Upper Approximation Reduction Based on Intuitionistic Fuzzy $\mathcal{T}$ Equivalence Information Systems

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Abstract. Attribute reduction, as one research problem, has played an important role in rough set theory. In this paper, the concept of upper approximation reduction is introduced in intuitionistic fuzzy $\mathcal{T}$ equivalence information systems. Moreover, rough set approach to upper approximation reduction is presented, and the judgement theorems and discernibility matrices are obtained in intuitionistic fuzzy $\mathcal{T}$ equivalence information systems. An example illustrates the validity of the approach, and shows that it is an efficient tool for knowledge discovery in intuitionistic fuzzy $\mathcal{T}$ equivalence information systems.

Keywords: Discernability matrix, Intuitionistic fuzzy relation, Intuitionistic fuzzy rough sets, Upper approximation reduction.

1 Introduction

The theory of rough sets, proposed by Pawlak [9,10], is a powerful mathematical approach to deal with inexact, uncertain or vague knowledge. It has been successfully applied to various fields of artificial intelligence such as pattern recognition, machine learning, and automated knowledge acquisition.

One important application of rough sets theory is attribute reduction in databases. For a data set with discrete attribute values, this can be done by reducing the number of redundant attributes and find a subset of the original attribute set that contains the same information as the original one. Then, people have been attempting to find all reducts. Much study on this area has been reported and many useful results were obtained [2,5,11,7,12].

In 1986, Atanassov [1] introduced the concept of intuitionistic fuzzy (IF) set. Combining IF set theory and rough set theory may result in a new hybrid mathematical structure for the requirement of knowledge-handling systems. The existing researches on IF rough sets are mainly concentrated on the approximation of IF sets. For example, according to fuzzy rough sets in the sense of Ntheda and Majumda [8], Jena and Ghosh [6] independently proposed the concept of

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the IF rough set in which the lower and upper approximations are both IF sets. Zhou and Wu [13] explored a general framework for the study of various relation-based IF rough approximation operators when the IF triangular norm $\mathcal{T} = \min$. Though Zhou and Wu [14] present a general framework for the study of relation based $(\mathcal{I}, \mathcal{T})$-IF rough sets by using constructive and axiomatic approachs, the reduction of IF rough sets based on IF $\mathcal{T}$ equivalence relation has not been considered. In this paper, we introduce formal concepts of upper approximation reductions with IF rough sets. The method using discernibility matrix to compute all the attribute reductions is developed.

The rest of this paper is structured as follows: to facilitate our discussion, some preliminary concepts are briefly recalled in Section 2. In Section 3, upper approximation reduction is proposed for the IF information systems. Moreover, the judgement theorems and discernibility matrices are obtained, from which we can provide an approach to attribute reductions in IF $\mathcal{T}$ equivalence information systems. In Section 4, an example illustrates the validity of this method, which shows that the method is effective in complicated information systems.

2 Intuitionistic Fuzzy Rough Sets and Intuitionistic Fuzzy $\mathcal{T}$ Equivalence Information Systems

In this section we mainly review the basic contents of IF information systems and IF rough sets based on IF $\mathcal{T}$ equivalence relation.

Definition 2.1.[4] Let $L^* = \{(\alpha_1, \alpha_2) \in I^2 | \alpha_1 + \alpha_2 \leq 1\}$. We define a relation $\leq_{L^*}$ on L^* as follows: for all $(\alpha_1, \alpha_2), (\beta_1, \beta_2) \in L^*$, $(\alpha_1, \alpha_2) \leq_{L^*} (\beta_1, \beta_2) \Leftrightarrow \alpha_1 \leq \beta_1$ and $\alpha_2 \geq \beta_2$.

