Optimistic Multi-granulation Fuzzy Rough Sets on Tolerance Relations

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***Abstract*—Based on the analysis of rough set model on a tolerance relation and the theory of fuzzy rough set, a new extended rough set model is constructed, which is the optimistic multi-granulation fuzzy rough sets based on tolerance relations. It follows the research on the properties of the lower and upper approximations of the new multi-granulation fuzzy rough set models based on tolerance relations. From which it can be found that the new extended model is a generalized rough set model based on tolerance relations from the perspective of granulation.**

Keywords- Tolerance relation; Fuzzy rough set; optimistic multi-granulation

I. INTRODUCTION

A Rough set theory, proposed by Pawlak[1-2], is a theory for the research of uncertainty management in a wide variety of applications related to artificial intelligence[3-5]. The theory has been applied many fields[6-8] successfully, which related an amount of imprecise, vague and uncertain information.

Zadeh firstly proposed the concept of granular computing and discussed issues of fuzzy information granulation in 1979[9]. Then the basic idea of information granulation had been applied to many fields including rough set[1-2]. In 1985, Hobbs proposed the concept of granularity. In the point view of granular computing, the classical Pawlak rough set and some expanded rough sets like tolerant models[10-13]are based on a single granulation.

Qian extended the Pawlak rough set to multi-granulation rough set models where the approximation operators are defined by multiple equivalence relations on the universe[10，14]. Associated tolerant rough set with the theory of fuzzy rough set with granular computing point view, we will propose a kind of optimistic multi-granulation fuzzy rough set models on tolerance relations. The main objective of this paper is to extend A. Skowron's[12]and J. Jarinen's[13]tolerance rough set model determined by a single tolerance relation to multi-granulation fuzzy rough sets where fuzzy set approximations are defined by multiple tolerance relations.

II. PRELIMINARIES

In this section, we will first review some basic concepts and notions in the theory of rough set on a tolerance relation, fuzzy rough sets and multi-granulation rough sets on the basis of equivalence relations. More can be found in [10, 13-15].

A tolerance information system[16] is an ordered triple $\mathcal{I}=(U,AT,\tau)$, where U is the non-empty finite set of objects known as universe; AT is the non-empty finite set of attributes; and τ is the mapping from power set AT into the family set $\mathcal{R}$ of tolerance relations satisfying reflexivity and symmetry on universe[13]. Note that a similarity relation only satisfies reflexivity, while a neighborhood operator is a mapping $n:U\to P(U), x\mapsto n(x)$. For $X\subseteq U$, denote $n(X)=\bigcup_{x\in X} n(x)$, then $n(X)$ is called as the neighborhood of the set X. The binary relations: serial relations, inverse serial relations, reflexive relations, symmetric relations, transitive relations, and Euclidean relations are all special neighborhood operators.

Let $\mathcal{I}=(U,AT,\tau)$ be a tolerance information system, for $B\subseteq AT$, $X\subseteq U$, $R_B\in\mathcal{R}$ is a relation with respect to the attributes set B. Denote

$$R_B(x_i)=\{x_j\in U\mid (x_i,x_j)\in R_B\}$$

where $i\in\{1,2,...,|U|\}$, then $R_B(x_i)$ will be called the tolerance class of x_i with respect to the tolerance relation R_B. Note that a tolerance relation can construct a covering instead of a partition of the universe U.

Let $\mathcal{I}=(U,AT,\tau)$ be a tolerance information system. The lower approximation and the upper approximation of a set $X\subseteq U$ with respect to a tolerance relation R_A are respectively defined by

$$\underline{R_A}(X)=\{x\in U\mid R_A(x)\subseteq X\},$$

$$\overline{R_A}(X)=\{x\in U\mid R_A(x)\cap X\neq\varnothing\}.$$

The set $Bnd_R(X)=\overline{R_A}(X)-\underline{R_A}(X)$ is called the boundary of X.

The set $\underline{R_A}(X)$ consists of elements which surely belong to X in view of the knowledge provided by R, while $\overline{R_A}(X)$ consists of elements which possibly belong to X. The boundary is the actual area of uncertainty. It consists of elements whose membership in X can not be decided when R-related objects can not be distinguished from each other.

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III. THE OPTIMISTIC MULTI-GRANULATION FUZZY ROUGH SETS ON TOLERANCE RELATION

In this section, we will make researches about optimistic multi-granulation fuzzy rough set which are on the problem of the rough approximations of a fuzzy set on multiple tolerance relations.

At first, we will propose a fuzzy rough set model (in brief FRS) based on a tolerance relation in the following.

