Optimistic Multi-Granulation Fuzzy Rough Set Model Based on Triangular Norm

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Abstract. With granular computing point of view, the generalized T-fuzzy rough set model is based on a single fuzzy granulation in a T-fuzzy approximation space. This paper is devoted to the construction and study of the multi-granulation rough set based on triangular norm by defining the optimistic multi-granulation T-fuzzy lower and upper approximation operators in the generalized T-fuzzy approximation space. It is obvious that the generalize T-fuzzy lower and upper approximation operators defined on (U, R) are obtained as a special case of these operators. The main properties of the T-fuzzy lower and upper approximation operators are also studied.

Keywords: Rough set, Multi-granulation, T-fuzzy similarity relation, Triangular norm, Residual implication.

1 Introduction

The theory of rough sets, proposed by Pawlak [2], is a powerful mathematical approach to deal with inexact, uncertain or vague knowledge. And the fuzzy set theory also offers a wide variety of techniques for analyzing imprecise data. It seems quite natural to extend the Pawlak rough set by combining methods developed within both theories to construct hybrid structures. Such structures, called fuzzy rough sets and rough fuzzy sets, have been proposed in the literature[3,4].

On the other hand, the majority of studies on rough sets have been concerned on the point view of granular computing. Zadeh firstly proposed the concept of granular computing and discussed issues of fuzzy information granulation in 1979 [5]. In the point view of granular computing, the classical Pawlak rough set is based on a single granulation which can be regarded as an equivalence relation on the universe induced from an indiscernibility relation. However, when the rough set is based on many granulations induced from several relations, we can have some cases as follow:

Case 1. There exists a granulation at least such that the elements surely belong to the concept.

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Case 2. There are some granulations such that the elements surely belong to the concept.

Case 3. All of the granulations such that the elements surely belong to the concept.

Case 4. There exists a granulation at least such that the elements possibly belong to the concept.

Case 5. There are some granulations such that the elements possibly belong to the concept.

Case 6. All of the granulations such that the elements possibly belong to the concept.

For the need of some practical issues, Qian and Xu extended the Pawlak rough set to multi-granulation rough set models where the approximation operators are defined by multiple equivalence relations on the universe [6,7,8]. On the basic, many researchers have been extended the multi-granulation rough set to the generalized multi-granulation rough sets[1,9,10].

Moreover, more generalizations of fuzzy rough sets were defined by using a residual implication and a triangular norm on $[0, 1]$ to define the lower and upper approximation operators. Several authors also have proposed a kind of implication[11], weak fuzzy partitions on the universe. In this paper, we intent to generalize the multi-granulation rough sets theory by using the concepts of a residual implication and a triangular norm on $[0, 1]$. In the following section, we recall some concepts and lemmas to be used in this paper. In Section 3, we proposed the definitions for the optimistic multi-granulation T-fuzzy lower and upper approximation operators and basic properties are studied. Section 4 concludes this article.

2 The Rough Set Based on Triangular Norm

Let U be a nonempty and finite set. The Cartesian product of U with U is denoted by $U \times U$. The classes of all crisp (fuzzy, respectively) subsets of U denoted by $P(U)$ ($F(U)$, respectively). Following[11], a binary operator T on the unit interval $I = [0, 1]$ is said to be a triangular norm, if $\forall a, b, c, d \in I$, we have

$(1) T(a, b) = T(b, a)$, $\quad (2) T(a, 1) = a$,

$(3) a \leq c, b \leq d \Rightarrow T(a, b) \leq T(c, d)$, $\quad (4) T(T(a, b), c) = T(a, T(b, c))$.

A fuzzy relation R from U to U is a fuzzy subset of $U \times U$, i.e., $R \in F(U \times U)$, and $R(x, y)$ is called the degree of relation between x and y. R is said to be reflexive on U, iff $\forall x \in U$, $R(x, x) = 1$; R is said to be symmetric on U, iff $\forall x \in U$, $R(x, y) = R(y, x)$; R is said to be T transitive on U, iff $\forall x, y, z \in U$, $R(x, x) \geq T(R(x, y), R(y, z))$. If R is reflexive, symmetric and T transitive on U, we then say that R is a T-fuzzy similarity relation on U.

Now, we define the following binary operator on I:

$$\theta(a, b) = \sup\{c \in I | T(a, c) \leq b\},$$

θ is called the residual implication based on a triangular norm T.

