Probabilistic Rough Set Model Based on Dominance Relation

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Abstract. Unlike Pawlak rough set, probabilistic rough set models allow a tolerance inaccuracy in lower and upper approximations. Dominance relation cannot establish probability measure space for the universe. In this paper, the basic set assignment function, namely partition function is introduced into our work, which can transform the non-probability measure generated by dominance relation into a probability measure space. The probabilistic rough set model is established based on dominance relation, and explained clearly through an example.

Keywords: Dominance relation, Probabilistic rough set, Partition function, Probability space.

1 Introduction

The notion of probabilistic rough set approximations was first introduced by Wong and Ziarko [11], expressed through a pair of lower and upper approximations. The acceptance of probabilistic rough sets is merely due to the fact that they are defined by using probabilistic information and are more general and flexible. The introduction of probability enables the models to treat the universe of objects as samples from a much larger universe [8]. Yao presented a decision making method based on probabilistic rough set, which is called decision-theoretic rough set, where decision rules obtained from positive region, negative region and boundary region [15], [17]. Essentially, the decision-theoretic rough set is a special case of probabilistic rough set. The two thresholds in the probabilistic rough set model can be directly and systematically calculated by minimizing the decision costs with Bayesian decision procedure, which gives a brief semantics explanation in practical applications with minimum decision risks. Bayesian decision theory deals with making decisions with

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minimum risk based on observed evidence. The probabilistic rough set has much more wider application after introducing the Bayesian decision principle.

Since the decision-theoretic rough set was proposed by Yao in 1990 [16], it has attracted much more attentions. Yao gave a decision theoretic framework for approximating concepts in 1992 [14]. Azam et al. proposed a threshold configuration mechanism for reducing the overall uncertainty of probabilistic regions in the probabilistic rough sets [1]. Jia et al. developed an optimization representation of decision-theoretic rough set model and raised an optimization problem [5]. Yu et al. applied decision-theoretic rough set model for automatically determining the number of clusters with much smaller time cost [18]. Liu et al. combined the logistic regression and the decision-theoretic rough set into a new classification approach [7].

The original rough set theory does not consider attributes with preference ordered domain, relations in the rough set theory are not equivalence relations. It is vital to propose an extension rough set theory called the dominance-based rough set approach [3] to take account into the ordering properties of criteria. The innovation is mainly based on substitution of the indiscernibility relation by a dominance relation. Recently, several studies have been made on properties and algorithmic implementations of dominance-based rough set approach [10]. Nevertheless, with the dominance-based rough set approach proposed by Greco et al. [3], only a limited number of methods use dominance-based rough set approach to acquire knowledge from inconsistent ordered information systems , but they did not clearly point out the semantic explanation of unknown values. Then Shao et al. further explored an extension of the dominance relation in an inconsistent ordered information system [9]. Many researchers have enriched the ordered theories and obtained many achievements. For instance, Xu et al. constructed a method of attribute reduction based on evidence theory in ordered information system [12], and others [2], [13].

Probabilistic rough set is based on an equivalence relation. However, in real life, one may often consider the rank of attributes. So we need to extend the probabilistic rough set theory by considering dominance relation. Relevantly, Greco et al. discussed a Bayesian decision theory for dominance-based rough set model in 2007 [4]. Kusunoki et al. studied an empirical risk associated with the classification function [6]. These approach want to take account into costs of misclassification in fixing parameters of the dominance-based rough set approach, while didn't transact the essence of issue about how to establish a probability measure space through a dominance relation. When we use the probabilistic rough set theory by considering a dominance relation, we may be face with problems that the dominance relation can't induce probability measure spaces. It is important that one solves this issue. Our objective is to explore how to establish probabilistic rough set model based on dominance relation. The rest of this paper is organized as follows. Some preliminary concepts about the rough set model based on dominance relation and probabilistic rough set are briefly reviewed in Section 2. In Section 3, we developed the probabilistic rough set based on dominance relation by using the partition function. Finally, Section 4 gets the conclusions.

2 Preliminaries

In this section, we review some basic concepts about rough sets based on dominance relation [3], probabilistic approaches to rough set theory.