Then the relation $\leq_{L^*}$ is a partial ordering on L^* and the pair $(L^*, \leq_{L^*})$ is a complete lattice with the smallest element $0_{L^*} = (0, 1)$ and the greatest element $1_{L^*} = (1, 0)$. The meet operator $\wedge$, join operator $\vee$ and complement operator $\sim$ on $(L^*, \leq_{L^*})$ which are linked to the ordering $\leq_{L^*}$ are, respectively, defined as follows: for all $(\alpha_1, \alpha_2), (\beta_1, \beta_2) \in L^*$, $(\alpha_1, \alpha_2) \wedge (\beta_1, \beta_2) = (\min(\alpha_1, \beta_1), \max(\alpha_2, \beta_2))$, $(\alpha_1, \alpha_2) \vee (\beta_1, \beta_2) = (\max(\alpha_1, \beta_1), \min(\alpha_2, \beta_2))$. $\sim (\alpha_1, \alpha_2) = (\alpha_2, \alpha_1)$.

Definition 2.2.[4] An IF t-norm on L^* is an increasing, commutative, associative mapping $\mathcal{T} : L^* \times L^* \rightarrow L^*$ satisfying $\mathcal{T}(1_{L^*}, \alpha) = \alpha$ for all $\alpha \in L^*$. An IF t-conorm on L^* is an increasing, commutative, associative mapping $\mathcal{S} : L^* \times L^* \rightarrow L^*$ satisfying $\mathcal{T}(0_{L^*}, \alpha) = \alpha$ for all $\alpha \in L^*$.

Definition 2.3.[4] An IF negator on L^* is a decreasing mapping $\mathcal{N} : L^* \rightarrow L^*$ satisfying $\mathcal{N}(0_{L^*}) = 1_{L^*}$ and $\mathcal{N}(1_{L^*}) = 0_{L^*}$. If $\mathcal{N}(\mathcal{N}(\alpha)) = \alpha$ for all $\alpha \in L^*$, then $\mathcal{N}$ is called an involutive IF negator.

The mapping $\mathcal{N}_s$, defined as $\mathcal{N}_s(\alpha_1, \alpha_2) = (\alpha_2, \alpha_1)$, $\forall (\alpha_1, \alpha_2) \in L^*$, is called the standard IF negator.

An IF t-norm $\mathcal{T}$ and an IF t-conorm $\mathcal{S}$ on L^* are said to be dual with respect to an IF negator $\mathcal{N}$ if

$$\mathcal{T}(\mathcal{N}(\alpha),\mathcal{N}(\beta)) = \mathcal{N}(\mathcal{S}(\alpha,\beta)), \quad \mathcal{S}(\mathcal{N}(\alpha),\mathcal{N}(\beta)) = \mathcal{N}(\mathcal{T}(\alpha,\beta)) \ \ \forall \alpha,\beta \in L^*.$$

Definition 2.4.[1] Let a set U be fixed. An IF set $\widetilde{A}$ in U is an object having the form $\widetilde{A} = \{\langle x, \mu_{\widetilde{A}}(x), \nu_{\widetilde{A}}(x)\rangle | x \in U\}$, where $\mu_{\widetilde{A}} : U \to I$ and $\nu_{\widetilde{A}} : U \to I$ satisfy $0 \le \mu_{\widetilde{A}}(x) + \nu_{\widetilde{A}}(x) \le 1$ for all $x \in U$, $\mu_{\widetilde{A}}(x)$ and $\nu_{\widetilde{A}}(x)$ are called the degree of membership and the degree of non-membership of the element $x \in U$ to $\widetilde{A}$, respectively. The family of all IF subsets of U is denoted by $IF(U)$. The complement of an IF set $\widetilde{A}$ is defined by $\sim \widetilde{A} = \{\langle x, \nu_{\widetilde{A}}(x), \mu_{\widetilde{A}}(x)\rangle | x \in U\}$. The IF universe set is $\widetilde{1}_U = \widetilde{(1,0)} = \widetilde{1}_{L^*} = \{\langle x, 1, 0\rangle | x \in U\}$ and the IF empty set is $\widetilde{1}_\emptyset = \widetilde{(0,1)} = \widetilde{0}_{L^*} = \{\langle x, 0, 1\rangle | x \in U\}$.

