2012 Fourth International Symposium on Information Science and Engineering

The intuitionistic fuzzy rough sets model over two different universes

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Abstract-**Recently, much attention has been given to the rough set models based on two universes of discourse. And many rough set models on two universes have been developed from different views. In this paper, a novel model, the intuitionistic fuzzy rough set model over two different universes is firstly proposed from the intuitionistic point of view. We study some important properties of approximation operators.**

Keywords- Intuitionistic fuzzy set; rough degree; rough set; two different universes

I. INTRODUCTION

Rough set theory, originally proposed by Pawlak in the early 1980s[6], as a useful tool for treating with uncertainty or imprecision information, has been successfully applied in the fields of artificial intelligence[2,5-7].

Moreover, the study on the intuitionistic fuzzy rough set model over one universe was done, and it has become one of the hottest researches in recent years for authors. Shen et al.[10] researched the variable precision rough set model over two universes and investigated the properties. Yan et al.[12]studied on the model of rough set over dual-universe. More details about recent advancements of rough set model over two universes can be found in the literatures [3-4, 8-9, 11-13]. In this paper, we will discuss the intuitionistic fuzzy rough set models over two universes in the generalized approximation space.

II. PRELIMINARIES

In this section, we will review some necessary definitions and concepts required in the sequel of this paper.

Definition 2.1[1] Let U be an ordinary nonempty set. An intuitionistic fuzzy set $\tilde{A}$ in U is an object having the form

$$\tilde{A} = \{< x, \mu_{\tilde{A}}(x), \nu_{\tilde{A}}(x) >| x \in U\},$$

where $\mu_{\tilde{A}} : U \to [0,1]$ and $\nu_{\tilde{A}} : U \to [0,1]$ satisfy $0 \le \mu_{\tilde{A}}(x) + \nu_{\tilde{A}}(x) \le 1$ for all $x \in U$.

$\mu_{\tilde{A}}(x)$ and $\nu_{\tilde{A}}(x)$ are, respectively, called the degrees of membership and non-membership of the element x to $\tilde{A}$.

The complement of an intuitionistic fuzzy set $\tilde{A}$ is denoted by $\sim \tilde{A} = \{< x, \nu_{\tilde{A}}(x), \mu_{\tilde{A}}(x) >| x \in U\}$.

Definition 2.2 Let $\tilde{A}$ be an intuitionistic fuzzy set over U and $(\alpha, \beta) \in L$, $L = \{(\alpha, \beta) | \alpha \in [0,1], \beta \in [0,1], \alpha + \beta \le 1\}$, the (α, β)-level cut set of $\tilde{A}$, denoted by $\tilde{A}_\alpha^\beta$, is defined as follows:

$$\tilde{A}_\alpha^\beta = \{x \in U \mid \mu_{\tilde{A}}(x) \ge \alpha, \nu_{\tilde{A}}(x) \le \beta\}.$$

Definition 2.3 Let (U, V, R) be a generalized approximation space, for any $A \in F(U)$, denote

$$\underline{R}_U(A)(y) = \min\{A(x) \mid x \in R_p(y)\},$$

$$\overline{R}_U(A)(y) = \max\{A(x) \mid x \in R_p(y)\}, y \in V,$$

then $\underline{R}_U(A)$ and $\overline{R}_U(A)$ are called the lower and upper approximations of fuzzy set A in $F(U)$.

III. INTUITIONISTIC FUZZY ROUGH SET MODEL OVER TWO UNIVERSES

In this section, we will introduce the intuitionistic fuzzy rough set model over the different universes.

Definition 3.1[11] Let R be a crisp binary relation on the universe U and V, then

(1) R is serial if for any $x \in U$, $\exists y \in V$, s.t. $R(x, y) = 1$,

(2) R is reverse serial if for any $y \in V$, $\exists x \in U$, s.t. $R(x, y) = 1$.

Definition 3.2 Let (U, V, R) be a generalized approximation space, for any $\tilde{A} \in IF(U)$, $\tilde{B} \in IF(V)$, denote

$$\underline{R}_U(\tilde{A}) = \{< y, \mu_{\underline{R}_U(\tilde{A})}(y), \nu_{\underline{R}_U(\tilde{A})}(y) >| y \in V\},$$

$$\overline{R}_U(\tilde{A}) = \{< y, \mu_{\overline{R}_U(\tilde{A})}(y), \nu_{\overline{R}_U(\tilde{A})}(y) >| y \in V\},$$

$$\underline{R}_V(\tilde{B}) = \{< x, \mu_{\underline{R}_V(\tilde{B})}(x), \nu_{\underline{R}_V(\tilde{B})}(x) >| x \in U\},$$

