The First Type of Graded Rough Set Based on Rough Membership Function

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***Abstract*—The extension of classical rough set model is a very hot and interesting topic. In this paper, our aim is to present the first type of graded rough set (FGRS) based on rough membership function. The concepts of $k-$regions, $k-$rough degree, etc., are proposed firstly, and some of important properties are investigated in this rough set model. Moreover, the model has the corresponding properties with classical rough set model. In addition, relative $k-$reduction is considered and an example is used to illustrate its validity. By comparing, one can find that the model is a generalization of variable precision rough set model.**

***Index Terms*—General binary relation; Graded rough set; Precision coefficient; Rough membership function; Variable precision rough set**

I. Introduction

The rough set theory, which was proposed by Pawlak[1], is an extension of the classical set theory and can be regarded as a soft computing tool to deal with the uncertainty or imprecision information. It was well known that this theory is based upon the classification mechanism, from which the classification can be viewed as an equivalence relation and knowledge granule induced by the equivalence relation be a partition of universe. Its main idea is to induce the decision making of problem or rules of classification by knowledge reduction. On account of the special thought and novelty methods contained in this theory, the research has recently roused great interest in the theoretical and application fronts, such as machine learning, pattern recognition, data analysis, and so on.

In Pawlak's original rough set theory, partition or equivalence (indiscernibility) relation is an important and primitive concept. But, partition or equivalence relation is still restrictive for many applications. One of these limitations is that the classification dealt with must be completely correct or determinately, in other words, it only consider completely "containing" or "belonging to". Another is that the object set is known and the results obtained can only be used to some special objects. To address this issue, several interesting and meaningful extensions to equivalence relation have been proposed in the past, such as tolerance relations[2], similarity relations[3], and so on[4]. Particularly, W. Zikrao [5] established the variable precision rough set model (VPRS) with a majority inclusion threshold β. The main idea of VPRS is that it admits the existence of some wrong classification ratio to such an extent that it is capable to get some useful information from irrelevant information. Up to now, various theories and methods have been proposed to deal with the graded or precision of inclusion and some useful works have been done in [6-13].

In order to apply this theory to reflect the uncertainty measurement of knowledge, in this paper, the first type of graded rough set (FGRS) based on rough membership function in the sense of general binary relation is proposed by considering one model of the definition of rough approximation. Furthermore, some characterization and properties of this rough set model are discussed carefully, which is very useful for future research works such as uncertainty measurement of knowledge, extracting rules, and so on.

II. Preliminaries

The following recalls necessary concepts required in our work. Detailed description of the theory can be found in[14].

Definition 2.1 A database system is a triple $K = (U, A, F)$, where $U = \{u_1, u_2, \cdots, u_n\}$ is set of objects called universe, $A = \{a_1, a_2, \cdots, a_m\}$ is set of attributes, $F = \{f_l \mid f_l : U \to V_l (l \leq m)\}$ is relation set of U and A, and $V_l = \{f_l(u) \mid u \in U\}$ is the domain of $a_l \in A$.

A database system $K = (U, A, F)$ can be called a database system with decision or decision system if A can be divided into set C of condition attributes and set D of decision attributes such that $A = C \bigcup D$ and $C \bigcap D = \emptyset$.

Any subset R of $U \times U$ is called a relation on U, and for any $(x, y) \in U \times U$, if $(x, y) \in R$, then we say x has relation R with y, and denote this relationship as xRy. Hence, for a database system $K = (U, A, F)$, $R_B = \{(x, y) \in U \times U \mid f_l(x) * f_l(y),\ \forall\ a_l \in B\}$ represents a binary relation with respect to B on U, where " $*$ " be " $\leq$ ", " $\geq$ " or " $=$ ". $[u_i]_B = \{u_j \mid (u_i, u_j) \in R_B\}$ can be called relationship class of x and $U/R_B = \{[u_i]_B \mid \forall u_i \in U\}$ be one classification or knowledge on U.

