Attribute Reduction in Multi-source Decision Systems

Yanting Guo and Weihua Xu(✉)

School of Mathematics and Statistics, Chongqing University of Technology, Chongqing 400054, People's Republic of China
1290237984@qq.com, chxuwh@gmail.com

Abstract. Data processing for information from different sources is a hot research topic in the contemporary data. Attribute reduction methods of multi-source decision systems (MSDS) are proposed in this paper. Firstly, based on the integrity of original effective information preservation, a consistent attribute reduction of the multi-source decision system is proposed. Secondly, in the case of a certain loss of original effective information, data is compressed by the fusion of conditional entropy. Then attribute reduction preserving knowledge unchanged are studied in the decision system obtained by fusion. Accordingly, examples are introduced to further elaborate the theory proposed in this paper.

Keywords: Attribute reduction · Conditional entropy fusion · Multi-source decision system

1 Introduction

Rough set theory proposed by Pawlak [6] is an important mathematical tool to deal with imprecise, inconsistent and incomplete information. In order to meet people's various requirements, many extended rough set models have been proposed, such as the fuzzy rough set and the rough fuzzy rough set, the variable precision rough set model, and other models [8,10].

Rough set theory has been widely applied in many fields, such as machine learning, knowledge discovery, data mining, decision support and analysis, information security, networking, cloud computing and biological information processing [2].

Attribute reduction is one of the core content in rough set, which has been made great development. Based on different criteria, various reduction methods are proposed in classical and generalized rough set models. According to the quantitative criteria, attribute reduction is mainly divided into two categories: qualitative reduction and quantitative reduction. From the perspective of qualitative criteria, Pawlak proposed an attribute reduction which keeps the positive region unchanged [7]. Slezak [9] provided a generalized reduction which keeps the generalized decision under the generalized decision function. Mi et al. [5] investigated the β lower distribution reduction and β upper distribution reduction in

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the variable precision rough model. Yao and Zhao [12]investigated several different quantitative reductions in the decision-theoretic rough model. Attribute reduction is mainly to solve the problem of high-dimensional data computation complexity and accuracy.

With the development of information technology, massive data is released every day, and the volume of data is fairly large. It is important for us to efficiently acquire knowledge from information derived from different sources (namely information boxes). There is no doubt that attribute reduction can eliminate the influence of the redundant and irrelevant attributes on the calculation process and the final results. Therefore, the research of attribute reduction based on multi-source decision systems (MSDS) is of great significance.

In order to make an accurate decision, without losing any effective information is the highest requirement of data processing. Based on the consideration of integrity of information preservation, for all the source, we hope to find a common attribute reduction (namely consistent attribute reductions of MSDS) to eliminate redundant attributes. If the amount of information can not be compressed in the case of keeping the integrity of information violence, we need to appropriately reduce the standards for the preservation of the original information. Therefore, in the case of a certain loss of information, multi-source information fusion has important significance.

By integrating different sources of data, the deficiency of the single data can be made up, so as to realize the mutual complement and mutual confirmation of various data sources. In this way, the application scope of data is expanded and the accuracy of analysis can be improved. In many circumstances, integrating all information from different sources is necessary. There are some related studies on multi-source information fusion. In particular, Khan [3] based on views of membership of objects studied rough set theory and notions of approximates in Multiple-Source Approximation Systems (MSAS). Besides, Md and Khan [4] proposed a modal logic for Multiple-source Tolerance Approximation Spaces (MTAS) based on the principle of only considering the information of sources have about objects.

This paper mainly study attribute reduction of multi-source information systems (MSDS) which have the same universe and attributes and different information functions (namely Isomorphic multi-source information systems). It should be pointed out that isomorphism multi-source information systems refers to the same cardinality of the partition generated by attribute set on the universe in each information system. For heterogeneous information systems, we just need to find the ultimate goal as a middle bridge to establish the relationship between different sources. Because the more information you have on the same thing, learned knowledge should be more accurate. A higher goal may be required for the isomorphic information system in addition to finding the middle bridge. Therefore, attribute reduction of multi-source information systems (MSDS) which have the same universe and attributes and different information functions is important. Next, the most important issue is how to make full use of the information provided by each source in Multi-Source Decision Systems

(MSDS). Then in the various original information preservation requirements, attribute reduction methods of multi-source information systems (MSDS) are proposed.