Let $\mathcal{I}=(U,AT,\tau)$ be a tolerance information system, $A\subseteq AT$. For the fuzzy set $X\in F(U)$, denote

$$\underline{R_A}(X)(x)=\wedge\{X(y)\mid y\in R_A(x)\},$$

$$\overline{R_A}(X)(x)=\vee\{X(y)\mid y\in R_A(x)\},$$

where " $\vee$ "means "max" and " $\wedge$ "means "min", then $\underline{R_A}(X)$ and $\overline{R_A}(X)$ are the lower and upper approximation of the fuzzy set X on the tolerance relation R with respect to the subset of attributes A. If $\underline{R_A}(X)\neq\overline{R_A}(X)$, then the fuzzy set X is a fuzzy rough set on the tolerance relation. We can easily find that this model will be the fuzzy rough set model we have introduced in the part 2.2 if the above relation R is an equivalence relation.

Definition 3.1 Let $\mathcal{I}=(U,AT,\tau)$ be a tolerance information system, $A_i\subseteq AT, 1\le i\le m$. For the fuzzy set $X\in F(U)$, denote

$$\underline{OR_{\sum_{i=1}^m A_i}}(X)(x)=\vee_{i=1}^m\{\wedge\{X(y)\mid y\in R_{A_i}(x)\}\},$$

$$\overline{OR_{\sum_{i=1}^m A_i}}(X)(x)=\wedge_{i=1}^m\{\vee\{X(y)\mid y\in R_{A_i}(x)\}\},$$

then $\underline{OR_{\sum_{i=1}^m A_i}}(X)$ and $\overline{OR_{\sum_{i=1}^m A_i}}(X)$ are respectively called the optimistic multi-granulation lower approximation and upper approximation of X on the tolerance relations $R_{A_i}(1\le i\le m)$. X is a multi-granulation fuzzy rough set on the tolerance relations $R_{A_i}(1\le i\le m)$ if and only if $\underline{OR_{\sum_{i=1}^m A_i}}(X)\neq\overline{OR_{\sum_{i=1}^m A_i}}(X)$. Otherwise, X is a multi-granulation fuzzy definable set on the tolerance relations $R_{A_i}(1\le i\le m)$. The boundary of the set X is defined as

$$Bnd^O_{R_{\sum_{i=1}^m A_i}}(X)=\overline{OR_{\sum_{i=1}^m A_i}}(X)\cap(\sim\underline{OR_{\sum_{i=1}^m A_i}}(X)).$$

It can be found that the OMGFRS on the tolerance relations $R_{A_i}(1\le i\le m)$ will be degenerated into fuzzy rough set when $A_i=A_j, i\neq j$ and $R_{A_i}(x)$ are equivalence classes with respect to the subsets of attributes $A_i(1\le i\le m)$. That is to say, a fuzzy rough set model is a special instance of the OMGFRS on the tolerance relations. Besides, this model can also been turned the OMGRS if the relations are equivalence relations and the considered set is a crisp one. What's more, the OMGFRS on tolerance relations will be degenerated into a rough set model on tolerance relation if $A_i=A_j, i\neq j$ and the considered concept X is a crisp set.

Just from Definition 3.1, we can obtain some properties of the OMGFRS in a tolerance information system.

Proposition 3.1 Let $\mathcal{I}=(U,AT,\tau)$ be a tolerance information system, $A_i\subseteq AT, 1\le i\le m$ and $X\in F(U)$. Then the following properties hold.

(1) $\underline{OR_{\sum_{i=1}^m A_i}}(X)\subseteq X\subseteq\overline{OR_{\sum_{i=1}^m A_i}}(X)$;

(2) $\underline{OR_{\sum_{i=1}^m A_i}}(\sim X)=\sim\overline{R_{\sum_{i=1}^m A_i}}(X)$,

$\overline{OR_{\sum_{i=1}^m A_i}}(\sim X)=\sim\underline{OR_{\sum_{i=1}^m A_i}}(X)$;

(3) $\underline{OR_{\sum_{i=1}^m A_i}}(U)=\overline{OR_{\sum_{i=1}^m A_i}}(U)=U$,

$\underline{OR_{\sum_{i=1}^m A_i}}(\varnothing)=\overline{OR_{\sum_{i=1}^m A_i}}(\varnothing)=\varnothing$;

(4) $\underline{OR_{\sum_{i=1}^m A_i}}(X)\supseteq\underline{OR_{\sum_{i=1}^m A_i}}(\underline{OR_{\sum_{i=1}^m A_i}}(X))$,

$\overline{OR_{\sum_{i=1}^m A_i}}(X)\subseteq\overline{OR_{\sum_{i=1}^m A_i}}(\overline{OR_{\sum_{i=1}^m A_i}}(X))$.

