k/\beta$, it is evident that $(posR^{\geq}_{\beta\vee k}\widetilde{X})(x)=\bigwedge_{y\in[x]_R^{\geq}}\{\widetilde{X}(y): k/(1-\beta)<|[x]_R^{\geq}|<k/\beta, \sum_{y\in[x]_R^{\geq}}\widetilde{X}(y)\geq|[x]_R^{\geq}|-k\}$.

When $|[x]_R^{\geq}|\leq k/(1-\beta)$, there are $(posR^{\geq}_{\beta\vee k}\widetilde{X})(x)=\bigwedge_{y\in[x]_R^{\geq}}\{\widetilde{X}(y): |[x]_R^{\geq}|\leq k/(1-\beta), \sum_{y\in[x]_R^{\geq}}\widetilde{X}(y)>\beta|[x]_R^{\geq}|\}$.

So the conclusion about $(posR^{\geq}_{\beta\vee k}\widetilde{X})(x)$ is true. And other conclusions can be similarly obtained. □

Theorem 3.5 Let $I^{\geq}=(U,A,V,F)$ be an ordered information system, $\widetilde{X}$ and $\widetilde{Y}$ are fuzzy sets of U, and $R^{\geq}$ is the dominance relation of $I^{\geq}$. The following assertions are valid, where $\alpha,\beta\in[0,1], k,l\in N_{0.5}$.

(1) $\overline{R^{\geq}_{\beta\vee k}}\varnothing=\underline{R^{\geq}_{\beta\vee k}}\varnothing=\varnothing$, $\underline{R^{\geq}_{\beta\vee k}}U=U$.

(2)If $\widetilde{X}\subseteq\widetilde{Y}$, then $\overline{R^{\geq}_{\beta\vee k}}\widetilde{X}\subseteq\overline{R^{\geq}_{\beta\vee k}}\widetilde{Y}$, $\underline{R^{\geq}_{\beta\vee k}}\widetilde{X}\subseteq\underline{R^{\geq}_{\beta\vee k}}\widetilde{Y}$.

(3) $\overline{R^{\geq}_{\beta\vee k}}(\widetilde{X}\cup\widetilde{Y})\supseteq\overline{R^{\geq}_{\beta\vee k}}\widetilde{X}\cup\overline{R^{\geq}_{\beta\vee k}}\widetilde{Y}$, $\underline{R^{\geq}_{\beta\vee k}}(\widetilde{X}\cup\widetilde{Y})\supseteq\underline{R^{\geq}_{\beta\vee k}}\widetilde{X}\cup\underline{R^{\geq}_{\beta\vee k}}\widetilde{Y}$.

(4) $\overline{R^{\geq}_{\beta\vee k}}(\widetilde{X}\cap\widetilde{Y})\subseteq\overline{R^{\geq}_{\beta\vee k}}\widetilde{X}\cap\overline{R^{\geq}_{\beta\vee k}}\widetilde{Y}$, $\underline{R^{\geq}_{\beta\vee k}}(\widetilde{X}\cap\widetilde{Y})\subseteq\underline{R^{\geq}_{\beta\vee k}}\widetilde{X}\cap\underline{R^{\geq}_{\beta\vee k}}\widetilde{Y}$.

(5)When $\beta\geq\alpha$, $k\geq l$, there are

$\overline{R^{\geq}_{\beta\vee k}}\widetilde{X}\subseteq\overline{R^{\geq}_{\beta\vee l}}\widetilde{X}$, $\overline{R^{\geq}_{\beta\vee k}}\widetilde{X}\subseteq\overline{R^{\geq}_{\alpha\vee k}}\widetilde{X}$,

$\overline{R^{\geq}_{\beta\vee k}}\widetilde{X}\subseteq\overline{R^{\geq}_{\alpha\vee l}}\widetilde{X}$, $\underline{R^{\geq}_{\beta\vee k}}\widetilde{X}\supseteq\underline{R^{\geq}_{\beta\vee l}}\widetilde{X}$,

$\underline{R^{\geq}_{\beta\vee k}}\widetilde{X}\supseteq\underline{R^{\geq}_{\alpha\vee k}}\widetilde{X}$, $\underline{R^{\geq}_{\beta\vee k}}\widetilde{X}\supseteq\underline{R^{\geq}_{\alpha\vee l}}\widetilde{X}$.

Proof The conclusions are obvious. □

4 CASE ANALYSIS

Let $I^{\geq}=(U,A,V,F)$ be an ordered information system, where $U=\{x_1,x_2,\cdots,x_{20}\}$ is composed of 20 patients with influenza, $A=\{a_1,a_2,a_3,a_4\}$ is the attribute set that represents headache, fever, cough, and muscle pain, and $V=\{1,2,3\}$ is the set that represents the extent of the disease. Let 1 indicate a lighter degree of the disease; 2 indicate a moderate degree of the disease, and 3 indicate a heavier degree of the disease. Detailed statistical data is shown in Table 4.1.

