# A Generalized Multi-granulation Rough Set Approach

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**Abstract.** A generalized multi-granulation rough set is proposed in this paper. In the new model, supporting characteristic function is defined and a parameter called information level is introduced to investigate that an object supports a concept precisely under majority granulations. Moreover, some important properties are discussed on the new multi-granulation rough set. And it can be found that the proposed model is more valid than old multiple granulation rough set models and Pawlak rough set model.

**Keywords:** Information level, Lower and upper approximation sets, Multi-granulation rough set, Supporting characteristic function, Majority granulations.

## 1 Introduction

The rough set theory proposed by Z. Pawlak ([2]) in 1980's is a useful soft computing tool for reasoning from data and a new mathematical approach to handle imprecision, vagueness and uncertainty in data analysis. As this theory has been applied to various fields such as medicine, engineering, management, economy, finance and security, many generalized rough set models are developed and studied ([1, 3, 7–12]).

Let $I = (U, A, V, f)$ be an information system. $B \subseteq A$ is an attribute subset. The equivalence relation corresponding to the attribute subset $B$ is still denoted by itself. For an arbitrary set $X \subseteq U$, it may be impossible to describe $X$ precisely using the equivalence classes $[x]_B = \{y \in U | f(x, a) = f(y, a), \forall a \in A\}$, that is, $X$ can't be equal to the combination of some equivalence classes. In this case, one can depict the concept $X$ by a pair of sets so called lower and upper approximation sets which are precise with respect to $B$. And the pair of sets can be defined as

$$\underline{B}(X) = \{x \in U | [x]_B \subseteq X\},\ \overline{B}(X) = \{x \in U | [x]_B \cap X \neq \varnothing\}.$$

$X$ is fine if and only if $\underline{B}(X) = \overline{B}(X)$, otherwise $X$ is rough if and only if $\underline{B}(X) \neq \overline{B}(X)$. The sets $\underline{B}(X)$ and $\overline{B}(X)$ are called, respectively, the lower approximation set and upper approximation set of $X$([1–3, 7, 12]).

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## 2 Multi-granulation Rough Set

Multi-granulation rough set model (MGRS) was studied as an expanding of Pawlak rough set model in references [4–6, 10]. An equivalence class of an object with respect to an attribute subset is a granularity in the view of granular computing. And a partition of the universe is a granular space. Then the classical rough set model is a single granulation rough set model (SGRS) and the granular space in this model is induced by the indiscernibility relation of attribute set. In cases referred in referrence [6](Case1, Case2 and Case3), there are limitations in SGRS for dealing with practical problems, and MGRS now can be used to solve these problems.

In MGRS, unlike SGRS, a concept is approximated through multiple partitions of $U$ induced by multiple equivalence relations. And we have a brief introduction of MGRS in this section. As we have studied multi-granulation rough set further, we now in this section illustrate the two forms of multi-granulation rough set in our submitted paper [10].

**Definition 2.1.**([6, 10]) Let $I = (U, A, V, f)$ be an information system, $X \subseteq U$ and $P = \{P_i \subseteq A | P_i \cap P_j = \varnothing (i \neq j), i, j \leq l\}$. The lower and upper approximation sets of $X$ with respect to $P$ can be defined by following.

$$\underline{OM}(X) = \{x \in U | \vee ([x]_{P_i} \subseteq X), i \leq l\},$$
$$\overline{OM}(X) = \{x \in U | \wedge ([x]_{P_i} \cap X \neq \varnothing), i \leq l\}.$$

where "$\vee$" means the logical operator "OR" and "$\wedge$" means the logical operator "AND".

$X$ is definable if and only if $\underline{OM}(X) = \overline{OM}(X)$; otherwise $X$ is rough if and only if $\underline{OM}(X) \neq \overline{OM}(X)$. This model can be called the optimistic multi-granulation rough set model, denoted by OMGRS. And $\underline{OM}(X)$ and $\overline{OM}(X)$ are called, respectively, optimistic lower and upper approximation sets.

From the above definition, the operators " $\vee$ " and " $\wedge$ " can be exchanged between the lower approximation set and the upper approximation set. Corresponding to OMGRS, the pessimistic multi-granulation rough set model, denoted by PMGRS, can be defined in the following .