Definition 2.2 Let $K = (U, A, F)$ be a database system, $X \subseteq U$ and $B \subseteq A$. The two sets

$$\begin{aligned} \underline{R_B}(X) &= \{u \in U \mid [u]_B \subseteq X\} \\ \overline{R_B}(X) &= \{u \in U \mid [u]_B \bigcap X \neq \emptyset\} \end{aligned} \tag{1}$$

are called the lower and the upper approximation of X, respectively. Moreover, if $\underline{R_B}(X) = \overline{R_B}(X)$, then X is exact set, otherwise, X is rough set.

Definition 2.3 Let $K = (U, A, F)$ be a database system, $X \subseteq U$ and $B \subseteq A$. Rough degree of X with respect to B, denoted by $\rho_B(X)$, is defined as

$$\rho_B(X) = 1 - \frac{|\underline{R_B}(X)|}{|\overline{R_B}(X)|},$$

where $\rho_B(X) \in [0, 1]$.

Definition 2.4 Let $K = (U, A, F)$ be a database system, $X \subseteq U$ and $B \subseteq A$. The positive region, negative region and boundary region of X with respect to B, denoted by $pos_B(X)$, $neg_B(X)$ and $bn_B(X)$ respectively, are defined as

$$pos_B(X) = \underline{R_B}(X),$$
$$neg_B(X) = U - \overline{R_B}(X),$$
$$bn_B(X) = \overline{R_B}(X) - \underline{R_B}(X).$$

Definition 2.5 Let $K = (U, A, F)$ be a database system, $X \subseteq U$ and $B, C \subseteq A$. The positive and negative region of R_C with respect to R_B, denoted by $pos_B(R_C)$ and $neg_B(R_C)$ respectively, are defined as

$$pos_B(R_C) = \bigcup_{X \in U/R_C} pos_B(X),$$
$$neg_B(R_C) = \bigcup_{X \in U/R_C} neg_B(X).$$

An attribute $a_l \in B$ is dispensable in B if $U/R_B = U/R_{B-\{a_l\}}$, otherwise a_l is indispensable in B. The collection of all indispensable attributes in A is called core of A and denoted by $core(A)$. We say that $B \subseteq A$ is independent if every attribute in B is indispensable, otherwise B is dependent. The subset $B \subseteq A$ is called a reduction in A if B is independent and $U/R_B = U/R_A$, which is denoted by $red(B)$.

III. The First Type of Graded Rough Set Based on Rough Membership Function

In this section, we start to investigate the first type of graded rough set based on rough membership function in database systems based on general binary relation. At first, the concept of rough membership function is proposed and some important properties are discussed.

A. Rough membership function

Definition 3.1[15] Let $K = (U, A, F)$ be a database system based on binary relation, $X \subseteq U$ and $B \subseteq A$. For every $u \in U$, the rough membership function of u with respect to R_B is defined as

$$\mu_X^B(u) = \frac{|[u]_B \bigcap X|}{|[u]_B|}.$$

As can be seen from above definition, we have that the properties of rough membership function listed as follows without any proof are trivial.

Proposition 3.1 Let $K = (U, A, F)$ be a database system based on binary relation and $B \subseteq A$. If $X \subseteq Y$, then $\mu_X^B(u) \leq \mu_Y^B(u)$ for every $u \in U$.

Proposition 3.2 Let $K = (U, A, F)$ be a database system based on binary relation and $B \subseteq A$. $\mu_X^B(u) + \mu_{\sim X}^B(u) = 1$ holds for any $X \subseteq U$.

Proposition 3.3 Let $K = (U, A, F)$ be a database system based on binary relation and $B \subseteq A$. For any $X \subseteq U$, we have that

(1) $\mu_X^B(u) = 1$ if and only if $u \in pos_B(X)$.
(2) $\mu_X^B(u) = 0$ if and only if $u \in neg_B(X)$.
(3) $0 < \mu_X^B(u) < 1$ if and only if $u \in bn_B(X)$.