Lemma 2.1. Let T is a lower semi-continuous triangular norm, $\forall a, b, c \in I$ then the residual implication based on a triangular norm T satisfies the following properties.

$(\theta 1)\theta(a,1) = 1, \quad \theta(1,a) = a$ $\qquad (\theta 2) a \le b \Rightarrow \theta(c,a) \le \theta(c,b)$
$(\theta 3) a \le b \Rightarrow \theta(a,c) \ge \theta(b,c)$ $\qquad (\theta 4) T(\theta(a,c), \theta(c,b)) \le \theta(a,b)$
$(\theta 5)\theta(a \vee b, c) = \theta(a,c) \wedge \theta(b,c)$ $\qquad (\theta 6)\theta(a, b \wedge c) = \theta(a,b) \wedge \theta(a,c)$
$(\theta 7) a \le b \Leftrightarrow \theta(a,b) = 1$ $\qquad (\theta 8)\theta(a, \theta(b,c)) = \theta(b, \theta(a,c))$
$(\theta 9)\theta(T(a,b),c) = \theta(a, \theta(b,c))$ $\qquad (\theta 10) T(\theta(T(a,b),c),a) \le \theta(b,c)$
$(\theta 11) \bigwedge_{a \in I} \theta(T(b, \theta(c,a)), a) = \theta(b,c)$ $\qquad (\theta 12)\theta(\theta(a,b),b) \ge a$
$(\theta 13) \bigwedge_{b \in I} \theta(\theta(a,b),b) = a$ $\qquad (\theta 14) T(\theta(a,b),c) \le \theta(a, T(b,c))$
$(\theta 15) \bigwedge_{b \in I} \theta(\theta(a,b), \theta(c,b)) = \theta(c,a)$ $\qquad (\theta 16)\theta(a,b) \le \theta(T(a,c), T(b,c))$
$(\theta 17)\theta(a, b \vee c) = \theta(a,b) \vee \theta(a,c)$ $\qquad (\theta 18) a \le \theta(b, T(a,b))$
$(\theta 19)\theta(a \wedge b, c) = \theta(a,c) \vee \theta(b,c)$ $\qquad (\theta 20)\theta(a \wedge b, c) \ge \theta(a,c) \wedge \theta(b,c)$

Definition 2.1. Let U be a finite and nonempty sets called the universe, and R be a T-fuzzy similarity relation from U to U. The pair (U, R) is called a generalized T-fuzzy approximation space. For any $A \in F(U)$, we define two fuzzy set-theoretic operators from $F(U)$ to $F(U)$:

$$\underline{R}(A)(x) = \bigwedge_{y \in U} \theta(R(x,y), A(y)), \quad \overline{R}(A)(x) = \bigvee_{y \in U} T(R(x,y), A(y)), \quad x \in U.$$

Where $\underline{R}$ and $\overline{R}$ are referred to as the generalized T-fuzzy lower and upper approximation operators. The pair $(\underline{R}(A), \overline{R}(A))$ is called the generalized T-fuzzy rough set of A.

The following proposition reflects the relationships between $\underline{R}$ and $\overline{R}$.

Proposition 2.1. Let (U, R) be a fuzzy approximation space, for $\forall A, B \in F(U), (x,y) \in U \times U$, then

(1) $\underline{R}(A) \subseteq A \subseteq \overline{R}(A)$.
(2) $\underline{R}(A \cap B) = \underline{R}(A) \cap \underline{R}(B), \quad \overline{R}((A) \cup B) = \overline{R}(A) \cup \overline{R}(B)$.
(3) $\underline{R}(A \cup B) \supseteq \underline{R}(A) \cup \underline{R}(B), \quad \underline{R}(A \cap B) \subseteq \underline{R}(A) \cap \overline{R}(B)$.
(4) $A \subseteq B \Rightarrow \underline{R}(A) \subseteq \underline{R}(B), \overline{R}(A) \subseteq \overline{R}(B)$.
(5) $\underline{R}(\underline{R}(A)) = \underline{R}(A), \overline{R}(\overline{R}(A)) = \overline{R}(A)$.
(6) $\overline{R}(\underline{R}(A)) = \underline{R}(A), \underline{R}(\overline{R}(A)) = \overline{R}(A)$.
(7) $\overline{R}(A) = A \Leftrightarrow \overline{R}(A) = A$.