Definition 2.5.[3] An IF binary relation $\widetilde{R}$ on U is an IF subset of $U \times U$, namely, $\widetilde{R}$ is given by $\widetilde{R} = \{\langle (x,y), \mu_{\widetilde{R}}(x,y), \nu_{\widetilde{R}}(x,y)\rangle | (x,y) \in U \times U\}$, where $\mu_{\widetilde{R}} : U \times U \to I$ and $\nu_{\widetilde{R}} : U \times U \to I$, $0 \le \mu_{\widetilde{R}}(x,y) + \nu_{\widetilde{R}}(x,y) \le 1$ for all $(x,y) \in U \times U$. $IFR(U \times U)$ will be used to denote the family of all IF relations on U.

An IF $\mathcal{T}$ equivalence relation $\widetilde{R}$ is an IF relation on U which is reflexive ($\widetilde{R}(x,x) = \widetilde{1}$), symmetric ($\widetilde{R}(x,y) = \widetilde{R}(y,x)$) and $\mathcal{T}$ transitive ($\widetilde{R}(x,z) \ge_{L^*} \mathcal{T}(\widetilde{R}(x,y), \widetilde{R}(y,z))$), for every $x,y,z \in U$.

An IF information system is an ordered quadruple $I = (U, AT, V, f)$, where $U = \{x_1, x_2, \cdots, x_n\}$ is a non-empty finite set of objects, $AT = \{a_1, a_2, \cdots, a_p\}$ is a non-empty finite set of attributes, $V = \bigcup_{a \in AT} V_a$ and V_a is a domain of attribute a, $f : U \times AT \to V$ is a function such that $f(a,x) \in V_a$, for each $a \in AT$, $x \in U$, called an information function, where V_a is an IF set of universe U. That is $f(a,x) = (\mu_a(x), \nu_a(x))$, for all $a \in AT$, where $\mu_a : U \to [0,1]$ and $\nu_a : U \to [0,1]$ satisfy $0 \le \mu_a(x) + \nu_a(x) \le 1$, for all $x \in U$. μ_a and ν_a are, respectively, called the degree of membership and the degree of non-membership of the element $x \in U$ to attribute a. We denote $\widetilde{a}(x) = (\mu_a(x), \nu_a(x))$, then it is clear that $\widetilde{a}$ is an IF set of U.

Definition 2.6. An IF $\mathcal{T}$ equivalence information system is an ordered quintuple $\widetilde{\mathcal{I}} = (U, AT, V, f, \digamma)$, where (U, AT, V, f) is an IF information system, $\digamma$ is the mapping from power set AT into the family set $\widetilde{\mathbf{R}}$ of IF $\mathcal{T}$ equivalence relation. Let $\widetilde{\mathcal{I}} = (U, AT, V, f, \digamma)$ be an IF $\mathcal{T}$ equivalence information system, for any $A \subseteq AT$, $a \in A$, $\widetilde{R}_a \in \widetilde{\mathbf{R}}$ be an IF $\mathcal{T}$ equivalence relation with respect to attribute a, denoted as $\widetilde{R}_A = \bigcap_{a \in A} \widetilde{R}_a$.

Definition 2.7.[14] Let $\widetilde{\mathcal{I}} = (U, AT, V, f, \digamma)$ be an IF $\mathcal{T}$ equivalence information system. $\widetilde{X} \in IF(U)$ and $A \subseteq AT$, the $\mathcal{T}$-upper and $\mathcal{S}$-lower approximations of $\widetilde{X}$ with respect to IF $\mathcal{T}$ equivalence relation $\widetilde{R}_A$ are respectively defined by

$$\overline{\widetilde{R}_A}(\widetilde{X})(x) = \bigvee_{y \in U} \mathcal{T}(\widetilde{R}_A(x,y), \widetilde{X}(y)), \quad \forall x \in U,$$

$$\underline{\widetilde{R}_A}(\widetilde{X})(x) = \bigwedge_{y \in U} \mathcal{S}(\sim \widetilde{R}_A(x,y), \widetilde{X}(y)), \quad \forall x \in U.$$

From the definition of IF rough approximation, the following important properties in IF $\mathcal{T}$ equivalence information systems have been proved.