From above proposition, we see that $\mu_X^B(u)$ represents a characteristic function of $u \in U$, and from which the fuzzy set on U, denoted by $F_X^B = \{(u, \mu_X^B(u)) \mid u \in U\}$, can be constructed.

Proposition 3.4 Let $K = (U, A, F)$ be a database system based on binary relation and $B \subseteq A$. For any $X, Y \subseteq U$, we have that

(1) $F_{X \bigcup Y}^B \supseteq F_X^B \bigcup F_Y^B$.
(2) $F_{X \bigcup Y}^B = F_X^B \bigcup F_Y^B$ if either $X \subseteq Y$ or $Y \subseteq X$.

Proposition 3.5 Let $K = (U, A, F)$ be a database system based on binary relation and $B \subseteq A$. For any $X, Y \subseteq U$, we have that

(1) $F_{X \bigcap Y}^B \subseteq F_X^B \bigcap F_Y^B$.
(2) $F_{X \bigcap Y}^B = F_X^B \bigcap F_Y^B$ if either $X \subseteq Y$ or $Y \subseteq X$.

B. The first type of graded rough set

In Pawlak's rough set theory, the rough approximation may be redefined by another form, listed as follows, besides the form in definition 2.1.

$$\begin{aligned} \underline{R_B}(X) &= \bigcup\{[u]_B \mid [u]_B \subseteq X\} \\ \overline{R_B}(X) &= \bigcup\{[u]_B \mid [u]_B \bigcap X \neq \emptyset\} \end{aligned} \tag{2}$$

In fact, these two forms are equal in the sense of equivalence relation but (1) and (2) are not equal to each other if the relationship is not an equivalence relation, and in which case a respective research of them is worthwhile. Thus, our work in this paper is to study an extended rough set model based on the form of equation (1) in the sense of precision coefficient $k(\in (0.5, 1])$, and be called first type of graded rough set, i.e. FGRS. Note that for no ambiguous, the rough approximations without consideration of precision coefficient have the form as defined in definition 2.1.

Definition 3.2 Let $K = (U, A, F)$ be a database system based on binary relation, $X \subseteq U$ and $B \subseteq A$. The $k-$lower and $k-$upper approximation of X based on rough membership function then may be defined as

$$\begin{aligned} \underline{R_B^k}(X) &= \{u \in U \mid \mu_X^B(u) \geq k\} \\ \overline{R_B^k}(X) &= \{u \in U \mid \mu_X^B(u) > 1 - k\} \end{aligned} \tag{3}$$

Moreover, X is $k-$exact if and only if $\underline{R_B^k}(X) = \overline{R_B^k}(X)$, otherwise, X is $k-$rough.

By comparing with classical rough set model, one can obtain the following property.

Theorem 3.1 Let $K = (U, A, F)$ be a database system based on binary relation, $X \subseteq U$ and $B \subseteq A$, then the following properties are trivial.