3 Optimistic Multi-Granulation Fuzzy Rough Set Based on Triangular Norm

In this section, we will study the optimistic multi-granulation fuzzy rough set based on triangular norm which is on the rough approximation problem in a generalized T-fuzzy approximation space.

Assumed that $(U, R_{A_1}, R_{A_2}, \cdots, R_{A_n})$ is a generalized T-fuzzy approximation space, if U is a finite and nonempty universe, and $\forall R_{A_i}$ is a T-fuzzy similarity relation from U to U.

In the following, we will give the definition of the optimistic multi-granulation rough set based on triangular norm.

Definition 3.1. Let $(U, R_{A_1}, R_{A_2}, \cdots, R_{A_n})$ be a generalized T-fuzzy approximation space. For any $X \in F(U)$, we can define the optimistic multi-granulation T-fuzzy lower and upper approximation of X as follows

$$\underline{OM_{\sum_{i=1}^{n} A_i}}(X)(x) = \bigvee_{i=1}^{n}(\bigwedge_{u \in U} \theta(R_{A_i}(u, x), X(u))),$$

$$\overline{OM_{\sum_{i=1}^{n} A_i}}(X)(x) = \bigwedge_{i=1}^{n}(\bigvee_{u \in U} T(R_{A_i}(u, x), X(u))),$$

where "$\bigvee$" means "max", "$\bigwedge$" means "min", θ and T are defined in Section 2. $\underline{OM_{\sum_{i=1}^{n} A_i}}$ and $\overline{OM_{\sum_{i=1}^{n} A_i}}$ are referred to as the generalized optimistic multi-granulation T-fuzzy lower and T upper approximation operators. The pair $(\underline{OM_{\sum_{i=1}^{n} A_i}}(X), \overline{OM_{\sum_{i=1}^{n} A_i}}(X))$ is called the generalized optimistic multi-granulation T-fuzzy rough set of X.

In the following, we employ an example to illustrate the above concepts.

Example 3.1. Let (U, R_A, R_B) be a generalized T-fuzzy approximation space, where $U = \{x_1, x_2, x_3, x_4, x_5\}$,

$$R_A = \begin{pmatrix} 1 & 0.4 & 0.8 & 0.5 & 0.5 \\ 0.4 & 1 & 0.4 & 0.4 & 0.4 \\ 0.8 & 0.4 & 1 & 0.5 & 0.5 \\ 0.5 & 0.4 & 0.5 & 1 & 0.6 \\ 0.5 & 0.4 & 0.5 & 0.6 & 1 \end{pmatrix}, \quad R_B = \begin{pmatrix} 1 & 0.8 & 0.8 & 0.2 & 0.8 \\ 0.8 & 1 & 0.85 & 0.2 & 0.85 \\ 0.8 & 0.85 & 1 & 0.2 & 0.9 \\ 0.2 & 0.2 & 0.2 & 1 & 0.2 \\ 0.8 & 0.85 & 0.9 & 0.2 & 1 \end{pmatrix}.$$

Taking $T(x, y) = \min(x, y)$, $X = (0.5, 0.3, 0.3, 0.6, 0.5)$.

It is not difficult to verify the fuzzy relation R_A and R_B are both T-fuzzy similar relations. So we can obtain the generalized optimistic multi-granulation T-fuzzy lower and upper approximation of X as follow

$$\underline{OM_{A+B}}(X) = (0.3, 0.3, 0.3, 0.6, 0.3), \quad \overline{OM_{A+B}}(X) = (0.5, 0.4, 0.5, 0.6, 0.5).$$

From the definitions of the optimistic multi-granulation T-fuzzy lower and upper approximation operators, it is possible to deduce the following properties by use of mathematical induction.

Proposition 3.1. Let $(U, R_{A_1}, R_{A_2}, \cdots, R_{A_n})$ be a generalized T-fuzzy approximation space, R_{A_i}, $i \in \{1, 2, 3..., n\}$ be the different T-fuzzy similarity relations, $\forall X, Y \in F(U)$. Then the optimistic multi-granulation T-fuzzy lower approximation operator has the following properties.

(1) $\underline{OM}_{\sum\limits_{i=1}^{n} A_i}(X) \subseteq X$.

(2) $\underline{OM}_{\sum\limits_{i=1}^{n} A_i}(\underline{OM}_{\sum\limits_{i=1}^{n} A_i}(X)) = \underline{OM}_{\sum\limits_{i=1}^{n} A_i}(X)$.