At first, a part of the patients were randomly selected as the research object, namely $X=\{x_1,x_5,x_7,x_{11},x_{17},x_{19},x_{20}\}$. The extent of the patient with influenza can be determined by above symptoms. It is a fuzzy concept that the extent of the patient with influenza. According to the result of medical examination and doctor diagnosis, the extent of seven patients with influenza is followed by 0.95, 0.82, 0.76, 0.88, 0.54, 0.45 and 0.64. The fuzzy set formed by the above evaluation can be denoted by $\widetilde{A}$, namely $\widetilde{A}=\{(x_1,0.95),(x_5,0.82),(x_7,0.76),(x_{11},0.88),(x_{17},0.54),(x_{19},0.45),(x_{20},0.64)\}$. Then the object classification based on dominance relations can be calculated.

Table 4.1 Medical diagnosis statistics

U	headache	fever	cough	Muscle pain
x_1	3	3	2	3
x_2	3	3	2	2
x_3	1	2	1	2
x_4	2	1	2	2
x_5	2	2	2	2
x_6	2	2	3	3
x_7	3	2	3	1
x_8	1	1	1	2
x_9	2	3	3	3
x_{10}	2	3	1	2
x_{11}	3	1	3	3
x_{12}	1	2	1	1
x_{13}	2	2	2	1
x_{14}	1	2	2	1
x_{15}	1	2	2	2
x_{16}	3	1	2	2
x_{17}	3	3	1	1
x_{18}	3	3	2	1
x_{19}	2	1	1	1
x_{20}	1	3	3	1

Might as well choose $\beta = 0.3$ and $k = 1$, the values of $\left|[x_i]^{\geq}\right|$, $\sum_{y\in[x]^{\geq}_R}\widetilde{X}(y)$, $c([x]^{\geq}_R,\widetilde{X})$ and $\left|[x_i]^{\geq}\right| - \sum_{y\in[x]^{\geq}_R}\widetilde{X}(y)$ are shown in Table 4.2.

Case 1 The required values can be calculated by the theorem 3.1. The detailed calculation process is as follows:

(1) When $\beta = 0.3$, there are

$$\overline{R^{\geq}_{\beta}}\widetilde{A} = \{(x_1,0.95),(x_2,0.95),(x_4,0.95),(x_5,0.95),(x_7,0.76),(x_{11},0.88),(x_{13},0.95),(x_{16},0.95),(x_{17},0.95),(x_{18},0.95),(x_{19},0.95),(x_{20},0.64)\},$$

$$\underline{R^{\geq}_{\beta}}\widetilde{A} = \{(x_1,0.95),(x_7,0.76),(x_{11},0.88)\}.$$

When $k = 1$, there are

$$\overline{R^{\geq}_{k}}\widetilde{A} = \{(x_4,0.95),(x_8,0.95),(x_{12},0.95),(x_{13},0.95),(x_{14},0.95),(x_{19},0.95)\},$$

$$\underline{R^{\geq}_{k}}\widetilde{A} = \{(x_1,0.95),(x_7,0.76),(x_{11},0.88)\}.$$

(2) According to the results obtained from (1) and theorem 3.1, the following conclusions can be obtained, namely

$$\overline{R^{\geq}_{0.3\vee 1}}\widetilde{A} = \{(x_1,0.95),(x_2,0.95),(x_4,0.95),(x_5,0.95),(x_7,0.76),(x_8,0.95),(x_{11},0.88),(x_{12},0.95),(x_{13},0.95),(x_{14},0.95),(x_{16},0.95),(x_{17},0.95),(x_{18},0.95),(x_{19},0.95),(x_{20},0.64)\},$$

$$\underline{R^{\geq}_{0.3\vee 1}}\widetilde{A} = \{(x_1,0.95),(x_7,0.76),(x_{11},0.88)\}.$$

(3) Combined with the results obtained by step (2), the following conclusions can be obtained by the definition 3.4.