(L_1) $\underline{\underline{R_B^k}}(\emptyset) = \emptyset$,
(U_1) $\overline{\overline{R_B^k}}(\emptyset) = \emptyset$;
(L_2) $\underline{\underline{R_B^k}}(U) = U$,
(U_2) $\overline{\overline{R_B^k}}(U) = U$;
(L_3) $\underline{\underline{R_B^k}}(X) =\sim \overline{\overline{R_B^k}}(\sim X)$,
(U_3) $\overline{\overline{R_B^k}}(X) =\sim \underline{\underline{R_B^k}}(\sim X)$;
(L_4) $\underline{\underline{R_B^k}}(X \bigcup Y) \supseteq \underline{\underline{R_B^k}}(X) \bigcup \underline{\underline{R_B^k}}(Y)$,
(U_4) $\overline{\overline{R_B^k}}(X \bigcap Y) \subseteq \overline{\overline{R_B^k}}(X) \bigcap \overline{\overline{R_B^k}}(Y)$;
(L_5) $\underline{\underline{R_B^k}}(X \bigcap Y) \subseteq \underline{\underline{R_B^k}}(X) \bigcap \underline{\underline{R_B^k}}(Y)$,
(U_5) $\overline{\overline{R_B^k}}(X \bigcup Y) \supseteq \overline{\overline{R_B^k}}(X) \bigcup \overline{\overline{R_B^k}}(Y)$;
(L_6) $l \leq k \Longrightarrow \underline{\underline{R_B^k}}(X) \subseteq \underline{\underline{R_B^l}}(X)$,
(U_6) $l \leq k \Longrightarrow \overline{\overline{R_B^l}}(X) \subseteq \overline{\overline{R_B^k}}(X)$;
(L_7) $X \subseteq Y \Longrightarrow \underline{\underline{R_B^k}}(X) \subseteq \underline{\underline{R_B^k}}(Y)$,
(U_7) $X \subseteq Y \Longrightarrow \overline{\overline{R_B^k}}(X) \subseteq \overline{\overline{R_B^k}}(Y)$;
(8) $\underline{\underline{R_B^k}}(X) \subseteq \overline{\overline{R_B^k}}(X)$.

Proof: It is straightforward. ■

In fact, one can find that equation (3) is equal to equation (1) when precision coefficient $k = 1$.

Definition 3.3 Let $K = (U, A, F)$ be a database system based on binary relation and $k \in (0.5, 1]$. For any $X \subseteq U$ and $B \subseteq A$, $k-$positive region, $k-$negative region and $k-$boundary region of X with respect to R_B, denoted by $pos_B^k(X)$, $neg_B^k(X)$ and $bn_B^k(X)$ respectively, are defined as

$$pos_B^k(X) = \{u \in U \mid \mu_X^B(u) \geq k\},$$
$$neg_B^k(X) = \{u \in U \mid \mu_X^B(u) \leq 1 - k\},$$
$$bn_B^k(X) = \{u \in U \mid 1 - k < \mu_X^B(u) < k\}.$$

Theorem 3.2 Let $K = (U, A, F)$ be a database system based on binary relation and $k \in (0.5, 1]$. For any $X \subseteq U$ and $B \subseteq A$, we have that X is $k-$exact if and only if $bn_B^k(X) = \emptyset$, otherwise X is $k-$rough.

Proof: It can be proved by definitions 3.2 and 3.3. ■

Theorem 3.3 Let $K = (U, A, F)$ be a database system based on binary relation and $k \in (0.5, 1]$. For any $X \subseteq U$ and $B \subseteq A$, we have that

(1) If $k_1 < k$ and X is $k-$exact, then X is k_1-exact.
(2) If $k_2 > k$ and X is $k-$rough, then X is k_2-rough.

Proof: It can be proved easily by definition 3.2 and theorem 3.2. ■

If X is $k-$rough for any $k \in (0.5, 1]$, then we call X is absolute $k-$rough, otherwise X is relative $k-$rough. In particular, for every relative $k-$rough set X, there must exist $k_0 \in (0.5, 1]$ such that X is k_0-exact.

From above, the following results hold.

Theorem 3.4 Let $K = (U, A, F)$ be a database system based on binary relation and $k \in (0.5, 1]$. For any $X \subseteq U$ and $B \subseteq A$, we have that

(1) $pos_B^k(X) = neg_B^k(\sim X)$,
(2) $bn_B^k(X) = bn_B^k(\sim X)$,
(3) If $bn_B^k(X) = \emptyset$, then $pos_B^k(X) \bigcup neg_B^k(X) = U$,

where $\sim X = U - X$.