Theorem 2.1.[14] Let $\widetilde{\mathcal{I}} = (U, AT, V, f, \mathcal{F})$ be an IF $\mathcal{T}$ equivalence information system. $\widetilde{X}, \widetilde{Y} \in IF(U)$ and $A \subseteq AT$ then the $\mathcal{T}$-upper and $\mathcal{S}$-lower approximations satisfy the following properties.

(1) $\underline{\widetilde{R}_A}(\sim \widetilde{X}) = \sim\overline{\widetilde{R}_A}(\widetilde{X})$, $\overline{\widetilde{R}_A}(\sim \widetilde{X}) =\sim \underline{\widetilde{R}_A}(\widetilde{X})$.
(2) $\underline{\widetilde{R}_A}(\widetilde{X}) \subseteq \widetilde{X} \subseteq \overline{\widetilde{R}_A}(\widetilde{X})$.
(3) $\underline{\widetilde{R}_A}(\widetilde{X} \cap \widetilde{Y}) = \underline{\widetilde{R}_A}(\widetilde{X}) \cap \underline{\widetilde{R}_A}(\widetilde{Y})$, $\overline{\widetilde{R}_A}(\widetilde{X} \cup \widetilde{Y}) = \overline{\widetilde{R}_A}(\widetilde{X}) \cup \overline{\widetilde{R}_A}(\widetilde{Y})$.
(4) $\widetilde{X} \subseteq \widetilde{Y} \Rightarrow \underline{\widetilde{R}_A}(\widetilde{X}) \subseteq \underline{\widetilde{R}_A}(\widetilde{Y})$ and $\overline{\widetilde{R}_A}(\widetilde{X}) \subseteq \overline{\widetilde{R}_A}(\widetilde{Y})$.
(5) $\underline{\widetilde{R}_A}(\widetilde{X} \cup \widetilde{Y}) \supseteq \underline{\widetilde{R}_A}(\widetilde{X}) \cup \underline{\widetilde{R}_A}(\widetilde{Y})$, $\overline{\widetilde{R}_A}(\widetilde{X} \cap \widetilde{Y}) \subseteq \overline{\widetilde{R}_A}(\widetilde{X}) \cup \overline{\widetilde{R}_A}(\widetilde{Y})$.
(6) $\underline{\widetilde{R}_A}(\widetilde{\alpha}) = \widetilde{\alpha}$, $\overline{\widetilde{R}_A}(\widetilde{\alpha}) = \widetilde{\alpha}$, for $\alpha = (\alpha_1, \alpha_2) \in L^*$.
In particular, $\underline{\widetilde{R}_A}(\widetilde{1}_\emptyset) = \overline{\widetilde{R}_A}(\widetilde{1}_\emptyset) = \widetilde{1}_\emptyset$, $\underline{\widetilde{R}_A}(\widetilde{1}_U) = \overline{\widetilde{R}_A}(\widetilde{1}_U) = \widetilde{1}_U$.
(7) $\underline{\widetilde{R}_A}(\underline{\widetilde{R}_A}(\widetilde{X})) = \underline{\widetilde{R}_A}(\widetilde{X})$, $\overline{\widetilde{R}_A}(\overline{\widetilde{R}_A}(\widetilde{X})) = \overline{\widetilde{R}_A}(\widetilde{X})$.

3 Upper Approximation Reduction in IF $\mathcal{T}$ Equivalence Information Systems with Decision

In this section we define upper approximation reduction with respect to single IF decision class; we also develop the method based on discernibility matrix to compute all the upper approximation reductions.

An IF $\mathcal{T}$ equivalence information system with decision, is a special case of an IF $\mathcal{T}$ equivalence information system $\widetilde{\mathcal{I}} = (U, AT \cup D, V, f, \mathcal{F})$, where $\widetilde{D} = \{\widetilde{D}_k | k = 1, 2, \cdots, n\}$, $\widetilde{D}_k$ is an IF set of U called IF decision class.