This paper provides the following innovations: (1) based on the integrity of information preservation, a consistent attribute reduction of MSDS is proposed (2) in the case of a certain loss of information, data is compressed by the fusion of conditional entropy (3) research on attribute reduction keeping the knowledge unchanged in the MSDS.

The study of this paper is organized as follows. Some basic concepts in Pawlak rough set theory are briefly reviewed in Sect. 2. In Sect. 3, definitions of Multi-Source Decision Systems and consistent attribute reduction are proposed. Under the consideration of various original effective information preservation requirements, attribute reduction methods of MSDS are proposed. Section 4 concludes this paper by bringing some remarks and discussions.

2 Preliminaries

In this section, some basic concepts about rough set theory, decision systems and uncertainty measures are reviewed.

Rough set theory proposed by Pawlak is an important tool for knowledge learning. Suppose U be a nonempty and finite set of objects, which is called the universe of discourse, and R be an equivalence relation of $U \times U$. The equivalence relation R induces a partition of U, denoted by $U/R = \{[x]_R | x \in U\}$, where $[x]_R$ represents the equivalence class of x with regard to R. Then (U, R) is the Pawlak approximation space. For an arbitrary subset X of U can be characterized by a pair of upper and lower approximations which are [6]:

$$\overline{R}(X) = \{x \in U | [x]_R \cap X \neq \emptyset\},$$
$$\underline{R}(X) = \{x \in U | [x]_R \subseteq X\}.$$

And $pos(X) = \underline{R}(X), neg(X) = \sim \overline{R}(X), bnd(X) = \overline{R}(X) - \underline{R}(X)$ are called the positive region, negative region, and boundary region of X, respectively. Objects belong to positive region $pos(X)$, whose equivalence class is definitely contained in the set X. Objects belong to negative region $neg(X)$, whose equivalence class is definitely not contained in the set X. And boundary region $bnd(X)$ is composed of objects whose equivalence class may be contained in the set X.

Let $K = (U, \{R_i\}_{i \in \tau})$ be a knowledge base, where $\{R_i\}_{i \in \tau}$ is a family of equivalence relations of $U \times U$ and τ is an index set. In the knowledge base K, when some knowledge is deleted, the classification ability of knowledge base K is not weakened. In the process of knowledge processing, deleting redundant knowledge can reduce the amount of computation. When a patient visits a doctor, the doctor does not require the patient to do the whole body examination first, then gives the diagnostic conclusion. Otherwise, it will delay the patient lots of time and will greatly increase the patient's medical expenses. Therefore, the knowledge reduction is an important aspect of rough set theory. Attribute reduction is helpful to eliminate the influence of the redundant and irrelevant attributes on the calculation process and the final results.

Let $K = (U, \{R_i\}_{i\in\mathcal{T}})$ be a knowledge base, $P \subseteq \widetilde{R} = \{R_i\}_{i\in\mathcal{T}}$ and $r \in P$.

- If $IND(P/r) = IND(P)$, then r is not necessary or redundant in P; otherwise, r is necessary in P. It should be noted that the indiscernibility relation $IND(P)$ generated by P is the intersection of all the equivalence relations in P;
- If for arbitrary $r \in P$, r is necessary in P, then P is called independent; otherwise, P is not independent;
- If P is independent and $IND(P) = IND(\widetilde{R})$, then P is called a reduction of $\widetilde{R}$.

A decision system $I = (U, A, V, f)$ is a quadruple [1], where U is a nonempty and finite universe; $A = C \cup D$ is the set composed of condition attribute set C and decision attribute set D, and $C \cap D = \emptyset$; V is the union of attribute value domain, i.e., $V = \cup_{a\in A} V_a$; $f : U \times A \to V$ is an information function,i.e., $\forall x \in U$, $a \in A$, that $f(x, a) \in V_a$, where $f(x, a)$ is the value of the object x under attribute a. Generally, let $D = \{d\}$. Unless otherwise specified, all decision systems in this paper are defined as the shown.