Proof: It can be proved by definitions 3.1 and 3.3. ■

Theorem 3.5 Let $K = (U, A, F)$ be a database system based on binary relation and $k \in (0.5, 1]$. For any $X \subseteq U$ and $B \subseteq A$, we have that

(1a) $\underline{R_B}(X) \subseteq \underline{\underline{R_B^k}}(X)$,
(1b) $\overline{\overline{R_B^k}}(X) \subseteq \overline{R_B}(X)$,
(2a) $bn_B^k(X) \subseteq bn_B(X)$,
(2b) $neg_B(X) \subseteq neg_B^k(X)$.

Proof: Without loss of generality, we only prove (1a). For any $u \in \underline{R_B}(X)$, we have $[u]_B \subseteq X$. From definition 3.1 we have that $\mu_X^B(u) = 1 \geq k$. Hence $u \in \underline{\underline{R_B^k}}(X)$, that is $\underline{R_B}(X) \subseteq \underline{\underline{R_B^k}}(X)$. ■

Corollary 3.1 Let $K = (U, A, F)$ be a database system based on binary relation and $k = 1$. For any $X \subseteq U$ and $B \subseteq A$, we have that

(1a) $\underline{R_B}(X) = \underline{\underline{R_B^k}}(X)$,
(1b) $\overline{\overline{R_B^k}}(X) = \overline{R_B}(X)$,
(2a) $bn_B^k(X) = bn_B(X)$,
(2b) $neg_B(X) = neg_B^k(X)$.

From theorem 3.5 and corollary 3.1, one can find that $k-$positive region and $k-$negative region of X are decreasing with respect to k while $k-$boundary region of X is increasing with respect to k. In particular, when k approaches 0.5, i.e. $k \to 0.5$, the following property holds.

Definition 3.4 Let $K = (U, A, F)$ be a database system based on binary relation, $X \subseteq U$ and $B \subseteq A$. The absolute boundary, that is $k \to 0.5$, is denoted by

$$bn_B^{0.5}(X) = \{u \in U \mid \mu_X^B(u) = \frac{1}{2}\}.$$

From above definition we have that

Proposition 3.6 Let $K = (U, A, F)$ be a database system based on binary relation, $X \subseteq U$ and $B \subseteq A$. If $k \to 0.5$, we have that

(1a) $\underline{\underline{R_B^{0.5}}}(X) = \bigcup^{k} \underline{\underline{R_B^k}}(X)$,
(1b) $\overline{\overline{R_B^{0.5}}}(X) = \bigcap^{k} \overline{\overline{R_B^k}}(X)$,
(2a) $bn_B^{0.5}(X) = \bigcap^{k} bn_B^k(X)$,
(2b) $neg_B^{0.5}(X) = \bigcup^{k} neg_B^k(X)$.

Example 3.1 Consider one database system.

Table 1. Database system

U	a_1	a_2	a_3	a_4	a_5
u_1	1	2	1	1	1
u_2	3	2	2	4	3
u_3	1	1	2	2	1
u_4	2	1	3	1	2
u_5	3	3	2	3	4
u_6	3	2	3	4	4

From above table we can obtain a dominance relation R_B with respect to $B = \{a_1, a_2, a_3, a_4, a_5\}$ and $[u_i]_B = \{u_j \mid f_l(u_j) \geq f_l(u_i)\ (\forall a_l \in B)\}$.

If take $X = \{u_1, u_3, u_5, u_6\}$, $Y = \{u_2, u_4\}$ and $k = 0.6$, by computing we have that

$$\underline{R_B^{0.6}}(X) = \{u_1, u_3, u_5, u_6\},$$
$$\overline{R_B^{0.6}}(X) = U,$$
$$pos_B^{0.6}(X) = \{u_1, u_3, u_5, u_6\},$$
$$neg_B^{0.6}(X) = \emptyset,$$
$$bn_B^{0.6}(X) = \{u_2, u_4\};$$

and

$$\underline{R_B^{0.6}}(Y) = \emptyset,$$
$$\overline{R_B^{0.6}}(Y) = \{u_2, u_4\},$$
$$pos_B^{0.6}(Y) = \emptyset,$$
$$neg_B^{0.6}(Y) = \{u_1, u_3, u_5, u_6\},$$
$$bn_B^{0.6}(Y) = \{u_2, u_4\}.$$

Hence, we can obtain

$$pos_B^{0.6}(X) = neg_B^{0.6}(Y),\ bn_B^{0.6}(X) = bn_B^{0.6}(Y).$$

If let $k = 1$, by computing we have that
$\underline{R_B}(X) = \underline{R_B^1}(X)$, $\overline{R_B^1}(X) = \overline{R_B}(X)$,
$bn_B^1(X) = bn_B(X)$, $neg_B(X) = neg_B^1(X)$.