**Definition2.2.**([10]) Let $I = (U, A, V, f)$ be an information system, $X \subseteq U$ and $P = \{P_i \subseteq A | P_i \cap P_j = \varnothing (i \neq j), i, j \leq l\}$. The pessimistic lower and upper approximation sets of $X$ with respect to $P$ can be defined as follows.

$$\underline{PM}(X) = \{x \in U | \wedge ([x]_{P_i} \subseteq X), i \leq l\},$$
$$\overline{PM}(X) = \{x \in U | \vee ([x]_{P_i} \cap X \neq \varnothing), i \leq l\}.$$

$X$ is definable if and only if $\underline{PM}(X) = \overline{PM}(X)$, otherwise $X$ is rough if and only if $\underline{PM}(X) \neq \overline{PM}(X)$. $\underline{PM}(X)$ and $\overline{PM}(X)$ are called,respectively, pessimistic lower and upper approximation sets.

As generalizations of Pawlak rough set model, we merely show the relations of OMGRS, PMGRS and SGRS in the next proposition. Other descriptions on multi-granulation rough set can be reviewed in references [4–6, 10].

**Proposition 2.1.**([6, 10]) Let $I = (U, A, V, f)$ be an information system, $X \subseteq U$ and $P = \{P_i \subseteq A | P_i \cap P_j = \varnothing (i \neq j), i, j \leq l\}$. The following propositions hold.

(1) $\underline{OM}(X) = \bigcup_{i=1}^{l} \underline{P_i}(X)$;

(2) $\overline{OM}(X) = \bigcap_{i=1}^{l} \overline{P_i}(X)$;

(3) $\underline{PM}(X) = \bigcap_{i=1}^{l} \underline{P_i}(X)$;

(4) $\overline{PM}(X) = \bigcup_{i=1}^{l} \overline{P_i}(X)$;

(5) $\underline{PM}(X) \subseteq \underline{OM}(X)$;

(6) $\overline{OM}(X) \subseteq \overline{PM}(X)$.

## 3 A Generalized Multi-Granulation Rough Set

The two forms of multi-granulation rough set model illustrated in Section 2 are only special ones . From these two forms, we propose a more generalized and logical one in this section.

In order to present the generalized multi-granulation rough set model, we define a function called supporting characteristic function firstly.

**Definition 3.1.** Let $I = (U, A, V, f)$ be an information system, $X \subseteq U$ and $P = \{P_i \subseteq A | P_i \cap P_j = \varnothing (i \neq j), i, j \leq l\}$. Characteristic function $S_X^{P_i}(x)$, describing the inclusion relation between the class $[x]_{P_i}$ and the concept $X$, is defined as follows.

$$S_X^{P_i}(x) = \begin{cases} 1, & [x]_{P_i} \subseteq X \\ 0, & else \end{cases} \quad (i \leq l).$$

We call $S_X^{P_i}(x)$ supporting characteristic function of $x \in U$. It shows the object $x$ supports the concept $X$ precisely or not with respect to $P_i$.

**Proposition 3.1.** For any $x \in U$ and $P_i \in P$, the following properties of $S_X^{P_i}(x)$ hold.

(1) $S_{\sim X}^{P_i}(x) = \begin{cases} 1, & [x]_{P_i} \cap X = \varnothing \\ 0, & [x]_{P_i} \cap X \neq \varnothing \end{cases}$;

(2) $S_{\varnothing}^{P_i}(x) = 0,\ S_U^{P_i}(x) = 1$;

(3) $S_{X \cup Y}^{P_i}(x) \geq S_X^{P_i}(x) \vee S_Y^{P_i}(x)$;

(4) $S_{X \cap Y}^{P_i}(x) = S_X^{P_i}(x) \wedge S_Y^{P_i}(x)$;

(5) $X \subseteq Y \Rightarrow S_X^{P_i}(x) \leq S_Y^{P_i}(x)$;

(6) $X \subseteq Y \Rightarrow S_{\sim X}^{P_i}(x) \geq S_{\sim Y}^{P_i}(x)$.

where, "$\wedge$" and "$\vee$" are, respectively, operations "minimum" and "maximum" in this proposition.

*Proof.* (1) Since $[x]_{P_i} \subseteq \sim X \Leftrightarrow [x]_{P_i} \cap X = \varnothing$ and $[x]_{P_i} \nsubseteq \sim X \Leftrightarrow [x]_{P_i} \cap X \neq \varnothing$. So this proposition is obvious.