Definition 3.1. Let $\widetilde{\mathcal{I}} = (U, AT \cup D, V, f, \mathcal{F})$ be an IF $\mathcal{T}$ equivalence information system with decision, $\widetilde{D}_k \in \widetilde{D}$ be the IF decision class, and $B \subseteq AT$. If $\overline{\widetilde{R}_B}(\widetilde{D}_k)(x) = \overline{\widetilde{R}_{AT}}(\widetilde{D}_k)(x)$ for any $x \in U$, we say that B is an upper consistent set of AT relative to $\widetilde{D}_k$. Moreover, if any proper subset of B is not the upper approximation set, then B is called one upper approximation reduction of this IF information system.

Theorem 3.1. Let $\widetilde{\mathcal{I}} = (U, AT \cup D, V, f, \mathcal{F})$ be an IF $\mathcal{T}$ equivalence information system with decision, $\widetilde{D}_k \in \widetilde{D}$ be the IF decision class, $B \subseteq AT$. Attribute set B is an upper approximation consistent set if and only if for any $x_i, x_j \in U$, there must exist $a_r \in B$ such that $\mathcal{T}(\widetilde{R}_{a_r}(x_i, x_j), \widetilde{D}_k(x_j)) \leq \overline{\widetilde{R}_{AT}}(D_k)(x_i)$.

Proof. On the one hand, if $B \subseteq AT$, then $\widetilde{R}_B \supseteq \widetilde{R}_{AT}$, we can easily show that $\overline{\widetilde{R}_B}(\widetilde{D}_k)(x_i) \geq \overline{\widetilde{R}_{AT}}(\widetilde{D}_k)(x_i)$, for any $x_i \in U$.

On the other hand,

$$\begin{aligned}
&\overline{\widetilde{R}_B}(\widetilde{D}_k)(x_i) \leq \overline{\widetilde{R}_{AT}}(\widetilde{D}_k)(x_i)\\
\Leftrightarrow &\bigvee_{x_j \in U} \mathcal{T}(\widetilde{R}_B(x_i, x_j), D_k(x_j)) \leq \overline{\widetilde{R}_{AT}}(\widetilde{D}_k)(x_i)\\
\Leftrightarrow &\bigvee_{x_j \in U} \mathcal{T}(\bigwedge_{a_k \in B} \widetilde{R}_{a_k}(x_i, x_j), D_k(x_j)) \leq \overline{\widetilde{R}_{AT}}(\widetilde{D}_k)(x_i)\\
\Leftrightarrow &\bigvee_{x_j \in U} \bigwedge_{a_k \in B} \mathcal{T}(\widetilde{R}_{a_k}(x_i, x_j), D_k(x_j)) \leq \overline{\widetilde{R}_{AT}}(\widetilde{D}_k)(x_i)\\
\Leftrightarrow &\forall x_j \in U, \exists a_r \in B, \text{such that}, \mathcal{T}(\widetilde{R}_{a_r}(x_i, x_j), D_k(x_j)) \leq \overline{\widetilde{R}_{AT}}(\widetilde{D}_k)(x_i)
\end{aligned}$$

The theorem is proved.

Definition 3.2. Let $\widetilde{\mathcal{I}} = (U, AT \cup D, V, f, \mathcal{F})$ be an IF $\mathcal{T}$ equivalence information system with decision, $\widetilde{D}_k \in \widetilde{D}$ be the IF decision class, $B \subseteq AT$. For any $x_i, x_j \in U$, we denote
$\text{UDis}(x_i, x_j) = \{a_r \in AT | \mathcal{T}(\widetilde{R}_{a_r}(x_i, x_j), \widetilde{D}_k(x_j)) \leq \widetilde{R}_{AT}(D_k)(x_i)\}$,
$\text{UM} = (u_{ij})_{n \times n}$,

where $u_{ij} = \text{UDis}(x_i, x_j)$, then $\text{UDis}(x_i, x_j)$ is said to be upper approximation discernibility attribute set between objects x_i and x_j. And matrix UM is referred to as upper approximation discernibility matrix of the IF $\mathcal{T}$ equivalence information system with decision.