(3) $\underline{OM}_{\sum\limits_{i=1}^{n} A_i}(X \cap Y) \subseteq \underline{OM}_{\sum\limits_{i=1}^{n} A_i}(X) \cap \underline{OM}_{\sum\limits_{i=1}^{n} A_i}(Y)$.

(4) $X \subseteq Y \Rightarrow \underline{OM}_{\sum\limits_{i=1}^{n} A_i}(X) \subseteq \underline{OM}_{\sum\limits_{i=1}^{n} A_i}(Y)$.

(5) $\underline{OM}_{\sum\limits_{i=1}^{n} A_i}(X \cup Y) \supseteq \underline{OM}_{\sum\limits_{i=1}^{n} A_i}(X) \cup \underline{OM}_{\sum\limits_{i=1}^{n} A_i}(Y)$.

Proof. Since the number of the granulations is finite, we only prove the results are true in a generalized T-fuzzy approximation space (U, R_A, R_B) for convenience. It is obvious that all terms hold when $R_A = R_B$. When $R_A \neq R_B$, the proposition can be proved as follows.

(1) For any $x \in U$, we have

$$\begin{aligned}
\underline{OM_{A+B}}(X)(x) &= \bigwedge_{u \in U} \theta(R_A(u,x), X(u)) \vee \bigwedge_{u \in U} \theta(R_B(u,x), X(u)) \\
&\leq \theta(R_A(x,x), X(x)) \vee \theta(R_B(x,x), X(x)) \\
&= \theta(1, X(x)) \vee \theta(1, X(x)) \\
&= X(x)
\end{aligned}$$

(2) For any $x \in U$, we can obtain

$$\begin{aligned}
&\underline{OM_{A+B}}(\underline{OM_{A+B}}(X))(x) \\
&= \bigwedge_{u \in U} \theta(R_A(u,x), \underline{OM_{A+B}}(X)(u)) \vee \bigwedge_{u \in U} \theta(R_B(u,x), \underline{OM_{A+B}}(X)(u)) \\
&\geq \bigwedge_{u,v \in U} \theta(R_A(u,x), \theta(R_A(v,u), X(v)) \vee \bigwedge_{u,v \in U} \theta(R_B(u,x), \theta(R_B(v,u), X(v)) \\
&\geq \bigwedge_{u,v \in U} \theta(T(R_A(u,x), R_A(v,u)), X(v)) \vee \bigwedge_{u,v \in U} \theta(T(R_B(u,x), R_B(v,u)), X(v)) \\
&\geq \bigwedge_{v \in U} \theta(R_A(v,x), X(v)) \vee \bigwedge_{v \in U} \theta(R_B(v,x), X(v)) \\
&= \underline{OM_{A+B}}(X)(x)
\end{aligned}$$

So, $\underline{OM_{A+B}}(\underline{OM_{A+B}}(X)) \supseteq \underline{OM_{A+B}}(X)$

On the other hand, we can obtain $\underline{OM_{A+B}}(\underline{OM_{A+B}}(X)) \subseteq \underline{OM_{A+B}}(X)$ by (1). Therefore, (2) have been proved.

(3) For any $x \in U$, we have

$$\begin{aligned}
&\underline{OM_{A+B}}(X \cap Y)(x)\\
&= \bigwedge_{u\in U} \theta(R_A(u,x),(X\cap Y)(u)) \vee \bigwedge_{u\in U} \theta(R_B(u,x),(X\cap Y)(u))\\
&\leq [(\bigwedge_{u\in U} \theta(R_A(u,x),X(u))) \vee (\bigwedge_{u\in U} \theta(R_B(u,x),X(u)))] \wedge\\
&\quad [(\bigwedge_{u\in U} \theta(R_A(u,x),Y(u))) \vee (\bigwedge_{u\in U} \theta(R_B(u,x),Y(u)))]\\
&= \underline{OM_{A+B}}(X)(x) \wedge \underline{OM_{A+B}}(Y)(x)\\
&= \underline{OM_{A+B}}(X \cap Y)(x)
\end{aligned}$$

i.e., $\underline{OM_{A+B}}(X \cap Y) \subseteq \underline{OM_{A+B}}(X) \cap \underline{OM_{A+B}}(Y)$.