A partial relation from U to U meets reflexivity, antisymmetry and transitivity, including decreasing preference $R^{\preccurlyeq}$ and increasing preference $R^{\succcurlyeq}$. As the decreasing preference can be converted to increasing preference, in this paper we only consider the increasing preference, namely the dominance relation $R^{\succcurlyeq}$ without any loss of generality.

Let U be a universe of discourse, and $R^{\succcurlyeq}$ be a dominance relation on U. $U/R^{\succcurlyeq}$ is the set of dominance classes induced by a dominance relation $R^{\succcurlyeq}$, and $[x]_{R^{\succcurlyeq}}$ is called dominance class containing x. For an arbitrary set $X \subseteq U$, one can characterize X by a pair of lower and upper approximations which are defined as follows.

$$\underline{R^{\succcurlyeq}}(X) = \{x \in U | [x]_{R^{\succcurlyeq}} \subseteq X\},$$
$$\overline{R^{\succcurlyeq}}(X) = \{x \in U | [x]_{R^{\succcurlyeq}} \cap X \neq \emptyset\}.$$

The pair $(\underline{R^{\succcurlyeq}}(X), \overline{R^{\succcurlyeq}}(X))$ is called the dominance-based rough set of X with respect to (U, R). If $\underline{R}(X) \neq \overline{R}(X)$, then X is said to be a dominance-based rough set.

One can define P as probability measure if the set-valued function P maps from 2^U to $[0, 1]$, which can satisfy the two conditions: $P(U) = 1$; if $A \cap B = \emptyset$, then $P(A \cup B) = P(A) + P(B)$. And then P is a probability measure of $\sigma - algebra$ which is combined by the family subset of U.

Mathematically, one may introduce a probability function on $\sigma - algebra$ of a universal set to construct a probabilistic approximation space, with which relationships between concepts can be defined in probabilistic terms. We can estimate the conditional probability of a set given an equivalence class. With probabilistic theory, an equivalence class is in the lower approximation if and only if an element in the equivalence class has a high probability (i.e., greater than or equal to a threshold) to be in the set.

Given U as a non-empty and finite set of objects, where R is an equivalence relation in U. Denote $[x]_R$ as the equivalence class with respect to x. And P is a probability measure of $\sigma - algebra$ which is combined by the family subset of U. The triple $A_P = (U, R, P)$ is called probability approximation space.

Definition 2.1. [14] Let $0 \leq \beta < \alpha \leq 1$, for any $X \subseteq U$, the lower and upper approximations based on thresholds α, β with respect to $A_P = (U, R, P)$ are defined as follows

$$\underline{pr}_R^{(\alpha,\beta)}(X) = \{x \in U | P(X|[x]_R) \geq \alpha\},$$

$$\overline{pr}_R^{(\alpha,\beta)}(X) = \{x \in U | P(X|[x]_R) > \beta\}.$$

If $\underline{pr}_R^{(\alpha,\beta)}(X) = \overline{pr}_R^{(\alpha,\beta)}(X)$, then X is a definable set, otherwise X is a rough set.

Accordingly, the probabilistic positive, negative and boundary region are

$$pos(X) = \underline{pr}_R^{(\alpha,\beta)}(X) = \{x \in U | P(X|[x]_R) \geq \alpha\};$$
$$neg(X) = U - \overline{pr}_R^{(\alpha,\beta)}(X) = \{x \in U | P(X|[x]_R) \leq \beta\};$$
$$bn(X) = \overline{pr}_R^{(\alpha,\beta)}(X) - \underline{pr}_R^{(\alpha,\beta)}(X) = \{x \in U | \beta < P(X|[x]_R) < \alpha\}.$$

The parameters α, β in the probabilistic rough set theory above can be determined by special methods according to some additional conditions.

Based on the well-established Bayesian decision procedure, the decision-theoretic rough set model is derived from probability. That is to say, the decision-theoretic rough set model is a kind of probabilistic rough set model. The decision-theoretic rough set provides systematic methods for deriving the required thresholds on probabilistic rough set.

In real application of the probabilistic rough set models, we can obtain the thresholds α, β based on an intuitive understanding the levels of tolerance for errors. Just like we confirm the value of parameters α and β included in the Section 3. And the calculation methods of the conditional probability can also meet for demands in application.