Theorem 3.2. Let $\widetilde{\mathcal{I}} = (U, AT \cup D, V, f, \mathcal{F})$ be an IF $\mathcal{T}$ equivalence information system with decision, $\widetilde{D}_k \in \widetilde{D}$ be the IF decision class, $B \subseteq AT$. Attribute set B is an upper approximation consistent set if and only if $B \cap \text{UDis}(x_i, x_j) \neq \emptyset$ for all $x_i, x_j \in U$

Proof. It can be obtained from Theorem 3.1 and Definition 3.2.

Definition 3.3. Let $\widetilde{\mathcal{I}} = (U, AT \cup D, V, f, \mathcal{F})$ be an IF $\mathcal{T}$ equivalence information system with decision, $\widetilde{D}_k \in \widetilde{D}$ be the IF decision class, $B \subseteq AT$. UM be the upper approximation discernibility matrix of the IF $\mathcal{T}$ equivalence information system with decision $\widetilde{\mathcal{I}}$. Let

$$UF = \wedge \{\vee \{a | a \in \text{UDis}(x_i, x_j)\} | x_i, x_j \in U\}.$$

Then UF is called discernibility formulas of upper approximation in IF $\mathcal{T}$ equivalence information system with decision $\widetilde{\mathcal{I}}$.

Theorem 3.3. Let $\widetilde{\mathcal{I}} = (U, AT \cup D, V, f, \mathcal{F})$ be an IF $\mathcal{T}$ equivalence information system with decision, $\widetilde{D}_k \in \widetilde{D}$ be the IF decision class, $B \subseteq AT$. The minimal disjunctive normal form of discernibility formula of upper approximation is

$$UF = \bigvee_{k=1}^{p} (\bigwedge_{s=1}^{q_k} a_s).$$

Let $UB_k = \{a_s | s = 1, 2, \cdots, q_k\}$. Then $\{UB_k | k = 1, 2, \cdots, p\}$ is just set of all upper approximation reductions in IF $\mathcal{T}$ equivalence information system with decision $\widetilde{\mathcal{I}}$.

Proof. For any $x_i, x_j \in U$, by the definition of minimum disjunctive normal form, we have that UB_k is upper approximation consistent set. If one element of UB_k is reduced in $UF = \bigvee_{k=1}^{p} (UB_k)$, without loss of generality and the result denoted by UB'_k, then there exist x_{i_0}, $x_{j_0} \in U$ such that $UB'_k \cap \mathrm{UDis}(x_{i_0}, x_{j_0}) = \emptyset$. So, UB'_k is no an upper approximation consistent set. So UB_k is an upper approximation reduction in IF $\mathcal{T}$ equivalence information system with decision.

On the other hand, we known that the discernibility formula of upper approximation includes all $\mathrm{UDis}(x_i, x_j)$. Thus, there is not other upper approximation reduction except UB_k.

4 An Illustrated Example

In this section, we employ an example to illustrate our approach in this paper.

Example 4.1. Table 1 shows an IF information system, where $U = \{x_1, x_2, x_3, x_4, x_5, x_6, x_7, x_8, x_9, x_{10}\}$, $AT = \{a_1, a_2, a_3, a_4, a_5\}$. The membership degree and non-membership degree of every object are given in the following table.