$$\overline{R}_V(\tilde{B}) = \{< x, \mu_{\overline{R}_V(\tilde{B})}(x), \nu_{\overline{R}_V(\tilde{B})}(x) >| x \in U\},$$

where

$$\mu_{\underline{R}_U(\tilde{A})}(y) = \bigwedge_{x \in R_p(y)} \mu_{\tilde{A}}(x), \ \nu_{\underline{R}_U(\tilde{A})}(y) = \bigvee_{x \in R_p(y)} \nu_{\tilde{A}}(x),$$

$$\mu_{\overline{R}_U(\tilde{A})}(y) = \bigvee_{x \in R_p(y)} \mu_{\tilde{A}}(x), \ \nu_{\overline{R}_U(\tilde{A})}(y) = \bigwedge_{x \in R_p(y)} \nu_{\tilde{A}}(x),$$

$$\mu_{\underline{R}_V(\tilde{B})}(x) = \bigwedge_{y \in R_s(x)} \mu_{\tilde{B}}(y), \ \nu_{\underline{R}_V(\tilde{B})}(x) = \bigvee_{y \in R_s(x)} \nu_{\tilde{B}}(y),$$

$$\mu_{\overline{R}_V(\tilde{B})}(x) = \bigvee_{y \in R_s(x)} \mu_{\tilde{B}}(y), \ \nu_{\overline{R}_V(\tilde{B})}(x) = \bigwedge_{y \in R_s(x)} \nu_{\tilde{B}}(y),$$

978-0-7695-4951-4/13 $26.00 © 2013 IEEE
DOI 10.1109/ISISE.2012.74

then $\underline{R}_U(\tilde{A})$ and $\overline{R}_U(\tilde{A})$ are called the lower and upper approximations of intuitionistic fuzzy set $\tilde{A}$ in $IF(U)$, $\underline{R}_V(\tilde{B})$ and $\overline{R}_V(\tilde{B})$ are called the lower and upper approximations of intuitionistic fuzzy set $\tilde{B}$ in $IF(V)$. $(\underline{R}_U(\tilde{A}),\overline{R}_U(\tilde{A}))$ and $(\underline{R}_V(\tilde{B}),\overline{R}_V(\tilde{B}))$ are called the intuitionistic fuzzy rough sets over the universes U and V.

Furthermore, we also define the positive region $pos_{R_U}(\tilde{A}), pos_{R_V}(\tilde{B})$, negative region $neg_{R_U}(\tilde{A})$, $neg_{R_V}(\tilde{B})$ and boundary region $bn_{R_U}(\tilde{A}), bn_{R_V}(\tilde{B})$ of $\tilde{A},\tilde{B}$ about R_U, R_V on the universe U,V as follows, respectively:

$$pos_{R_U}(\tilde{A}) = \underline{R}_U(\tilde{A}),$$
$$neg_{R_U}(\tilde{A}) = \sim \overline{R}_U(\tilde{A}),$$
$$bn_{R_U}(\tilde{A}) = \overline{R}_U(\tilde{A}) \cap \sim \underline{R}_U(\tilde{A});$$
$$pos_{R_V}(\tilde{B}) = \underline{R}_V(\tilde{B}),$$
$$neg_{R_V}(\tilde{B}) = \sim \overline{R}_V(\tilde{B}),$$
$$bn_{R_V}(\tilde{B}) = \overline{R}_V(\tilde{B}) \cap \sim \underline{R}_V(\tilde{B}).$$

If for any $y \in V$ (respectively, $x \in U$), $\underline{R}_U(\tilde{A}) = \overline{R}_U(\tilde{A})$ (respectively, $\underline{R}_V(\tilde{B}) = \overline{R}_V(\tilde{B})$), then the intuitionistic fuzzy set $\tilde{A}$ (respectively, $\tilde{B}$) is a intuitionistic fuzzy definable set about the generalized approximation space (U,V,R). Otherwise the intuitionistic fuzzy set $\tilde{A}$ (respectively, $\tilde{B}$) is a rough set about the generalized approximation space, and $\tilde{A}$ (respectively, $\tilde{B}$) is called a rough intuitionistic fuzzy set.

Remark 3.1 In a generalized approximation space, we can find out that the lower and upper approximations of intuitionistic fuzzy set $\tilde{A} \in IF(U)$ belong to $IF(V)$, and the lower and upper approximations of intuitionistic fuzzy set $\tilde{B} \in IF(V)$ belong to $IF(U)$. This property is different from the lower and upper approximations over a universe. What's more, we can obtain the other properties as following.