$$posR^{\geq}_{0.3\vee 1}\widetilde{A} = \{(x_1,0.95),(x_7,0.76),(x_{11},0.88)\},$$

$$negR^{\geq}_{0.3\vee 1}\widetilde{A} = \{(x_1,0.05),(x_2,0.05),(x_3,1)(x_4,0.05),(x_5,0.05),(x_6,1),(x_7,0.24),(x_8,0.05),(x_9,1),(x_{10},1),(x_{11},0.12),(x_{12},0.05),(x_{13},0.05),(x_{14},0.05),(x_{15},1),(x_{16},0.05),(x_{17},0.05),(x_{18},0.05),(x_{19},0.05),(x_{20},0.36)\},$$

$$UbnR^{\geq}_{0.3\vee 1}\widetilde{A} = \{(x_1,0.05),(x_2,0.95),(x_4,0.95),(x_5,0.95),(x_7,0.24),(x_8,0.95),(x_{11},0.12),(x_{12},0.95),(x_{13},0.95),(x_{14},0.95),(x_{16},0.95),(x_{17},0.95),(x_{18},0.95),(x_{19},0.95),(x_{20},0.64)\},$$

$$LbnR^{\geq}_{0.3\vee 1}\widetilde{A} = \{(x_1,0.05),(x_7,0.24),(x_{11},0.12)\},$$

$$bnR^{\geq}_{0.3\vee 1}\widetilde{A} = \{(x_1,0.05),(x_2,0.95),(x_4,0.95),(x_5,0.95),(x_7,0.24),(x_8,0.95),(x_{11},0.12),(x_{12},0.95),(x_{13},0.95),(x_{14},0.95),(x_{16},0.95),(x_{17},0.95),(x_{18},0.95),(x_{19},0.95),(x_{20},0.64)\}.$$

Case 2 According to the theorem 3.4, the required values can be directly calculated. When $\beta = 0.3, k = 1$, there are

$$posR^{\geq}_{0.3\vee 1}\widetilde{A} = \{(x_1,0.95),(x_7,0.76),(x_{11},0.88)\},$$

$$negR^{\geq}_{0.3\vee 1}\widetilde{A} = \{(x_1,0.05),(x_2,0.05),(x_3,1),(x_4,0.05),(x_5,0.05),(x_6,1),(x_7,0.24),(x_8,0.05),(x_9,1),(x_{10},1),(x_{11},0.12),(x_{12},0.05),(x_{13},0.05),(x_{14},0.05),(x_{15},1),(x_{16},0.05),(x_{17},0.05),(x_{18},0.05),(x_{19},0.05),(x_{20},0.36)\},$$

$$UbnR^{\geq}_{0.3\vee 1}\widetilde{A} = \{(x_1,0.05),(x_2,0.95),(x_4,0.95),(x_5,0.95),(x_7,0.24),(x_8,0.95),(x_{11},0.12),(x_{12},0.95),(x_{13},0.95),(x_{14},0.95),(x_{16},0.95),(x_{17},0.95),(x_{18},0.95),(x_{19},0.95),(x_{20},0.64)\},$$

$$LbnR^{\geq}_{0.3\vee 1}\widetilde{A} = \{(x_1,0.05),(x_7,0.24),(x_{11},0.12)\},$$

$$bnR^{\geq}_{0.3\vee 1}\widetilde{A} = \{(x_1,0.05),(x_2,0.95),(x_4,0.95),(x_5,0.95),(x_7,0.24),(x_8,0.95),(x_{11},0.12),(x_{12},0.95),(x_{13},0.95),(x_{14},0.95),(x_{16},0.95),(x_{17},0.95),(x_{18},0.95),(x_{19},0.95),(x_{20},0.64)\}.$$

Obviously, calculation processes of two methods are different. The step of the first method is as follows: $\overline{R^{\geq}_{\beta}}\widetilde{A}$ and $\underline{R^{\geq}_{\beta}}\widetilde{A}$ are firstly calculated; then $\overline{R^{\geq}_{\beta\vee k}}\widetilde{A}$ and $\underline{R^{\geq}_{\beta\vee k}}\widetilde{A}$ are obtained; finally, the other regions are calculated. The second method is to directly use the result of the theorem to calculate. The results of the above two methods are consistent. In practical application, it is necessary to select appropriate methods to solve problems according to their own needs.

Table 4.2 The statistical data about dominant classes

x_i	$\lvert [x_i]^{\geq} \rvert$	$\sum_{y\in[x]_R^{\geq}} \widetilde{X}(y)$	$c([x]_R^{\geq},\widetilde{X})$	$\lvert [x_i]^{\geq} \rvert - \sum_{y\in[x]_R^{\geq}} \widetilde{X}(y)$
1	1	0.95	0.050	0.05
2	2	0.95	0.525	1.05
3	8	1.77	0.779	6.23
4	8	2.65	0.669	5.35
5	5	1.77	0.646	3.23
6	2	0	1.000	2.00
7	1	0.76	0.240	0.24
8	12	2.65	0.779	9.35
9	1	0	1.000	1.00
10	4	0.95	0.763	3.05
11	1	0.88	0.120	0.12
12	15	3.71	0.753	11.29
13	8	2.53	0.684	5.47
14	11	3.17	0.712	7.83
15	6	1.77	0.705	4.23
16	4	1.83	0.543	2.17
17	4	1.49	0.628	2.51
18	3	0.95	0.683	2.05
19	14	4.4	0.686	9.60
20	2	0.64	0.680	1.36