Uncertainty measures can help us to analyze the essential characteristics of data. Therefore, the uncertainty measure issue is an important research direction in rough set theory. The approximation accuracy proposed by Pawlak provides the percentage of possible correct decisions when classifying objects by employing the attribute set R. Let $I = (U, A, V, f)$ be a decision system, and $U/D = \{D_1, D_2, \cdots, D_m\}$ be a classification of universe U, and R be an attribute set. Then the approximation accuracy of U/D by R is defined as

$$\alpha_R(U/D) = \frac{\sum_{D_i\in U/D} |\underline{R}(D_i)|}{\sum_{D_i\in U/D} |\overline{R}(D_i)|}.$$

Dai et al. [1] proposed a reasonable uncertainty measure for incomplete decision systems. The uncertainty measure has the property of monotonicity, namely the finer the partition of universe U generated by indiscernibility relation is, the smaller the value of uncertainty measure is. That is to say, this measure can well reflect the uncertainty of incomplete information system. And what's more, when the incomplete decision system degenerates to a complete decision system, the property of monotonicity is still true. In a complete decision system $S = (U, A, V, f)$, conditional entropy of D with respect to $B (B \subseteq C)$ is defined to be

$$H(D|B) = -\sum_{i=1}^{|U|} p([x_i]_B) \sum_{j=1}^{m} p(D_j|[x_i]_B) logp(D_j|[x_i]_B)$$

where B is the conditional attribute subset of C, $p([x_i]_B) = |[x_i]_B|/|U|$, and $p(D_j|[x_i]_B) = |[x_i]_B \cap D_j|/|[x_i]_B|$.

3 Attribute Reduction of Multi-source Decision System

With the development of information technology, there are a large amount of information is collected every day. In particular, data about the same information can be obtained from different information sources. How to make full use of the information from different sources to efficiently acquire knowledge is very important. Multi-source information fusion will become a hot spot in the field of information research. The integration of information from different sources can get more comprehensive information to help make the right decision. In this paper, we study attribute reduction under multiple information sources which have the same universe and attributes and different information functions. That is to say, the research background is the multi-source decision system. It should be pointed out that this paper studies numerical decision systems.

First of all, decision systems from different sources can form a new information system which is called the Multi-Source Decision System. Detail descriptions are as follows:

Definition 3.1. A Multi-Source Decision System (MSDS) is defined to be the structure $MI = (U, \{I_i\}_{i \in N})$, where $\forall i \in N$, each $I_i = (U, C \cup D, V_i, f_i)$ be a decision system which represents the ith source of the Multi-Source Decision System and $N = \{1, 2, 3, \cdots\cdots\}$ denotes the number of sources. And $U/D = \{D_1, D_2, \cdots, D_m\}$ for each source S_i are identical.

A Multi-Source Decision System which includes s single information sources. Let the s pieces of single-source information system overlapping together can form a information box have s levels and it comes from our previous study [11].

3.1 The First Method of Attribute Reduction in the MSDS

Next, we study attribute reduction of the Multi-Source Decision System based on requirements of original information preservation. First and foremost, based on the consideration of the integrity of original effective information preservation, we proposed a consistent attribute reduction of MSDS.

Definition 3.2. Let $MI = (U, \{I_i\}_{i \in N})$ be a Multi-Source Decision System. And for $\forall i \in N$, $I_i = (U, C \cup D, V_i, f_i)$ be a decision system. For each S_i, if $\exists A \subseteq C$ such that $IND(A) = IND(C)$ and $B \subset A$ such that $IND(B) \neq IND(C)$, then A is called a consistent attribute reduction of the Multi-Source Decision System.

It is well known that all reductions of each decision system can be obtained by discernibility matrix. If a consistent attribute reduction of the MI can be obtained, then the information box can be compressed. And the amount of calculation can be reduced.

3.2 The Second Method of Attribute Reduction in the MSDS

In the case of a certain loss of original effective information, the fusion of information from different sources is key to data compression. There is no uniform standard about information fusion from multiple sources. Mean value fusion is the most common method of information fusion. In a Multi-Source Decision System $MI = (U, \{I_i\}_{i \in N})$, for $\forall x \in U$, $a \in A$, the value of x under attribute a is equal to $\sum_{i \in N} f_i(x, a)/|N|$.