C. $k-$rough degree

It is known that the roughness of every set is induced by the existence of boundary. The larger the boundary region is, the rougher the set is, and vice versa. From the ideas in[16], the concept of $k-$rough degree is proposed as follows and some properties of it are discussed.

Definition 3.5 Let $K = (U, A, F)$ be a database system based on binary relation and $k \in (0.5, 1]$. For any $X \subseteq U$ and $B \subseteq A$, the $k-$rough degree, denoted by $\rho_B^k(X)$, is defined as

$$\rho_B^k(X) = 1 - \frac{|\underline{R_B^k}(X)|}{|\overline{R_B^k}(X)|}.$$

Obviously $k-$rough degree reflects the unknown degree for the set X.

Corollary 3.2 Let $K = (U, A, F)$ be a database system based on binary relation and $k \in (0.5, 1]$. For any $X \subseteq U$ and $B \subseteq A$, we have that

(1) $0 \leq \rho_B^k(X) \leq 1$,
(2) X is $k-$exact if and only if $\rho_B^k(X) = 0$,
(3) X is $k-$rough if and only if $\rho_B^k(X) > 0$.

Theorem 3.6 Let $K = (U, A, F)$ be a database system based on binary relation and $0.5 < l \leq k \leq 1$. $\rho_B^l(X) \leq \rho_B^k(X)$ holds for any $X \subseteq U$ and $B \subseteq A$.

Proof: It can be proved by definition 3.5 and item (L_6),(U_6) of theorem 3.1. ■

Theorem 3.7 Let $K = (U, A, F)$ be a database system based on binary relation and $k \in (0.5, 1]$. $\rho_B^k(X) \leq \rho_B(X)$ holds for any $X \subseteq U$ and $B \subseteq A$.

Proof: By theorem 3.1 we have that

$$|\underline{R_B}(X)| \leq |\underline{R_B^k}(X)| \text{ and } |\overline{R_B}(X)| \geq |\overline{R_B^k}(X)|.$$

Hence

$$\frac{|\underline{R_B}(X)|}{|\overline{R_B}(X)|} \leq \frac{|\underline{R_B^k}(X)|}{|\overline{R_B^k}(X)|}.$$

With definition 3.5, we have that $\rho_B^k(X) \leq \rho_B(X)$ holds. ■

Example 3.2(Continued from Example 3.1) Take $X = \{u_3, u_5, u_6\}$ and $B = \{a_1, a_2, a_3, a_4\}$. If $k = 0.68$, by computing we have that

$$\rho_B^{0.68}(X) = \frac{1}{2}.$$

If $k = 0.76$, by computing we have that $\rho_B^{0.76}(X) = \frac{2}{3}$. Obviously, one can find that $\rho_B^{0.68}(X) \leq \rho_B^{0.76}(X)$.

IV. RELATIVE $k-$REDUCTION

In this section, we introduce the concept of relative $k-$reduction of the first graded rough set.