(2) According to Definition 3.1, one can have that
$\forall x \in U \Rightarrow [x]_{P_i} \nsubseteq \varnothing$, i.e., $S_{\varnothing}^{P_i}(x) = 0$; $\forall x \in U \Rightarrow [x]_{P_i} \subseteq U$, i.e., $S_U^{P_i}(x) = 1$.
This item is proved.

(3) As is known, "$Z \subseteq X$ or $Z \subseteq Y \Rightarrow (\nLeftarrow) Z \subseteq X \cup Y$" holds for any set $Z$. Thus, we have

$$\begin{aligned} S_X^{P_i}(x) \vee S_Y^{P_i}(x) = 1 \Leftrightarrow S_X^{P_i}(x) = 1 \text{ or } S_Y^{P_i}(x) = 1 \Leftrightarrow [x]_{P_i} \subseteq X \text{ or } [x]_{P_i} \subseteq Y \\ \Rightarrow (\nLeftarrow)[x]_{P_i} \subseteq X \cup Y \Leftrightarrow S_{X \cup Y}^{P_i}(x) = 1. \end{aligned}$$

If $X \cup Y = U$, then $S_{X\cup Y}^{P_i}(x) = S_U^{P_i}(x) = 1$ is obvious. If $X \cup Y \neq U$, then $\sim (X \cup Y) = \sim X \cap \sim Y \neq \varnothing$. So, we have that

$$\begin{aligned} S_{X\cup Y}^{P_i}(x) = 0 &\Leftrightarrow [x]_{P_i} \cap \sim (X \cup Y) \neq \varnothing \Leftrightarrow [x]_{P_i} \cap \sim X \cap \sim Y \neq \varnothing \\ &\Rightarrow (\nLeftarrow)[x]_{P_i} \cap \sim X \neq \varnothing \text{ and } [x]_{P_i} \cap \sim Y \neq \varnothing \\ &\Leftrightarrow S_X^{P_i}(x) = 0 \text{ and } S_Y^{P_i}(x) = 0 \Leftrightarrow S_X^{P_i}(x) \vee S_Y^{P_i}(x) = 0. \end{aligned}$$

That is to say that $S_{X \bigcup Y}^{P_i}(x) \geq S_X^{P_i}(x) \vee S_Y^{P_i}(x)$ holds for any $x \in U$.

(4) Since "$Z \subseteq X$ and $Z \subseteq Y \Leftrightarrow Z \subseteq X \cap Y$" holds for any set $Z$, we can obviously have

$$\begin{aligned} S_{X\cap Y}^{P_i}(x) = 0 &\Leftrightarrow [x]_{P_i} \cap \sim (X \cap Y) \neq \varnothing \Leftrightarrow [x]_{P_i} \cap (\sim X \cup \sim Y) \neq \varnothing \\ &\Leftrightarrow ([x]_{P_i} \cap \sim X) \cup ([x]_{P_i} \cap \sim Y) \neq \varnothing \\ &\Leftrightarrow [x]_{P_i} \cap \sim X \neq \varnothing \text{ or } [x]_{P_i} \cap \sim Y \neq \varnothing \\ &\Leftrightarrow S_X^{P_i}(x) = 0 \text{ or } S_Y^{P_i}(x) = 0 \Leftrightarrow S_X^{P_i}(x) \wedge S_Y^{P_i}(x) = 0; \end{aligned}$$

and

$$\begin{aligned} S_{X\cap Y}^{P_i}(x) = 1 &\Leftrightarrow [x]_{P_i} \subseteq X \cap Y \Leftrightarrow [x]_{P_i} \subseteq X \text{ and } [x]_{P_i} \subseteq Y \\ &\Leftrightarrow S_X^{P_i}(x) = 1 \text{ and } S_Y^{P_i}(x) = 1 \Leftrightarrow S_X^{P_i}(x) \wedge S_Y^{P_i}(x) = 1. \end{aligned}$$

Then, $S_{X\cap Y}^{P_i}(x) = S_X^{P_i}(x) \wedge S_Y^{P_i}(x)$ holds for any $x \in U$. This item is proved.