Due to limited pages, we only discuss the properties of the intuitionistic fuzzy sets on the universe U, the properties of intuitionistic fuzzy sets on the universe V can be obtained similarly.

(1) $\underline{R}_U(\tilde{A}) = \sim \overline{R}_U(\sim \tilde{A})$, $\overline{R}_U(\tilde{A}) = \sim \underline{R}_U(\sim \tilde{A})$;

(2) $\underline{R}_U(\tilde{A} \cap \tilde{A}') = \underline{R}_U(\tilde{A}) \cap \underline{R}_U(\tilde{A}')$,
$\overline{R}_U(\tilde{A} \cup \tilde{A}') = \overline{R}_U(\tilde{A}) \cup \overline{R}_U(\tilde{A}')$;

(3) $\tilde{A}' \subseteq \tilde{A} \Rightarrow \underline{R}_U(\tilde{A}') \subseteq \underline{R}_U(\tilde{A})$,
$\overline{R}_U(\tilde{A}') \subseteq \overline{R}_U(\tilde{A})$;

(4) $\underline{R}(\tilde{A} \cup \tilde{A}') \supseteq \underline{R}(\tilde{A}) \cup \underline{R}(\tilde{A}')$;
$\overline{R}(\tilde{A} \cap \tilde{A}') \subseteq \overline{R}(\tilde{A}) \cap \overline{R}(\tilde{A}')$

(5) $\underline{R}_U(\tilde{A}) \subseteq \overline{R}_U(\tilde{A})$.

Proof. We only need to prove the first part of each property as the similarity of the above properties.

(1)According to Definition 3.2, we can obtain

$$\underline{R}_U(\sim A) = \{< y, \bigwedge_{x \in R_p(y)} \mu_{\sim \tilde{A}}(x), \bigvee_{x \in R_p(y)} \nu_{\sim \tilde{A}}(x) > \mid y \in V\}$$
$$= \{< y, \bigwedge_{x \in R_p(y)} \nu_{\tilde{A}}(x), \bigvee_{x \in R_p(y)} \mu_{\tilde{A}}(x) > \mid y \in V\}$$
$$= \sim \overline{R}_U(\tilde{A})$$

So we can have $\overline{R}_U(\tilde{A}) = \sim \underline{R}_U(\sim \tilde{A})$.

The property $\underline{R}_U(A) = \sim \overline{R}_U(\sim A)$ can be proved similarly.

(2)We can have

$$\underline{R}_U(\tilde{A} \cap \tilde{A}')$$
$$= \{< y, \bigwedge_{x \in R_p(y)} \mu_{\tilde{A} \cap \tilde{A}'}(x), \bigvee_{x \in R_p(y)} \nu_{\tilde{A} \cap \tilde{A}'}(x) > \mid y \in V\}$$
$$= \{< y, \bigwedge_{x \in R_p(y)} (\mu_{\tilde{A}}(x) \wedge \mu_{\tilde{A}}(x)), \bigvee_{x \in R_p(y)} (\mu_{\tilde{A}}(x) \vee \mu_{\tilde{A}}(x)) > \mid y \in V\}$$
$$= \{< y, \bigwedge_{x \in R_p(y)} \mu_{\tilde{A}}(x) \wedge \bigwedge_{x \in R_p(y)} \mu_{\tilde{A}'}(x), \bigvee_{x \in R_p(y)} \nu_{\tilde{A}}(x) \vee \bigvee_{x \in R_p(y)} \nu_{\tilde{A}'}(x) > \mid y \in V\}$$
$$= \underline{R}_U(\tilde{A}) \cap \underline{R}_U(\tilde{A}')$$

Hence, we can obtain $\underline{R}_U(\tilde{A} \cap \tilde{A}') = \underline{R}_U(\tilde{A}) \cap \underline{R}_U(\tilde{A}')$.

(3)According to the definitions of intuitionistic fuzzy lower and fuzzy upper approximation, (3) holds.

(4)It is easy to prove by the property (3).

(5) $\forall y \in \underline{R}_U(\tilde{A})$, we can have

$$\mu_{\underline{R}_U(\tilde{A})}(y) = \bigwedge_{x \in R_p(y)} \mu_{\tilde{A}}(x) \le \bigvee_{x \in R_p(y)} \mu_{\tilde{A}}(x)$$
$$\nu_{\underline{R}_U(\tilde{A})}(y) = \bigvee_{x \in R_p(y)} \mu_{\tilde{A}}(x) \le \bigwedge_{x \in R_p(y)} \mu_{\tilde{A}}(x).$$

Therefore, $\underline{R}_U(A) \subseteq \overline{R}_U(A)$.