(4) Since $X \subseteq Y$, then for any $x \in U$, we have $X(x) \leq Y(x)$. Therefore,

$$\begin{aligned}
\underline{OM_{A+B}}(X)(x) &= \bigwedge_{u\in U} \theta(R_A(u,x),X(u)) \vee \bigwedge_{u\in U} \theta(R_B(u,x),X(u))\\
&\leq \bigwedge_{u\in U} \theta(R_A(u,x),Y(u)) \vee \bigwedge_{u\in U} \theta(R_B(u,x),Y(u))\\
&= \underline{OM_{A+B}}(Y)(x)
\end{aligned}$$

(5) According to the proposition (4), this item can be proved easily.

Proposition 3.2. Let $(U, R_{A_1}, R_{A_2}, \cdots, R_{A_n})$ be a generalized T-fuzzy approximation space, R_{A_i}, $i \in \{1,2,3...,n\}$ be the different T-fuzzy similarity relations. For $\forall X, Y \in F(U)$, the optimistic multi-granulation T-fuzzy upper approximation operator has the following properties.

(1) $X \subseteq \overline{OM_{\sum_{i=1}^n A_i}}(X)$.

(2) $\overline{OM_{\sum_{i=1}^n A_i}}(\overline{OM_{\sum_{i=1}^n A_i}}(X)) = \overline{OM_{\sum_{i=1}^n A_i}}(X)$.

(3) $\overline{OM_{\sum_{i=1}^n A_i}}(X \cup Y) \supseteq \overline{OM_{\sum_{i=1}^n A_i}}(X) \cup \overline{OM_{\sum_{i=1}^n A_i}}(Y)$.

(4) $X \subseteq Y \Rightarrow \overline{OM_{\sum_{i=1}^n A_i}}(X) \subseteq \overline{OM_{\sum_{i=1}^n A_i}}(Y)$.

(5) $\overline{OM_{\sum_{i=1}^n A_i}}(X \cap Y) \subseteq \overline{OM_{\sum_{i=1}^n A_i}}(X) \cap \overline{OM_{\sum_{i=1}^n A_i}}(Y)$.

Proof. Since the number of the granulations is finite, we only prove the results are true in a generalized T-fuzzy approximation space (U, R_A, R_B) for convenience. When $R_A \neq R_B$, the proposition can be proved as follows.

(1) For any $x \in U$,

$$\begin{aligned}
\overline{OM_{A+B}}(X)(x) &= \bigvee_{u\in U} T(R_A(u,x),X(u)) \wedge \bigvee_{u\in U} T(R_B(u,x),X(u))\\
&\geq T(R_A(x,x),X(x)) \wedge T(R_B(x,x),X(x))\\
&= X(x)
\end{aligned}$$

(2) For any $x \in U$,

$$\begin{aligned}
&\overline{OM_{A+B}}(\overline{OM_{A+B}}(X))(x)\\
&= \bigvee_{u\in U} T(R_A(u,x), \overline{OM_{A+B}}(X)(u)) \wedge \bigvee_{u\in U} T(R_B(u,x), \overline{OM_{A+B}}(X)(u))\\
&\leq \bigvee_{u,v\in U} T(R_A(u,x), T(R_A(v,u), X(v))) \wedge \bigvee_{u,v\in U} T(R_B(u,x), T(R_B(v,u), X(v)))\\
&= \bigvee_{u,v\in U} T(T(R_A(u,x), R_A(v,u)), X(v)) \wedge \bigvee_{u,v\in U} T(T(R_B(u,x), R_B(v,u)), X(v))\\
&\leq \bigvee_{v\in U} T(R_A(v,x), X(v)) \wedge \bigvee_{v\in U} T(R_B(v,x), X(v)) = \overline{OM_{A+B}}(X)(x)
\end{aligned}$$

Moreover, we have know $\overline{OM_{A+B}}(X) \subseteq \overline{OM_{A+B}}(\overline{OM_{A+B}}(X))$ by the proposition (1).