(5) The case is obvious if $[x]_{P_i} \nsubseteq X$ by Definition 3.1 and $S_X^{P_i}(x) = 0 \leq S_Y^{P_i}(x)$. If $[x]_{P_i} \subseteq X$, one can have that $[x]_{P_i} \subseteq X \subseteq Y$, i.e., $S_X^{P_i}(x) = 1 = S_Y^{P_i}(x)$. Then this item is proved.

(6) From (1), this item can be proved similarly as (5).

For any $x \in U$ and $X \subseteq U$, the number of equivalence classes $[x]_{P_i}$ satisfying $[x]_{P_i} \subseteq X$ can be represented as $\sum\limits_{i=1}^{l} S_X^{P_i}(x)$ by supporting characteristic function and the number of equivalence classes $[x]_{P_i}$ satisfying $[x]_{P_i} \cap X \neq \varnothing$ can be represented as $\sum\limits_{i=1}^{l} (1 - S_{\sim X}^{P_i}(x))$. Moreover, we have the following proposition.

**Proposition 3.2.** By supporting characteristic function, the lower and upper approximation sets in OMGRS and PMGRS can be represented, respectively, in the following form.

(1) $\underline{OM}(X) = \{x \in U | \frac{\sum_{i=1}^{l} S_X^{P_i}(x)}{l} > 0\};$

$\overline{OM}(X) = \{x \in U | \frac{\sum_{i=1}^{l} (1 - S_{\sim X}^{P_i}(x))}{l} \geq 1\}.$

(2) $\underline{PM}(X) = \{x \in U | \frac{\sum_{i=1}^{l} S_X^{P_i}(x)}{l} \geq 1\};$

$\overline{PM}(X) = \{x \in U | \frac{\sum_{i=1}^{l} (1 - S_{\sim X}^{P_i}(x))}{l} > 0\}.$

*Proof.* It can be proved easily from Definition 2.1, 2.2 and 3.1.

In the view of granular computing, models in the above proposition may be not always effective in practice. OMGRS may be so loose that the approximation sets can't describe concepts as precisely as possible. And PMGRS may be too strict to depict concepts on universe.

As a generalization of OMGRS and PMGRS, we will propose a new multi-granulation rough set model with a parameter $\beta \in (0.5, 1]$. We introduce this parameter to realize that the objects supporting a concept in majority granulations are included and the ones possibly describing the concept below the corresponding level are ignored. This model is presented in the definition below.

**Definition 3.2.** Let $I = (U, A, V, f)$ be an information system, $X \subseteq U$ and $P = \{P_i \subseteq A | P_i \cap P_j = \varnothing (i \neq j), i, j \leq l\}$. $S_X^{P_i}(x)$ is supporting characteristic function of $x$. For any $\beta \in (0.5, 1]$, the lower and upper approximation sets of $X$ with respect to $P$ are defined as follows.

$$\underline{P}(X)_\beta = \{x \in U | \frac{\sum_{i=1}^{l} S_X^{P_i}(x)}{l} \geq \beta\},$$

$$\overline{P}(X)_\beta = \{x \in U | \frac{\sum_{i=1}^{l} (1 - S_{\sim X}^{P_i}(x))}{l} > 1 - \beta\}.$$

$X$ is called definable if and only if $\underline{P}(X)_\beta = \overline{P}(X)_\beta$, otherwise $X$ is rough if and only if $\underline{P}(X)_\beta \neq \overline{P}(X)_\beta$. We denote this generalized multi-granulation rough set model by GMGRS and call $\beta$ the information level with respect to $P$.

GMGRS is a generalization of OMGRS and PMGRS. The approximations in these models can reflect this. In the following proposition, we present the relations between GMGRS and OMGRS (PMGRS).

**Proposition 3.3.** Let $I = (U, A, V, F)$ be an information system, $X \subseteq U$, $\beta \in (0.5, 1]$ and $P = \{P_i \subseteq A | P_i \cap P_j = \varnothing (i \neq j), i, j \leq l\}$. The lower and upper approximations in GMGRS have the following relation with those in OMGRS and PMGRS.

(1) $\underline{PM}(X) \subseteq \underline{P}(X)_\beta \subseteq \underline{OM}(X)$;
(2) $\overline{OM}(X) \subseteq \overline{P}(X)_\beta \subseteq \overline{PM}(X)$.

This proposition can be proved easily by Definition3.2 and Proposition3.2. Details will not be illustrated on these two properties.