It is well known that the more accurate data is, the more precise knowledge is. In order to obtain more accurate knowledge, we evaluate the accuracy of data collected under each attribute in the multi-source decision system. Therefore, our approach is to take every condition attribute as a basic point. For each condition attribute, the reliable source is selected by conditional entropy. So the importance of arbitrary condition attribute is characterized by conditional entropy in a decision system. The conditional entropy proposed by Dai can evaluate the importance of attributes [1]. The lower conditional entropy is, the more important the condition attribute will be. According to actual requirements, other uncertainty measure can be used to select the reliable source for each condition attribute.

Definition 3.3. Let $MI = (U, \{I_i\}_{i \in N})$ be a Multi-Source Decision System. The importance of any attribute a ($\forall a \in C$) in the Multi-Source Decision System is defined to be

$$d(a) = min_{i \in N}\{H(a|I_i)\}$$

where $H(a|I_i)$ denotes the conditional entropy of D with respect to a in the decision system I_i, which can be calculated by

$H(a|I_i) = -\sum_{k=1}^{|U|} p([x_k]_a) \sum_{h=1}^{m} p(D_h|[x_k]_a) logp(D_h|[x_k]_a).$

and

$p([x_k]_a) = |[x_k]_a|/|U|$, $p(D_h|[x_k]_a) = |[x_k]_a \cap D_h|/|[x_k]_a|$.

In a Multi-Source Decision System $MI = (U, \{I_i\}_{i \in \{1,2,\cdots,s\}})$, for $\forall a \in C$, there are r values can be calculated, namely $d(a|I_1), d(a|I_2), \cdots, d(a|I_s)$. By

Table 1. A Multi-source decision system

	1st source				2nd source				3rd source				4th source				d
	a_1	a_2	a_3	a_4	a_1	a_2	a_3	a_4	a_1	a_2	a_3	a_4	a_1	a_2	a_3	a_4	
x_1	1	2	2	1	1	2	2	1	1	2	1	1	1	2	2	1	1
x_2	1	2	1	1	1	2	2	1	1	2	1	1	1	2	1	1	1
x_3	1	1	2	1	1	1	2	1	1	1	1	1	1	1	2	1	0
x_4	0	1	1	1	1	1	1	1	0	1	2	1	0	1	2	0	1
x_5	2	1	1	2	0	1	1	1	1	1	1	1	2	2	1	1	0
x_6	0	1	1	0	0	1	2	0	0	1	1	0	1	1	2	0	1
x_7	1	1	2	1	2	2	2	1	1	2	1	1	1	2	1	1	0
x_8	1	1	1	0	2	1	1	0	1	1	1	0	1	1	1	0	1
x_9	2	1	1	0	2	1	1	1	2	1	2	1	2	1	2	1	0
x_{10}	1	1	1	0	1	1	1	1	0	1	2	1	0	1	2	0	0

comparing the size of these values, selecting the decision system with the minimum value as a reliable source of attribute a. Then a new restructuring decision system can be obtained. In the following, an example is introduced to illustrate the fusion process of multi-source information.

Example 3.2. There are four information sources about medical diagnosis, which can construct a Multi-Source Decision System with the same universe and attributes and different information functions., denoted by $MI = (U, \{I_1, I_2, I_3, I_4\})$. And $\forall i \in \{1,2,3,4\}$, $I_i = (U, C \cup D, V_i, f_i)$ be a decision system, where $U = \{x_1, x_2, \cdots, x_{10}\}$ is composed of ten patients, $C = \{a_1, a_2, a_3, a_4\}$ is the conditional attribute set, and $D = \{d\}$ is the decision attribute set. Specific data information is shown in Table 1.

Through discernibility matrix method, we can easily know that there are no redundant attributes in the information sources I_1,I_2,I_4, and the information source I_3 have a unique reduction namely $\{a_1, a_2, a_4\}$. According to Definition 3.2, therefore, there is no a consistent attribute reduction of MI can be obtained. So the fusion of four information sources need to be carried out. According to Definition 3.3, conditional entropy of each attribute in each source can be obtained, and detailed results are presented in the Table 2.