3 Probabilistic Rough Set Model Based on Dominance Relation

Probabilistic rough set models allow a tolerance inaccuracy in lower and upper approximations, or equivalently in the probabilistic positive, negative and boundary regions. When the relations are never equivalence relations but dominance relations, they will not produce the probability measure space. Here one can handle the dominance classes induced by the dominance relation with an operator to transform the non-probability measure into a probability measure space.

$R_A^{\succcurlyeq}$ is a dominance relation, $[x]_{R_A^{\succcurlyeq}}$ is the dominance class containing x. And $P(X|Y)$ is the conditional probability of whether concept X happens or not depends on Y. We get the following definition.

Definition 3.1. Let $R_A^{\succcurlyeq}$ be a dominance relation. The basic set assignment function j is from 2^U to 2^U, is defined as

$$j(X) = \{x \in U | [x]_{R_A^{\succcurlyeq}} = X\}, X \in 2^U.$$

Obviously, $x \in j(X) \Leftrightarrow [x]_{R_A^{\succcurlyeq}} = X$.

The basic set assignment function $j([x]_{R_A^{\succcurlyeq}})$ contains these two properties:

- $\bigcup\limits_{X \subseteq U} j(X) = U$;
- $For X \neq Y$, then $j(X) \bigcap j(Y) = \emptyset$.

It is easy to notice that the function $j([x]_{R_A^{\succcurlyeq}})$ is a partition function of the universe U, one can also call the partition function as set-valued mapping approximation operator. Accordingly, this operator transforms the triple $A_P = (U, R_A^{\succcurlyeq}, P)$, which is not a probability approximation space into probability measure approximation space.

Definition 3.2 Let $R_A^{\succcurlyeq}$ be a dominance relation. Set $0 \le \beta < \alpha \le 1$, for any $X \subseteq U$, the lower and upper approximations based on parameters α, β with respect to $A_P = (U, R_A^{\succcurlyeq}, P)$ are defined as follows

$$\underline{jpr}_{R_A^{\succcurlyeq}}^{(\alpha,\beta)}(X) = \{x \in U | P(X | j([x]_{R_A^{\succcurlyeq}})) \ge \alpha\},$$

$$\overline{jpr}_{R_A^{\succcurlyeq}}^{(\alpha,\beta)}(X) = \{x \in U | P(X | j([x]_{R_A^{\succcurlyeq}})) > \beta\}.$$

If $\underline{jpr}_{R_A^{\succcurlyeq}}^{(\alpha,\beta)}(X) = \overline{jpr}_{R_A^{\succcurlyeq}}^{(\alpha,\beta)}(X)$, then X is a definable set, otherwise X is a rough set.

Accordingly, the probabilistic positive, negative and boundary region are

$$pos(X) = \underline{jpr}_{R_A^{\succcurlyeq}}^{(\alpha,\beta)}(X) = \{x \in U | P(X | j([x]_{R_A^{\succcurlyeq}})) \ge \alpha\};$$
$$neg(X) = U - \overline{jpr}_{R_A^{\succcurlyeq}}^{(\alpha,\beta)}(X) = \{x \in U | P(X | j([x]_{R_A^{\succcurlyeq}})) \le \beta\};$$
$$bn(X) = \overline{jpr}_{R_A^{\succcurlyeq}}^{(\alpha,\beta)}(X) - \underline{jpr}_{R_A^{\succcurlyeq}}^{(\alpha,\beta)}(X) = \{x \in U | \beta < P(X | j([x]_{R_A^{\succcurlyeq}})) < \alpha\}.$$

An example is employed to present the probabilistic rough sets based on dominance relation.

Example 3.1 In Table 1, $U = \{x_1, x_2, \cdots, x_7\}$ is a universe which consists of 7 objects, a_1, a_2, a_3, a_4 are the conditional attributes. One uses A, B, C, D to denote the values of these attributes. Moreover, A $\ge$ B $\ge$ C $\ge$ D.