**Remark 1.** The relation of inclusion between $\underline{P}(X)_\beta$ and an arbitrary $\underline{P_i}(X)$ is uncertain. And so does it between $\overline{P}(X)_\beta$ and an arbitrary $\overline{P_i}(X)$.

Propositions of approximations in rough set theory are useful and important in theoretical research and practice. Thus, we will investigate some important properties as Pawlak rough set in the following.

**Proposition 3.4.** Let $I = (U, A, V, F)$ be an information system, $X \subseteq U$ and $P = \{P_i \subseteq A | P_i \cap P_j = \varnothing (i \neq j), i, j \leq l\}$. For any $\beta \in (0.5, 1]$, we have that

(1a) $\underline{P}(\sim X)_\beta = \sim \overline{P}(X)_\beta$;
(1b) $\overline{P}(\sim X)_\beta = \sim \underline{P}(X)_\beta$;
(2a) $\underline{P}(X)_\beta \subseteq X$;
(2b) $X \subseteq \overline{P}(X)_\beta$;
(3a) $\underline{P}(\varnothing)_\beta = \overline{P}(\varnothing)_\beta = \varnothing$;
(3b) $\underline{P}(U)_\beta = \overline{P}(U)_\beta = U$;
(4a) $X \subseteq Y \Rightarrow \underline{P}(X)_\beta \subseteq \underline{P}(Y)_\beta$;
(4b) $X \subseteq Y \Rightarrow \overline{P}(X)_\beta \subseteq \overline{P}(Y)_\beta$;
(5a) $\underline{P}(X \cap Y)_\beta \subseteq \underline{P}(X)_\beta \cap \underline{P}(Y)_\beta$;
(5b) $\overline{P}(X \cup Y)_\beta \supseteq \overline{P}(X)_\beta \cup \overline{P}(Y)_\beta$;
(6a) $\underline{P}(X \cup Y)_\beta \supseteq \underline{P}(X)_\beta \cup \underline{P}(Y)_\beta$;
(6b) $\overline{P}(X \cap Y)_\beta \subseteq \overline{P}(X)_\beta \cap \overline{P}(Y)_\beta$.
where, "$\sim$" is means the complementary operation of cantor sets.

*Proof.* (1a) Since $x \in \overline{P}(X)_\beta \Leftrightarrow \frac{\sum_{i=1}^{l} (1 - S_{\sim X}^{P_i}(x))}{l} > 1 - \beta$. Then,

$$x \in \sim \overline{P}(X)_\beta \Leftrightarrow \frac{\sum_{i=1}^{l} (1 - S_{\sim X}^{P_i}(x))}{l} \leq 1 - \beta \Leftrightarrow \frac{\sum_{i=1}^{l} S_{\sim X}^{P_i}(x)}{l} \geq \beta \Leftrightarrow x \in \underline{P}(\sim X)_\beta.$$

This item is proved. Item (1b) can be proved similarly as (1a).

(2a) For any $x \in \underline{P}(X)_\beta$, we have that $\frac{\sum_{i=1}^{l} S_X^{P_i}(x)}{l} \geq \beta > 0$. Then, there must exist $i \leq l$ such that $[x]_{P_i} \subseteq X$. Thus $x \in X$. $\underline{P}(X)_\beta \subseteq X$ is proved.

(2b) By $\sim \overline{P}(X)_\beta = \underline{P}(\sim X)_\beta \subseteq \sim X$, we can have $X \subseteq \overline{P}(X)_\beta$ directly.

(3a)(3b) From Proposition 3.1 $S_\varnothing^{P_i}(x) = 0$ and $S_U^{P_i}(x) = 1$ $(\forall x \in U)$, we have that

$$\underline{P}(\varnothing)_\beta = \{x \in U | \frac{\sum\limits_{i=1}^{l} S_{\varnothing}^{P_i}(x)}{l} = \frac{\sum\limits_{i=1}^{l} 0}{l} = 0 \geq \beta\} = \varnothing,$$

$$\underline{P}(U)_\beta = \{x \in U | \frac{\sum\limits_{i=1}^{l} S_{U}^{P_i}(x)}{l} = \frac{\sum\limits_{i=1}^{l} 1}{l} = 1 \geq \beta\} = U.$$

From the duality (1a)(1b), we can easily have $\overline{P}(\varnothing)_\beta = \varnothing$ and $\overline{P}(U)_\beta = U$.