Table 1. An IF information system

U	a_1	a_2	a_3	a_4	a_5
x_1	(0.3, 0.5)	(0.6, 0.4)	(0.5, 0.2)	(0.7, 0.1)	(0.5, 0.4)
x_2	(0.2, 0.7)	(0.1, 0.8)	(0.4, 0.5)	(0.1, 0.8)	(0.2, 0.8)
x_3	(0.2, 0.7)	(0.1, 0.8)	(0.4, 0.5)	(0.7, 0.1)	(0.2, 0.8)
x_4	(0.1, 0.8)	(0.1, 0.8)	(0.2, 0.7)	(0.1, 0.8)	(0.2, 0.8)
x_5	(0.9, 0.1)	(0.8, 0.1)	(0.8, 0.1)	(0.9, 0.0)	(0.7, 0.1)
x_6	(0.4, 0.6)	(0.8, 0.1)	(0.6, 0.3)	(0.9, 0.0)	(0.7, 0.1)
x_7	(0.3, 0.5)	(0.7, 0.3)	(0.5, 0.1)	(0.7, 0.1)	(0.6, 0.3)
x_8	(0.5, 0.3)	(0.8, 0.1)	(0.7, 0.1)	(1.0, 0.0)	(0.7, 0.1)
x_9	(0.6, 0.3)	(0.9, 0.0)	(0.7, 0.1)	(0.8, 0.2)	(0.8, 0.0)
x_{10}	(0.9, 0.1)	(0.9, 0.0)	(0.8, 0.1)	(0.6, 0.3)	(1.0, 0.0)

Every IF attribute a_k can define an IF $\mathcal{T}$ equivalence relation $\widetilde{R}_{a_k}$ as:

$$\widetilde{R}_{a_k}(x_i, x_j) = (\mu_{\widetilde{R}_{a_k}}(x_i, x_j), \nu_{\widetilde{R}_{a_k}}(x_i, x_j)),$$

where, $\mu_{\widetilde{R}_{a_k}}(x_i, x_j) = 1 - \max\{|\mu_{a_k}(x_i) - \mu_{a_k}(x_j)|, |\nu_{a_k}(x_i) - \nu_{a_k}(x_j)|\}$ and $\nu_{R_{a_k}}(x_i, x_j) = \frac{1}{2}(|\mu_{a_k}(x_i) - \mu_{a_k}(x_j)| + |\nu_{a_k}(x_i) - \nu_{a_k}(x_j)|)$. Consider the IF t-norm $\mathcal{T}$: $\mathcal{T}(\alpha, \beta) = (\max\{0, \alpha_1 + \beta_1 - 1\}, \min\{1, \alpha_2 + \beta_2\})$ for $\alpha = (\alpha_1, \alpha_2), \beta = (\beta_1, \beta_2) \in L^*$. Let $U = \{x_1, x_2, x_3, x_4, x_5, x_6, x_7, x_8, x_9, x_{10}\}$, $A = \{(0.6, 0.3), (0.3, 0.5), (0.9, 0.1), (0.6, 0.3), (0.7, 0.1), (0.2, 0.7), (0.4, 0.5), (0.5, 0.2), (0.7, 0.2), (1.0, 0)\}$,

$\widetilde{R}_{AT}$ is computed as follows:

$$\begin{pmatrix}
(1.0,0) & (0.3,0.65) & (0.5,0.45) & (0.3,0.65) & (0.4,0.5) & (0.7,0.25) & (0.9,0.1) & (0.7,0.25) & (0.6,0.35) & (0.4,0.5) \\
(0.3,0.65) & (1.0,0) & (0.3,0.65) & (0.8,0.2) & (0.2,0.8) & (0.2,0.8) & (0.3,0.65) & (0.1,0.85) & (0.2,0.8) & (0.2,0.8) \\
(0.5,0.45) & (0.3,0.65) & (1.0,0) & (0.3,0.65) & (0.3,0.7) & (0.3,0.7) & (0.4,0.55) & (0.3,0.7) & (0.2,0.8) & (0.2,0.8) \\
(0.3,0.65) & (0.8,0.2) & (0.3,0.65) & (1.0,0) & (0.2,0.8) & (0.2,0.8) & (0.3,0.65) & (0.1,0.85) & (0.2,0.8) & (0.2,0.8) \\
(0.4,0.5) & (0.2,0.8) & (0.3,0.7) & (0.2,0.8) & (1.0,0.5) & (0.5,0.5) & (0.4,0.5) & (0.6,0.3) & (0.7,0.25) & (0.7,0.3) \\
(0.7,0.25) & (0.2,0.8) & (0.3,0.7) & (0.2,0.8) & (0.5,0) & (1.0,0) & (0.8,0.15) & (0.7,0.2) & (0.7,0.25) & (0.5,0.5) \\
(0.9,0.1) & (0.3,0.65) & (0.4,0.55) & (0.3,0.65) & (0.4,0.15) & (0.8,0.15) & (1.0,0) & (0.7,0.2) & (0.7,0.25) & (0.4,0.5) \\
(0.7,0.25) & (0.1,0.85) & (0.3,0.7) & (0.1,0.85) & (0.6,0.2) & (0.7,0.2) & (0.7,0.2) & (1.0,0) & (0.8,0.2) & (0.6,0.35) \\
(0.6,0.35) & (0.2,0.8) & (0.2,0.8) & (0.2,0.8) & (0.7,0.25) & (0.7,0.25) & (0.7,0.25) & (0.8,0.2) & (1.0,0) & (0.7,0.25) \\
(0.4,0.5) & (0.2,0.8) & (0.2,0.8) & (0.2,0.8) & (0.7,0.3) & (0.5,0.5) & (0.4,0.5) & (0.6,0.35) & (0.7,0.25) & (1.0,0)
\end{pmatrix}.$$