5 CONCLUSIONS

In real life, the fuzzy phenomenon can be seen everywhere. How to accurately describe a fuzzy concept is a very meaningful research topic. From the perspective of logic relationships, this paper considers the relative quantization and absolute quantization information of approximation spaces. It is more important that the rough fuzzy set model based on logical disjunct operation of the variable precision and grade are proposed in the ordered information system. Through in-depth study, the basic structure of the model and its important properties are obtained. Finally, an illustrative example is introduced to further deepen the understanding of the model.

REFERENCES

[1] Z. Pawlak, Rough sets, International Journal of Computer and Information Sciences, Vol.11, No.5, 341-356, 1982.

[2] W. H. Xu, Ordered Information Systems and Rough Sets, Science Press, Beijing, China, 2013.

[3] K. Dembczyński, R. Pindur, R. Susmaga, Dominance-based rough set classifier without induction of decision rules, Electronic Notes in Theoretical Computer Science, Vol.82, No.4, 84-95, 2003.

[4] K. Dembczyński, R. Pindur, R. Susmaga, Generation of exhaustive Set of Rules within Dominance-based Rough Set Approach, Electronic Notes in Theoretical Computer Science, Vol.82, No.4, 96-107, 2003.

[5] Z. Pawlak, Rough Sets：Theoretical Aspects of Reasoning about Data, Kluwer Academic Publishers, USA, 1992.

[6] W. X. Zhang, Y. Liang, W. Z. Wu, Information Systems and Knowledge Discovery, Science Press, Beijing, China, 2004.

[7] W. X. Zhang, W. Z. Wu, J. Y. Liang, D. Y. Li, Rough Set Theory and Method, Science Press, Beijing, China, 2001.

[8] C. Liu, D. Miao, N. Zhang, Graded rough set model based on two universes and its properties, Knowledge-Based Systems, Vol.33, No.3, 65-72, 2012.

[9] Y. Yao, T. Lin, Graded rough set approximations based on nested neighborhood systems, Proceedings of 5th European Congress on intelligent techniques and Soft computing, Vol.1, 196-200, 1997.

[10] W. Xu, S. Liu, Q Wang , The First Type of Grade Rough Set Based on Rough Membership Function, 2010 Seventh International Conference System and Knowledge Discovery(FSKD2010), Vol.4,1922-1926, 2010.

[11] X. Y. Zhang, Z. W. Mo, Variable precision rough sets, Pattern Recognition and Artificial Intelligence, Vol.17, No.2, 151-155, 2004. (in Chinese)

[12] Y. Yao, Decision-theoretic rough set models, In: Proceeding of Rough Sets and Knowledge Technology, Lecture Notes in Artificial Intelligence, Vol.4481, No.2, 1-12, 2007.

[13] W. Ziarko, Variable precision rough set model, Journal of Computer and System Sciences, Vol.46, No.1, 39-59, 1993.

[14] S. Greco, B. Matarazzo, R. Slowinski, Parameterized rough set model using rough membership and Bayesian confirmation measures, International Journal of Approximate Reasoning, Vol.49, No.2, 285-300, 2008.

[15] J. Herbert, J. Yao, Game-theoretic rough sets, Fundamenta Informaticae, Vol.108, No.3-4, 267-286, 2011.

[16] J. B. Shen, Y. J. Lv, Promotion for rough set of variable precision and extent of rough set, Computer Engineering and Applications, Vol.44, No.36, 45-47, 2008. (in Chinese)

[17] Y. Yao, T. Lin, Generalization of rough sets using modal logics, Intelligent Automatic and Soft Computing, Vol.2, No.2, 103-119, 1996.

[18] W. Xu, W. Li, Double-quantitative decision-theoretic rough set, Information Sciences, Vol.316, 54-67, 2015.

[19] X. Zhang, Z. Mo, F. Xiong, W. Cheng, Comparative study of variable precision rough set model and graded rough set model, International Journal of Approximate Reasoning , Vol.53, No.2, 104-116, 2012.

[20] X. Y. Zhang, F. Xiong, Z. W. Mo, rough set model based on logical or operation of precision and grade, Pattern Recognition and Artificial Intelligence, Vol.22, No.5, 697-703, 2009. (in Chinese)

[21] J. H. Yu, W. H. Xu, Rough set based on logical disjunct operation of variable precision and grade in ordered information system, Computer Science and Technology, Vol.9, No.1, 112-118, 2015. (in Chinese)