Definition 3.3 Let (U,V,R) be a generalized approximation space, for any $\tilde{A} \in IF(U)$, $\tilde{B} \in IF(V)$, denote

$$\underline{R}_U(\tilde{A}_\alpha^\beta) = \{y \mid R_p(y) \subseteq \tilde{A}_\alpha^\beta\},$$
$$\overline{R}_U(\tilde{A}_\alpha^\beta) = \{y \mid R_p(y) \cap \tilde{A}_\alpha^\beta \ne \varnothing\},$$
$$\underline{R}_V(\tilde{B}_\alpha^\beta) = \{x \mid R_s(x) \subseteq \tilde{B}_\alpha^\beta\},$$
$$\overline{R}_V(\tilde{B}_\alpha^\beta) = \{x \mid R_s(x) \cap \tilde{B}_\alpha^\beta \ne \varnothing\},$$

where $\alpha, \beta \in [0,1]$, $\underline{R}_U(\tilde{A}_\alpha^\beta)$ and $\overline{R}_U(\tilde{A}_\alpha^\beta)$ are called the lower and upper approximation of $\tilde{A}_\alpha^\beta$ on the universe U, $\underline{R}_V(\tilde{B}_\alpha^\beta)$ and $\overline{R}_V(\tilde{B}_\alpha^\beta)$ are called the lower and upper approximation of $\tilde{B}_\alpha^\beta$ on the universe V.

Remark 3.2 In Definition 3.3, we give the concepts of the lower approximation and upper approximation of $\tilde{A}_{\alpha}^{\beta}$ about R on the universe U and V .Similarly ,we can also define the lower approximation and upper approximation of sets $\tilde{A}_{\alpha}, \tilde{A}_{\alpha+}, \tilde{A}^{\beta}, \tilde{A}^{\beta+}, \tilde{A}_{\alpha+}^{\beta}, \tilde{A}_{\alpha}^{\beta+}$, and $\tilde{A}_{\alpha+}^{\beta+}$ about R on the universe U and V . In the following discussion, without loss of generality, we only investigate the properties of $\underline{R}_U(\tilde{A}_{\alpha}^{\beta}), \overline{R}_U(\tilde{A}_{\alpha}^{\beta})$ and $\underline{R}_V(\tilde{B}_{\alpha}^{\beta}), \overline{R}_V(\tilde{B}_{\alpha}^{\beta})$. The corresponding properties can be extended to the other lower approximations and upper approximations, and we omit them here.

Proposition 3.2 Let (U,V,R) be a generalized approximation space, if $\alpha_1 > \alpha_2, \beta_1 < \beta_2$, we can obtain

$$\underline{R}_U(\tilde{A}_{\alpha_1}^{\beta_1}) \subseteq \underline{R}_U(\tilde{A}_{\alpha_2}^{\beta_2}), \ \overline{R}_U(\tilde{A}_{\alpha_1}^{\beta_1}) \subseteq \overline{R}_U(\tilde{A}_{\alpha_2}^{\beta_2}).$$

Proof. Since $\alpha_1 > \alpha_2, \beta_1 < \beta_2$, so $\tilde{A}_{\alpha_1}^{\beta_1} \subseteq \tilde{A}_{\alpha_2}^{\beta_2}$. For any $y \in \underline{R}_U(\tilde{A}_{\alpha_1}^{\beta_1})$, we can have $R_p(y) \subseteq \tilde{A}_{\alpha_1}^{\beta_1}$. Thus, $R_p(y) \subseteq \tilde{A}_{\alpha_2}^{\beta_2} \Leftrightarrow y \in \underline{R}_U(\tilde{A}_{\alpha_2}^{\beta_2})$. I.e., $\underline{R}_U(\tilde{A}_{\alpha_1}^{\beta_1}) \subseteq \underline{R}_U(\tilde{A}_{\alpha_2}^{\beta_2})$.

The properties $\overline{R}_U(\tilde{A}_{\alpha_1}^{\beta_1}) \subseteq \overline{R}_U(\tilde{A}_{\alpha_2}^{\beta_2})$ can be proved similarly.