Proof. Since the number of the granulations is finite, we only prove the results are true when the tolerance information system has two tolerance relations($A,B\subseteq AT$) for convenience in the following.

It is obvious that all terms hold when $A=B$. When $A\neq B$, the proposition can be proved as follows.

(1) According to the definition 3.1, it holds obviously.

(2) For any $x\in U$ and $A,B\subseteq AT$, since $\underline{R_A}(\sim X)=\sim\overline{R_A}(X)$ and $\underline{R_B}(\sim X)=\sim\overline{R_B}(X)$, then we have

$$\begin{aligned}\underline{OR_{A+B}}(\sim X)(x)&=\{\wedge\{1-X(y)\mid y\in R_A(x)\}\}\vee\{\wedge\{1-X(y)\mid y\in R_B(x)\}\}\\&=\{1-\vee\{X(y)\mid y\in R_A(x)\}\}\vee\{1-\vee\{X(y)\mid y\in R_B(x)\}\}\\&=1-\{\vee\{X(y)\mid y\in R_A(x)\}\}\wedge\{\vee\{X(y)\mid y\in R_B(x)\}\}.\\&=\sim\overline{OR_{A+B}}(X)(x)\end{aligned}$$

So $\overline{OR_{\sum_{i=1}^m A_i}}(\sim X)=\sim\underline{OR_{\sum_{i=1}^m A_i}}(X)$ can be proved similarly.

(3) Since for any $x\in U$, $U(x)=1$, then for any $A,B\subseteq U$,

$$\begin{aligned}\underline{OR_{A+B}}(U)(x)&=\{\wedge\{U(y)\mid y\in R_A(x)\}\}\vee\{\wedge\{U(y)\mid y\in R_B(x)\}\}\\&=1=U(x)\end{aligned}$$

and

$$\begin{aligned}\overline{OR_{A+B}}(U)(x)&=\{\vee\{U(y)\mid y\in R_A(x)\}\}\wedge\{\vee\{U(y)\mid y\in R_B(x)\}\}\\&=1=U(x)\end{aligned}$$

So $\underline{OR_{A+B}}(U)=\overline{OR_{A+B}}(U)=U$. The others can be proved similarly.

(4) Since $\underline{OR_{A+B}}(X)\subseteq X$, then for any $x\in U$, one has $\underline{OR_{A+B}}(X)(x)\le X(x)$. So by Definition 3.1, one can obtain $\underline{OR_{A+B}}(X)\supseteq\underline{OR_{A+B}}(\underline{OR_{A+B}}(X))$. The other can be proved similarly.

Proposition 3.2 Let $\mathcal{I}=(U,AT,\tau)$ be a tolerance information system, $A_i \subseteq AT, 1\le i\le m,$ $X,Y\in F(U)$. Then the following properties hold.

(1) $\underline{OR_{\sum_{i=1}^{m}A_i}}(X\cap Y)\subseteq \underline{OR_{\sum_{i=1}^{m}A_i}}(X)\cap \underline{OR_{\sum_{i=1}^{m}A_i}}(Y),$

$\overline{OR_{\sum_{i=1}^{m}A_i}}(X\cup Y)\supseteq \overline{OR_{\sum_{i=1}^{m}A_i}}(X)\cup \overline{OR_{\sum_{i=1}^{m}A_i}}(Y);$

(2) $X\subseteq Y \Rightarrow \underline{OR_{\sum_{i=1}^{m}A_i}}(X)\subseteq \underline{OR_{\sum_{i=1}^{m}A_i}}(Y),$

$X\subseteq Y \Rightarrow \overline{OR_{\sum_{i=1}^{m}A_i}}(X)\subseteq \overline{OR_{\sum_{i=1}^{m}A_i}}(Y);$

(3) $\underline{OR_{\sum_{i=1}^{m}A_i}}(X\cup Y)\supseteq \underline{OR_{\sum_{i=1}^{m}A_i}}(X)\cup \underline{OR_{\sum_{i=1}^{m}A_i}}(Y),$

$\overline{OR_{\sum_{i=1}^{m}A_i}}(X\cap Y)\subseteq \overline{OR_{\sum_{i=1}^{m}A_i}}(X)\cap \overline{OR_{\sum_{i=1}^{m}A_i}}(Y).$

Proof. All terms hold obviously when $A=B$ or $X=Y$. If $A\ne B$ and $X\ne Y$, the proposition can be proved as follows.