Suppose an IF decision is $\widetilde{D} = \{(1,0),(0.6,0.4),(0.5,0.4),(0.7,0.1),(0.4,0.3),(1,0),(0,1),(0.5,0.5),(0.3,0.5),(0.7,0.2)\}$, then $\overline{\widetilde{R}}_{AT}(\widetilde{D}) = \{(1.0,0),(0.6,0.3),(0.5,0.4),(0.7,0.1),(0.5,0.3),(1.0,0),(0.9,0.1),(0.7,0.2),(0.7,0.25),(0.7,0.2)\}$ and the upper approximation discernibility matrix of UM is as follows:

$$UM = \begin{pmatrix}
AT & AT & AT & AT & AT & AT & AT & AT & AT & AT \\
\{2,4,5\} & AT & AT & \{3\} & AT & \{2,4,5\} & AT & AT & AT & AT \\
\{2\} & \{4\} & AT & \{4\} & AT & \{2,5\} & AT & AT & AT & \{1,2,3,5\} \\
AT & AT & AT & AT & AT & AT & AT & AT & AT & AT \\
\{1\} & AT & AT & AT & AT & \{1\} & AT & AT & AT & \{4,5\} \\
AT & AT & AT & AT & AT & AT & AT & AT & AT & AT \\
\{2,5\} & AT & AT & AT & AT & AT & AT & AT & AT & AT \\
\{2,4,5\} & AT & AT & AT & AT & \{1\} & AT & AT & AT & AT \\
\{1,2,5\} & AT & AT & AT & AT & \{1\} & AT & AT & AT & \{1,3,4,5\} \\
\{1,2,3,5\} & AT & AT & AT & AT & \{1,4,5\} & AT & AT & AT & AT
\end{pmatrix}.$$

Where 1,2,3,4,5 means a_1, a_2, a_3, a_4, a_5. We can get that $\{a_1, a_2, a_3, a_4\}$ is the only reduction of AT.

5 Conclusions

Intuitionistic fuzzy rough sets are the extension of fuzzy rough sets to deal with both fuzziness and vagueness in data. It is more material and concise than fuzzy rough sets to describe the essence of fuzziness. The existing researches on intuitionistic fuzzy rough sets are mainly concentrated on the construction of approximation operators. Less effort has been put on the attributes reduction of databases with intuitionistic fuzzy rough sets. In this paper, the concept of upper approximation reduction has been constructed in intuitionistic fuzzy $\mathcal{T}$ equivalence information systems. Moreover, rough set approach to upper approximation reductions has been presented and the judgement theorems and discernibility matrices have been obtained in intuitionistic fuzzy $\mathcal{T}$ equivalence information systems. The effectiveness of the approach to attribute reduction has been demonstrated by an example.

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