According to the Definition 3.2, we can define two pairs of intuitionistic fuzzy sets as follows:

$$\underline{R}'_U(\tilde{A}) = \{< y, \mu_{\underline{R}'_U(\tilde{A})}(y), \nu_{\underline{R}'_U(\tilde{A})}(y) > | \ y \in V\},$$

$$\overline{R}'_U(\tilde{A}) = \{< y, \mu_{\underline{R}'_U(\tilde{A})}(y), \nu_{\underline{R}'_U(\tilde{A})}(y) > | \ y \in V\},$$

where

$$\mu_{\underline{R}'_U(\tilde{A})}(y) = \vee\{\alpha \mid y \in \underline{R}_U(\tilde{A}_{\alpha}^{\beta})\} = \vee\{\alpha \mid R_p(y) \subseteq \tilde{A}_{\alpha}^{\beta}\},$$

$$\nu_{\underline{R}'_U(\tilde{A})}(y) = \wedge\{\beta \mid y \in \underline{R}_U(\tilde{A}_{\alpha}^{\beta})\} = \wedge\{\beta \mid R_p(y) \subseteq \tilde{A}_{\alpha}^{\beta}\};$$

$$\mu_{\overline{R}'_U(\tilde{A})}(y) = \vee\{\alpha \mid y \in \overline{R}_U(\tilde{A}_{\alpha}^{\beta})\} = \vee\{\alpha \mid R_p(y) \cap \tilde{A}_{\alpha}^{\beta} \neq \phi\},$$

$$\nu_{\overline{R}'_U(\tilde{A})}(y) = \wedge\{\beta \mid y \in \overline{R}_U(\tilde{A}_{\alpha}^{\beta})\} = \wedge\{\beta \mid R_p(y) \cap \tilde{A}_{\alpha}^{\beta} \neq \phi\}.$$

Then we can obtain the properties in the following.

Proposition 3.3 Let (U,V,R) be a generalized approximation space, for any $\tilde{A} \in IF(U)$, then

$$\underline{R}_U(\tilde{A}) = \underline{R}'_U(\tilde{A}), \ \overline{R}_U(\tilde{A}) = \overline{R}'_U(\tilde{A}).$$

Proof. For any $y \in V$, denote

$$\alpha_1 = \mu_{\underline{R}_U(A)}(y) = \bigwedge_{x \in R_p(y)} \mu_{\tilde{A}}(x),$$

$$\beta_1 = \nu_{\underline{R}_U(A)}(y) = \bigvee_{x \in R_p(y)} \nu_{\tilde{A}}(x);$$

$$\alpha_2 = \vee\{\alpha \mid R_p(y) \subseteq \tilde{A}_{\alpha}^{\beta}\}, \quad \beta_2 = \wedge\{\beta \mid R_p(y) \subseteq \tilde{A}_{\alpha}^{\beta}\}.$$

Let α, β satisfy $R_p(y) \subseteq \tilde{A}_{\alpha}^{\beta}$, if $x \in R_p(y)$, then $\mu_{\tilde{A}(x)} \geq \alpha$, $\nu_{\tilde{A}(x)} \leq \beta$ and $\bigwedge_{x \in R_p(y)} \mu_{\tilde{A}}(x) \geq \alpha$, $\bigvee_{x \in R_p(y)} \nu_{\tilde{A}}(x) \leq \beta$. So $\alpha_1 \geq \alpha, \beta_1 \leq \beta$, therefore $\alpha_1 \geq \alpha_2, \beta_1 \leq \beta_2$.

On the other hand, for any $\alpha > \alpha_2, \beta < \beta_2$, according to the definition of α_2, β_2 , we can know that there exists $x \in R_p(y)$, s.t. $x \notin \tilde{A}_{\alpha}^{\beta}$, i.e., $\alpha_1 \leq \mu_{\tilde{A}}(x) < \alpha, \beta_1 \geq \nu_{\tilde{A}}(x) > \beta$, thus $\alpha > \alpha_1, \beta < \beta_1$, by the arbitrary of $\alpha > \alpha_2$ and $\beta < \beta_2$, we can obtain $\alpha_2 \geq \alpha_1, \beta_2 \leq \beta_1$. Hence $\underline{R}_U(\tilde{A}) = \underline{R}'_U(\tilde{A})$.

The properties $\overline{R}_U(\tilde{A}) = \overline{R}'_U(\tilde{A})$ can be proved similarly.

IV. CONCLUSIONS

In this paper, we have introduced intuitionistic fuzzy rough set model over two different universes in the generalized approximation space (U,V,R) . Meantime, we gave the definitions and properties about the intuitionistic fuzzy rough sets.

ACKNOWLEDGMENT

This work is supported by National Natural Science Foundation of China (No.61105041, 71071124 and 11001227), National Natural Science Foundation of CQ CSTC (No. cstc2011jjA40037), Science and Technology Program of Board of Education of Chongqing (KJ120805) and Graduate Innovation Foundation of Chongqing University of Technology (No.YCX2011312).

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