Definition 4.1 Let $K = (U, A, F)$ be a database system based on binary relation and $k \in (0.5, 1]$. For any $X \subseteq U$ and $B, C \subseteq A$, the $k-$positive region of R_C with respect to R_B, denoted by $pos_B^k(R_C)$, is defined as

$$pos_B^k(R_C) = \bigcup_{X \in U/R_C} pos_B^k(X).$$

Definition 4.2 Let $K = (U, C \bigcup D, F)$ be a database system based on binary relation, $P \subseteq C$ and $Q \subseteq D$. relative $k-$dependent of P with respect to Q, denoted by $\gamma(P, Q, k)$, is defined by

$$\gamma(P, Q, k) = \frac{|pos_P^k(R_Q)|}{|U|}.$$

Definition 4.3 Let $K = (U, C \bigcup D, F)$ be a database system based on binary relation and $k \in (0.5, 1]$. $a_l \in P \subseteq C$ is dispensable in P with respect to $Q \subseteq D$ if $pos_P^k(R_Q) = pos_{P-\{a_l\}}^k(R_Q)$, otherwise a_l is indispensable. We say that $P \subseteq C$ is relative $k-$independent if every attribute in P is indispensable, otherwise P is relative $k-$dependent.

From above, one can find that relative $k-$dependent, which can be regarded as an evaluation for classification ability of objects with precision coefficient, extends the thought of rough dependent while it is not equal to rough dependent. Moreover, it can not be regarded as partial dependent because some of its properties are weaker than the properties of functional dependent such as the property of transitive. But we have that relative $k-$dependent is equal to rough dependent when precision coefficient $k = 1$.

Definition 4.4 Let $K = (U, C \bigcup D, F)$ be a database system based on binary relation and $k \in (0.5, 1]$. $P \subseteq C$ ia a relative $k-$reduction of C with respect to $Q \subseteq D$, denoted by $red_Q(P, k)$, if and only if

(1) P is relative $k-$independent with respect to $Q \subseteq D$,
(2) $\gamma(P, Q, k) = \gamma(C, Q, k)$.

The purpose of relative $k-$reduction of C is to find a minimal subset of C to keep every decision rule invariant.

Example 4.1 Consider one database system.

Table 2. Decision system

U	a_1	a_2	a_3	d
u_1	Y	Y	0	N
u_2	N	Y	1	Y
u_3	Y	Y	2	Y
u_4	N	Y	0	N
u_5	N	N	1	N
u_6	N	Y	2	Y
u_7	N	N	1	Y
u_8	N	Y	2	N

Assuming that the relationship is equivalence relation and precision coefficient $k = 0.75$, then by computing we have that

$$pos_C^k(R_{\{d\}}) = \{u_1, u_2, u_4\}$$

while

$$\begin{aligned} &pos_{\{a_1\}}^k(R_{\{d\}}) = \emptyset, \\ &pos_{\{a_2\}}^k(R_{\{d\}}) = \emptyset, \\ &pos_{\{a_3\}}^k(R_{\{d\}}) = \{u_1, u_4\}, \\ &pos_{\{a_1,a_2\}}^k(R_{\{d\}}) = \emptyset, \\ &pos_{\{a_1,a_3\}}^k(R_{\{d\}}) = \{u_1, u_3, u_4\}, \\ &pos_{\{a_2,a_3\}}^k(R_{\{d\}}) = \{u_1, u_2, u_4\}. \end{aligned}$$

Thus, $\{a_1, a_3\}$ and $\{a_2, a_3\}$ is the relative $k-$reduction of C, that is,

$$red_{\{d\}}(C, 0.75) = \{a_1, a_3\} \bigvee \{a_2, a_3\}.$$

Furthermore, one can obtain that relative core of C is $\{a_3\}$.

V. Conclusion

It is well-known that rough set theory has been regarded as a mathematical tool to deal with the uncertainty or imprecision information. But, it is still restrictive for many applications. Thus, inspired by Ziarko's model of variable precision rough set approach, the rough membership function based first type of graded rough set in database systems based on binary relation is proposed in this paper and some of its properties are discussed in detail. Compared with classical rough set, one can find that the graded rough set model defined in this paper is not only the generalization of graded rough set, but also the generalization of variable precision rough set model based on equivalence relation. In the future we will further study other types of graded rough sets, knowledge reduction and so on.

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