(1) For any $x\in U$, $A,B\subseteq AT$ and $X,Y\in F(U)$,

$\underline{OR_{A+B}}(X\cap Y)(x)=\{\wedge\{(X\cap Y)(y)\mid y\in R_A(x)\}\}$

$\vee\{\wedge\{(X\cap Y)(y)\mid y\in R_B(x)\}\}$

$=\{\wedge\{X(y)\wedge Y(y)\mid y\in R_A(x)\}\}$

$\vee\{\wedge\{X(y)\wedge Y(y)\mid y\in R_B(x)\}\}$

$=\{\underline{R_A}(X)(x)\wedge \underline{R_A}(Y)(x)\}\vee\{\underline{R_B}(X)(x)\wedge \underline{R_B}(Y)(x)\}$

$\le\{\underline{R_A}(X)(x)\vee \underline{R_B}(X)(x)\}\wedge\{\underline{R_A}(Y)(x)\vee \underline{R_B}(Y)(x)\}$

$=\underline{OR_{A+B}}(X)(x)\wedge \underline{OR_{A+B}}(Y)(x).$

Then $\underline{OR_{A+B}}(X\cap Y)\subseteq \underline{OR_{A+B}}(X)\cap \underline{OR_{A+B}}(Y).$

So $\overline{OR_{\sum_{i=1}^{m}A_i}}(X\cup Y)\supseteq \overline{OR_{\sum_{i=1}^{m}A_i}}(X)\cup \overline{OR_{\sum_{i=1}^{m}A_i}}(Y)$ can be proved similarly.

(2) Since for any $x\in U$, we have $X(y)\le Y(y)$. Then the properties hold obviously by Definition 3.1.

(3) Since $X\subseteq X\cup Y$, and $Y\subseteq X\cup Y$, then $\underline{OR_{A+B}}(X)\subseteq \underline{OR_{A+B}}(X\cup Y)$ and $\underline{OR_{A+B}}(Y)\subseteq \underline{OR_{A+B}}(X\cup Y)$. So the property $\underline{OR_{A+B}}(X\cup Y)\supseteq \underline{OR_{A+B}}(X)\cup \underline{OR_{A+B}}(Y)$ obviously holds.

Denote $\underline{OR_{\sum_{i=1}^{m}A_i}}(X)_\alpha$ and $\overline{OR_{\sum_{i=1}^{m}A_i}}(X)_\beta$ as the α-cut set of the optimistic multi-granulation lower approximation and the β-cut set of the optimistic multi-granulation upper approximation of X on tolerance relations $R_{A_i}, 1\le i\le m$.

Proposition 3.3 Let $\mathcal{I}=(U,AT,\tau)$ be a tolerance information system, $A_i\subseteq AT, 1\le i\le m,$ and $X,Y\in F(U)$. For any $0<\beta\le\alpha\le 1$, we have

(1) $\underline{OR_{\sum_{i=1}^{m}A_i}}(X\cap Y)_\alpha\subseteq \underline{OR_{\sum_{i=1}^{m}A_i}}(X)_\alpha\cap \underline{OR_{\sum_{i=1}^{m}A_i}}(Y)_\alpha,$

$\overline{OR_{\sum_{i=1}^{m}A_i}}(X\cup Y)_\beta\supseteq \overline{OR_{\sum_{i=1}^{m}A_i}}(X)_\beta\cup \overline{OR_{\sum_{i=1}^{m}A_i}}(Y)_\beta;$

(2) $X\subseteq Y\Rightarrow \underline{OR_{\sum_{i=1}^{m}A_i}}(X)_\alpha\subseteq \underline{OR_{\sum_{i=1}^{m}A_i}}(Y)_\alpha,$

$X\subseteq Y\Rightarrow \overline{OR_{\sum_{i=1}^{m}A_i}}(X)_\beta\subseteq \overline{OR_{\sum_{i=1}^{m}A_i}}(Y)_\beta;$

(3) $\underline{OR_{\sum_{i=1}^{m}A_i}}(X\cup Y)_\alpha\supseteq \underline{OR_{\sum_{i=1}^{m}A_i}}(X)_\alpha\cup \underline{OR_{\sum_{i=1}^{m}A_i}}(Y)_\alpha,$

$\overline{OR_{\sum_{i=1}^{m}A_i}}(X\cap Y)_\beta\subseteq \overline{OR_{\sum_{i=1}^{m}A_i}}(X)_\beta\cap \overline{OR_{\sum_{i=1}^{m}A_i}}(Y)_\beta.$

Proof. It is easy to prove by Proposition 3.2.

IV. CONCLUSIONS

The contribution of this paper is having constructed fuzzy rough set on tolerance relations associated with granular computing called optimistic multi-granulation fuzzy rough set models on tolerance relations, in which the set approximation operators are defined on the basis of multiple tolerance relations. More properties of the fuzzy rough set on tolerance relations are discussed.

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