Table 1. An information table

U	a_1	a_2	a_3	a_4
x_1	B	C	C	D
x_2	C	B	B	A
x_3	B	B	C	B
x_4	A	D	A	C
x_5	C	B	B	A
x_6	B	A	D	B
x_7	B	C	C	D

Here we consider all of these four conditions: a_1, a_2, a_3, a_4, accordingly, $R^{\succcurlyeq}$ is the dominance relation induced by these four attributes. Then one can obtain that the dominance classes are as following

$[x_1]_{R^{\succcurlyeq}} = \{x_1, x_3, x_7\}$, $[x_2]_{R^{\succcurlyeq}} = \{x_2, x_5\}$, $[x_3]_{R^{\succcurlyeq}} = \{x_3\}$, $[x_4]_{R^{\succcurlyeq}} = \{x_4\}$, $[x_5]_{R^{\succcurlyeq}} = \{x_2, x_5\}$, $[x_6]_{R^{\succcurlyeq}} = \{x_6\}$, $[x_7]_{R^{\succcurlyeq}} = \{x_1, x_3, x_7\}$.

It is obvious that these seven classes form a covering of the universe, but not a partition. Accordingly, one may use the partition function j. Then we can get

$$j(X_1) = \{x_1, x_7\},$$
$$j(X_2) = \{x_2, x_5\},$$
$$j(X_3) = \{x_3\},$$
$$j(X_4) = \{x_4\},$$
$$j(X_5) = \{x_6\}.$$

These five sets, namely $j(X_1)$, $j(X_2)$, $j(X_3)$, $j(X_4)$ and $j(X_5)$ form a partition of the universe U.

Given $X = \{x_2, x_3, x_5\}$. We assume that $\alpha = 2/3, \beta = 1/4$. Conditional probability is $P(X|Y)$, where

$$P(X|Y) = \frac{|X \bigcap Y|}{|Y|}.$$

Then the conditional probabilities with respect to $R^{\succcurlyeq}$ are shown as following:
$P(X|j([x_1]_{R^{\succcurlyeq}})) = 1/3$, $P(X|j([x_7]_{R^{\succcurlyeq}})) = 1/3$,
$P(X|j([x_2]_{R^{\succcurlyeq}})) = 1$, $P(X|j([x_5]_{R^{\succcurlyeq}})) = 1$,
$P(X|j([x_3]_{R^{\succcurlyeq}})) = 1$,
$P(X|j([x_4]_{R^{\succcurlyeq}})) = 0$,
$P(X|j([x_6]_{R^{\succcurlyeq}})) = 0$.

The lower and upper approximations based on parameters α, β with respect to $A_P = (U, R^{\succcurlyeq}, P)$ are computed as

$$\underline{jpr}_{R^{\succcurlyeq}}^{(\frac{2}{3},\frac{1}{4})}(X) = \{x \in U | P(X|j([x]_{R^{\succcurlyeq}})) \geq 2/3\} = \{x_2, x_3, x_5\},$$
$$\overline{jpr}_{R^{\succcurlyeq}}^{(\frac{2}{3},\frac{1}{4})}(X) = \{x \in U | P(X|j([x]_{R^{\succcurlyeq}})) > 1/4\} = \{x_1, x_2, x_3, x_5, x_7\}.$$

And then the probabilistic positive, negative and boundary region are

$$pos(X) = \underline{jpr}_{R^{\succcurlyeq}}^{(\frac{2}{3},\frac{1}{4})}(X) = \{x_2, x_3, x_5\};$$
$$neg(X) = U - \overline{jpr}_{R^{\succcurlyeq}}^{(\frac{2}{3},\frac{1}{4})}(X) = \{x_4, x_6\};$$
$$bn(X) = \overline{jpr}_{R^{\succcurlyeq}}^{(\frac{2}{3},\frac{1}{4})}(X) - \underline{jpr}_{R^{\succcurlyeq}}^{(\frac{2}{3},\frac{1}{4})}(X) = \{x_1, x_7\}.$$

Through the basic set assignment function, namely the partition function j, one can easily achieve the probability approximation space.

4 Conclusions

By considering the probabilistic rough sets based on dominance relation, the basic set assignment function, namely partition function is introduced into our work. The dominance relation results in a non-probability measure space. By the basic set assignment function, we can transact the covering of universe U induced by a dominance relation into a partition of the universe U. This paper presents a partition function to construct a probability measure combining the probability and rough set theory, and proposes the probabilistic rough set based on dominance relation. In the future work, we can do further and relevant studies about the probabilistic rough set model based on dominance relation.

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