Table 2. Conditional entropy

source	a_1	a_2	a_3	a_4
S_1	1.0837	1.8388	1.7020	1.2124
S_2	1.0999	1.7020	1.4614	1.8388
S_3	1.3325	1.7020	1.7020	1.8388
S_4	1.1156	1.5654	1.5654	1.3859

The smaller conditional entropy is, the more important the condition attribute will be. By comparing these values, selecting the decision system with the minimum value as a reliable source of each attribute. The reliable source of attribute a_1 is the first information source I_1, the reliable source of attribute a_2 is the fourth information source I_4, the reliable source of attribute a_3 is the second information source I_2 and the reliable source of attribute a_4 is the first information source I_1. Based on conditional entropy fusion, a new restructuring decision system can be obtained, namely Table 3.

Then attribute reduction of the new system is carried out. According the definition of knowledge reduction, we can get

$U/C = \{\{x_1, x_2, x_7\}, \{x_3\}, \{x_4\}, \{x_5\}, \{x_6\}, \{x_8, x_{10}\}, \{x_9\}\}$;
$U/\{C - \{a_1\}\} = \{\{x_1, x_2, x_7\}, \{x_3\}, \{x_4\}, \{x_5\}, \{x_6\}, \{x_8, x_9, x_{10}\}$;
$U/\{C - \{a_2\}\} = \{\{x_1, x_2, x_3, x_7\}, \{x_4\}, \{x_5\}, \{x_6\}, \{x_8, x_{10}\}, \{x_9\}\}$;
$U/\{C - \{a_3\}\} = \{\{x_1, x_2, x_7\}, \{x_3\}, \{x_4\}, \{x_5\}, \{x_6\}, \{x_8, x_{10}\}, \{x_9\}\}$;
$U/\{C - \{a_4\}\} = \{\{x_1, x_2, x_7\}, \{x_3\}, \{x_4\}, \{x_5\}, \{x_6\}, \{x_8, x_{10}\}, \{x_9\}\}$.

Table 3. The new system after entropy fusion

U	a_1	a_2	a_3	a_4	d
x_1	1	2	2	1	1
x_2	1	2	2	1	1
x_3	1	1	2	1	0
x_4	0	1	1	1	1
x_5	2	2	1	2	0
x_6	0	1	2	0	1
x_7	1	2	2	1	0
x_8	1	1	1	0	1
x_9	2	1	1	0	0
x_{10}	1	1	1	0	0

Therefore, a_1 and a_2 are necessary attributes of the new decision system. Further the following conclusions can be obtained, namely $IND(U/C) = IND(U/\{C - \{a_3\}\})$ and $IND(U/C) = IND(U/\{C - \{a_4\}\})$. That is to say, $\{a_1, a_2, a_3\}$ and $\{a_1, a_2, a_4\}$ are attribute reductions of the new decisiom system.

In a Multi-Source Decision System, in the case of a certain loss of original effective information, attribute reduction of the MSDS can be obtained after conditional entropy fusion.

3.3 The Effectiveness of Conditional Entropy Fusion

In order to evaluate the effectiveness of conditional entropy fusion, the approximation accuracy is used as a quantitative index to reflect the superiority of conditional entropy fusion method. The reason why we chose this uncertainty measure is that the conditional entropy has monotonicity [1]. Theoretical derivation guarantee that the proposed conditional entropy can be used as a reasonable uncertainty measure for decision system. The validity of the proposed measure is verified by experiments on some real-life data sets. The finer the partition of universe U generated by indiscernibility relation is, the smaller value of the uncertainty measure is. In the process of conditional entropy fusion, we select the information source with the minimum conditional entropy as the reliable source of each condition attribute. Therefore, the uncertainty of decision system obtained by fusion is relatively small. Next, followed by Example 3.2, classification ability of each information source, conditional entropy fusion and mean fusion is compared by approximation accuracy.

Firstly, the mean fusion information is provided in Table 4.

Let $U = \{x_1, x_2, \cdots, x_{10}\}$, $C = \{a_1, a_2, a_3, a_4\}$ and $U/d = \{D_1, D_2\}$, where $D_1 = \{x_1, x_2, x_4, x_6, x_8\}$ and $D_1 = \{x_3, x_5, x_7, x_9, x_{10}\}$. According to the information of Tables 2, 4 and 5, the lower and upper approximations of decision classes can be obtained.