(4a) For any $x \in \underline{P}(X)_\beta$, we have $\frac{\sum_{i=1}^{l} S_X^{P_i}(x)}{l} \geq \beta$. Since $X \subseteq Y$, one can have $S_X^{P_i}(x) \leq S_Y^{P_i}(x)$. Then,

$$\frac{\sum\limits_{i=1}^{l} S_Y^{P_i}(x)}{l} \geq \frac{\sum\limits_{i=1}^{l} S_X^{P_i}(x)}{l} \geq \beta.$$

So $x \in \underline{P}(Y)_\beta$ is obtained. Thus, this item is proved and item (4b) can be proved similarly.

(5a) From the propositions of $S_X^{P_i}(x)$, for any $x \in \underline{P}(X \cap Y)_\beta$, we have that

$$\begin{aligned} x \in \underline{P}(X \cap Y)_\beta \Leftrightarrow \frac{\sum\limits_{i=1}^{l} S_{X\cap Y}^{P_i}(x)}{l} &= \frac{\sum\limits_{i=1}^{l} S_X^{P_i}(x) \wedge \sum\limits_{i=1}^{l} S_Y^{P_i}(x)}{l} \geq \beta \\ &\Leftrightarrow \frac{\sum\limits_{i=1}^{l} S_X^{P_i}(x)}{l} \geq \beta \text{ and } \frac{\sum\limits_{i=1}^{l} S_Y^{P_i}(x)}{l} \geq \beta \\ &\Leftrightarrow x \in \underline{P}(X)_\beta \text{ and } x \in \underline{P}(Y)_\beta \\ &\Leftrightarrow x \in \underline{P}(X)_\beta \cap \underline{P}(Y)_\beta. \end{aligned}$$

(5b) From the duality property, this item can be proved easily by (5a).

(6a)(6b) can be proved directly by properties (4a) and (4b).

**Remark 2.** Propositions (4),(5) in Proposition 3.4 are not the same as SGRS, OMGRS and PMGRS. And the properties $\underline{P}(\underline{P}(X)_\beta)_\beta = \underline{P}(X)_\beta = \overline{P}(\underline{P}(X)_\beta)_\beta$ and $\overline{P}(\overline{P}(X)_\beta)_\beta = \overline{P}(X)_\beta = \underline{P}(\overline{P}(X)_\beta)_\beta$ don't hold in GMGRS.

For different information levels, one can consider the difference between approximations and the following properties hold.

**Proposition 3.5.** Let $I = (U, A, V, F)$ be an information system, $X \subseteq U$ and $P = \{P_i \subseteq A | P_i \cap P_j = \varnothing (i \neq j), i, j \leq l\}$. For any $\alpha \leq \beta$ and $\alpha, \beta \in (0.5, 1]$, the following propositions hold.

(1) $\underline{P}(X)_\beta \subseteq \underline{P}(X)_\alpha$,

(2) $\overline{P}(X)_\alpha \subseteq \overline{P}(X)_\beta$.

*Proof.* It can be proved easily by Definition 3.1 and Proposition 3.4.

In this section, we proposed a generalized multi-granulation rough set model and studied some important propositions. From these propositions, we can easily have that GMGRS is a more generalized and logical multi-granulation rough set model than ones refereed in Section 2. By the information level $\beta \in (0.5, 1]$, GMGRS has the ability to discover more affirmative information than PMGRS and leave out some useless possible knowledge in information systems. Furthermore, GMGRS popularize OMGRS and discover information more precise than OMGRS. Propositions studied in this section have important effect in practice and make it convenient to solve problems using the new model we propose.

## 4 Conclusions

We proposed a generalized multi-granulation rough set model denoted by GMGRS in this paper. And some important propositions were discussed in detail. GMGRS is a generalization of OMGRS and PMGRS. More useful information and descriptions can be employed in GMGRS to represent the knowledge precisely for supportting the concept in majority granulations. Correspondingly, many useless information and descriptions can be thrown off since they have so less effect on possible knowledge representation that they can be ignored in sense of multi-granulation.

From the paper, one can find that GMGRS is more valid than PMGRS and OMGRS in the view of multi-granulation. This model is a complement of multi-granulation rough set theory and may make great effect in practice.

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