(3)For any $x \in U$,

$$\begin{aligned}
&\overline{OM_{A+B}}(X \cup Y)(x)\\
&= \bigvee_{u\in U} T(R_A(u,x), X(u) \vee Y(u)) \wedge \bigvee_{u\in U} T(R_B(u,x), X(u) \vee Y(u))\\
&=[\bigvee_{u\in U} T(R_A(u,x), X(u)) \vee \bigvee_{u\in U} T(R_A(u,x), Y(u))] \wedge\\
&\quad [\bigvee_{u\in U} T(R_B(u,x), X(u)) \vee \bigvee_{u\in U} T(R_B(u,x), Y(u))]\\
&\geq [\bigvee_{u\in U} T(R_A(u,x), X(u)) \wedge \bigvee_{u\in U} T(R_B(u,x), X(u))] \vee\\
&\quad [\bigvee_{u\in U} T(R_A(u,x), Y(u)) \wedge \bigvee_{u\in U} T(R_B(u,x), Y(u))]\\
&= \overline{OM_{A+B}}(X \cup Y)(x)
\end{aligned}$$

(4) Since $X \subseteq Y$, then for any $x \in U$, we can have $X(x) \leq Y(x)$. So

$$\begin{aligned}
\overline{OM_{A+B}}(X)(x) &= \bigvee_{u\in U} T(R_A(u,x), X(u)) \wedge \bigvee_{u\in U} T(R_B(u,x), X(u))\\
&\leq \bigvee_{u\in U} T(R_A(u,x), Y(u)) \wedge \bigvee_{u\in U} T(R_B(u,x), Y(u))\\
&= \overline{OM_{A+B}}(Y)(x)
\end{aligned}$$

(5) This item can be proved by (4).

By the definitions of the optimistic multi-granulation T-fuzzy lower and upper approximation operators based on triangular norm, for $\forall X \in F(U)$, the relationships of the optimistic multi-granulation T-fuzzy lower approximation and upper approximation operators as follows:

$$\underline{OM_{\sum_{i=1}^{n} A_i}}(X) \subseteq X \subseteq \overline{OM_{\sum_{i=1}^{n} A_i}}(X).$$

4 Conclusions

In this paper, the generalized T-fuzzy rough set model based on triangular norm has been significantly extended. In this extension, the approximations of sets were defined by using multiply T-fuzzy similarity relations on the universe. It is obvious that the generalize T-fuzzy lower and upper approximation operators defined on (U, R) were obtained as a special case of these operators. More properties of the optimistic multi-granulation T-fuzzy rough set based on triangular norm were discussed. And we investigated the relationships between the approximation operators. The construction of the optimistic multi-granulation fuzzy rough set model over T-fuzzy similarity relations on the universe is meaningful in terms of the generalization of rough set theory.

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References

1. Khan, M. A., Ma, M.: A Modal Logic for Multiple-Source Tolerance Approximation Spaces. In: Banerjee, M., Seth, A. (eds.) Logic and Its Applications. LNCS (LNAI), vol. 6521, pp. 124–136. Springer, Heidelberg (2011)
2. Pawlak, Z.: Rough sets. International Journal of Computer and Information Sciences 11(5), 341–356 (1982)
3. Dubois, D., Prade, H.: Rough fuzzy sets and fuzzy rough sets. International Journal of General Systems 17, 191–208 (1990)
4. Dubois, D., Prade, H.: Putting rough sets and fuzzy sets together (1992)
5. Zadeh, L.A.: Fuzzy Sets and Information Granularity, Advances in fuzzy set theory and application. North Holland Publishing, Amstandam (1979)
6. Qian, Y.H., Liang, J.Y., Dang, C.Y.: Knowledge structure, knowledge granulation and knowledge distance in a knowledge base. International Journal of Approximate Reasoning 50, 174–188 (2009)
7. Qian, Y.H., Liang, J.Y., Yao, Y.Y., Dang, C.H.: MGRS: A multi-granulation rough set. Information Sciences 180, 949–970 (2010)
8. Xu, W.H., Zhang, X.Y., Zhang, W.X.: Two new types of multiple granulation rough set. Information Sciences (submitted)
9. Xu, W.H., Zhang, X.T., Wang, Q.R.: A Generalized Multi-granulation Rough Set Approach. In: Huang, D.-S., Gan, Y., Premaratne, P., Han, K. (eds.) ICIC 2011. LNCS, vol. 6840, pp. 681–689. Springer, Heidelberg (2012)
10. Xu, W.H., Wang, Q.R., Zhang, X.T.: Multi-granulation Fuzzy Rough Set Model on Tolerance Relations. In: Fourth International Workshop on Advanced Computational Intelligence, pp. 359-366 (2011)
11. Zhang, W.X., Wu, W.Z., Liang, J.Y., Li, D.Y.: Theory and method of rough sets (2001)