Table 4. The new system after mean fusion

U	a_1	a_2	a_3	a_4	d
x_1	1	2	1.75	1	1
x_2	1	2	1.25	1	1
x_3	1	1	1.75	1	0
x_4	0.25	1	1.5	0.75	1
x_5	1.25	1.25	1	1.25	0
x_6	0.25	1	1.5	0	1
x_7	1.25	1.75	1.5	1	0
x_8	1.25	1	1	0	1
x_9	2	1	1.5	0.75	0
x_{10}	0.5	1	1.5	0.5	0

According to Table 1, the lower and upper approximation sets of decision classes and the approximation accuracy of each source can be obtained. Detailed information is shown in Table 5.

Table 5. The approximation accuracy of each source in the MSDS

$Information$	1st source	2nd source
$\overline{C}(D_1)$	$\{x_1, x_2, x_4, x_6, x_8, x_{10}\}$	$\{x_1, x_2, x_4, x_6, x_8, x_{10}\}$
$\underline{C}(D_1)$	$\{x_1, x_2, x_4, x_6\}$	$\{x_1, x_2, x_6, x_8\}$
$\overline{C}(D_2)$	$\{x_3, x_5, x_7, x_8, x_9, x_{10}\}$	$\{x_3, x_4, x_5, x_7, x_9, x_{10}\}$
$\underline{C}(D_2)$	$\{x_3, x_5, x_7, x_9\}$	$\{x_3, x_5, x_7, x_9\}$
$\alpha_C(U/d)$	$\frac{2}{3}$	$\frac{2}{3}$
$Information$	3rd source	4th source
$\overline{C}(D_1)$	$\{x_1, x_2, x_4, x_6, x_7, x_8, x_{10}\}$	$\{x_1, x_2, x_4, x_6, x_7, x_8, x_{10}\}$
$\underline{C}(D_1)$	$\{x_6, x_8\}$	$\{x_1, x_6, x_8\}$
$\overline{C}(D_2)$	$\{x_1, x_2, x_3, x_4, x_5, x_7, x_9, x_{10}\}$	$\{x_2, x_3, x_4, x_5, x_7, x_9, x_{10}\}$
$\underline{C}(D_2)$	$\{x_3, x_5, x_9\}$	$\{x_3, x_5, x_9\}$
$\alpha_C(U/d)$	$\frac{1}{3}$	$\frac{3}{7}$

According to Tables 3 and 4, the lower and upper approximation sets of decision classes and the approximation accuracy of conditional entropy fusion and mean fusion can be obtained. Detailed information is shown in Table 6.

From the perspective of approximation accuracy, compared with the mean fusion, condition entropy fusion is more objective and more close to the essential characteristics of the MSDS, such as classification ability.

Table 6. The approximation accuracy of entropy fusion and mean fusion

$Information$	Conditional entropy fusion	Mean fusion
$\overline{C}(D_1)$	$\{x_1, x_2, x_4, x_6, x_8, x_{10}\}$	$\{x_1, x_2, x_4, x_6, x_8\}$
$\underline{C}(D_1)$	$\{x_1, x_2, x_4, x_6\}$	$\{x_1, x_2, x_4, x_6, x_8\}$
$\overline{C}(D_2)$	$\{x_3, x_5, x_7, x_8, x_9, x_{10}\}$	$\{x_3, x_5, x_7, x_9, x_{10}\}$
$\underline{C}(D_2)$	$\{x_3, x_5, x_7, x_9\}$	$\{x_3, x_5, x_7, x_9, x_{10}\}$
$\alpha_C(U/d)$	$\frac{2}{3}$	$\frac{1}{1}$

4 Conclusions

Attribute reduction of the Multi-Source Decision System is a hot topic in data processing. Based on the consideration of original effective information preservation, two methods of attribute reduction of the MSDS are proposed. In the case of no loss of original effective information, a consistent attribute reduction of the MSDS is proposed. In the case of a certain loss of original effective information, attribute reduction of the MSDS can be obtained after conditional entropy fusion. By attribute reduction, computation complexity of high-dimensional data can be simplified and the amount of computation can be reduced effectively. Therefore, the research on attribute reduction of the Multi-Source Decision System has great significance. This paper only proposes a framework for attribute reduction of the MSDS. In the future work, there are a lot of in-depth research needs to continue, such as selection of uncertainty measurement in fusion, quantitative reduction to meet user's requirements.

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