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Upper Approximation Reduction in Ordered Information System with Fuzzy Decision

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Abstract: Attributes reduction is one of the most important problems in rough set theory. In this paper, the concept of upper approximation reduction is constructed in ordered information systems with fuzzy decision. Moreover the judgment theorem and discernability matrix are obtained, in which case one can provide an approach to this reduction in ordered information systems with fuzzy decision. As an application of upper approximation reduction, examples are considered to illustrate the validity of some results obtained in our works.

Key words: Discernability matrix; Fuzzy decision; Upper approximation reduction; Ordered information systems; Rough set;

1. Introduction

Rough set theory, proposed by Z. Pawlak in the early 1980s[1], is an extension of classical set theory and can be regarded as a soft computing tool to deal with uncertainty or imprecision information. It was well known that this theory is based upon the classification mechanism, in which case the classification can be viewed as an equivalence relation and knowledge blocks induced by the equivalence relation be partition on universe. For this reason, it has been applied successfully and widely in pattern recognition[2], medical diagnosis[3] ,granular computing[4] and so on.

Attributes reduction, as one important portion of rough set researching, its main idea is not only to delete redundant attributes but also the classification ability of database to be not changed. But there exist many information systems in which the relation is not equivalence relation because of various reasons, such as noise or information losing. Hence, how to deal with this type of information systems has became a very hot topic in rough set theory. Many experts have researched attributes reduction by extending the equivalence relation to consistent relation, similar relation and dominance relation[5-7] and so on. Some useful works have been done in dominance relation based information systems [8-10].

At the same time, for decision makers, there may exist one case in that the decision objects are not certain or precision but fuzzy. So the fuzzy decision making must be take into consideration. Our work in this paper is to consider the attributes reduction in ordered information systems with fuzzy decision. Firstly, concept of upper approximation consistent set is proposed by comparing decision attribute values. What is more, upper approximation reduction and judgment theorem is introduced, from which one can define the discernability matrix and find that it is an useful approach to attributes reduction. As an application of upper approximation reduction, examples are considered to illustrate the validity of some results obtained in our works.

The rest of this paper is organized as follows. Some preliminary concepts required in our work are briefly recalled in section 2. In section 3 the upper approximation reduction in this information systems is investigated. The discernability matrix and reduction approach are defined in section 4. Section 5 concludes the paper.

2. Information System with fuzzy decision

In this section, we shall begin our work with some necessary concepts required in the sequel of our work. For more detailed description of the theory, please refer Refs.[11, 12].

Definition 2.1 An information system is a quaternion $I = (U, A \cup D, F, G)$, where

$U = \{u_1, u_2, \dots, u_n\}$ is set of objects;

$A = \{a_1, a_2, \dots, a_p\}$ is set of conditional attributes.

$D = \{d\}$ is set of decision attribute.

$F = \{f_k | U \to V_k, k \le p\}$ is relation set of U and A ,and $V_k = \{f_k(u_i) | u_i \in U\}$ is the domain of $a_k \in A$. $G = \{d | U \to V_d\}$ is the relation set of U and D , and $V_d = \{d(u_i) | u_i \in U\}$ is the domain of $d \in D$.

Information system with fuzzy decision is that the decision value of objects is expressed as fuzzy form, i.e. for any $u_i \in U$, $d(u_i) \in [0,1]$.

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Definition 2.2 Let $I=(U,A\cup D,F,G)$ be an information system and $B\subseteq A$, if denote

$$R_B^{\succeq}=\{(u_i,u_j)\in U\times U \mid f_k(u_i)\le f_k(u_j),\forall a_k\in B\},$$

then $R_B^{\succeq}$ means a dominance relation on U relate to B, in which case the information system is called ordered information system and denoted by $I^{\succeq}$. If an information system is based on dominance relation and the decision value of objects is fuzzy, then the information system is called ordered information system with fuzzy decision.

For convenience, the notation $I_f^{\succeq}$ is used to express ordered information systems with fuzzy decision, in which case the dominance class of any $u_i\in U$ is denoted as $[u_i]_B^{\succeq}=\{u_j|(u_i,u_j)\in R_B^{\succeq}\}$.

Proposition 2.1 Let $I_f^{\succeq}$ be an information system and $B\subseteq A$.

(1) $R_B^{\succeq}$ is reflexive, transitive, but not symmetric, so it is not an equivalence relation.

(2) If $B_1\subseteq B_2\subseteq A$, then $R_A^{\succeq}\subseteq R_{B_2}^{\succeq}\subseteq R_{B_1}^{\succeq}$.

(3) If $B_1\subseteq B_2\subseteq A$, then $[u_i]_A^{\succeq}\subseteq [u_i]_{B_2}^{\succeq}\subseteq [u_i]_{B_1}^{\succeq}$.

(4) If $u_j\in[u_i]_A^{\succeq}$, then $[u_j]_A^{\succeq}\subseteq[u_i]_A^{\succeq}$.

Definition 3.1 For an information system $I_f^{\succeq}$, the lower and upper approximation set of D relate to A is denoted by $\underline{A_D^{\succeq}}$ and $\overline{A_D^{\succeq}}$ respectively. And their membership function are defined by

$$\underline{A_D^{\succeq}}(u_i)=\min\{d(u_j)| u_j\in[u_i]_A^{\succeq}\},$$

$$\overline{A_D^{\succeq}}(u_i)=\max\{d(u_j)| u_j\in[u_i]_A^{\succeq}\}.$$

Example 2.1 Consider an ordered information system with fuzzy decision in Table 1.

Table 1.

U	a_1	a_2	a_3	d
u_1	1	2	1	0.5
u_2	3	2	2	0.3
u_3	1	1	2	0.7
u_4	2	1	3	0.9
u_5	3	3	2	0.1
u_6	3	2	3	0.6

By computing we have that

$d(u_1)=0.5$ $\quad d(u_2)=0.3$

$d(u_3)=0.7$ $\quad d(u_4)=0.9$

$d(u_5)=0.1$ $\quad d(u_6)=0.6$

$$[u_1]_A^{\succeq}=\{u_1,u_2,u_5,u_6\}$$
$$[u_2]_A^{\succeq}=\{u_2,u_5,u_6\}$$
$$[u_3]_A^{\succeq}=\{u_2,u_3,u_4,u_5,u_6\}$$
$$[u_4]_A^{\succeq}=\{u_4,u_6\}$$
$$[u_5]_A^{\succeq}=\{u_5\}$$
$$[u_6]_A^{\succeq}=\{u_6\}$$

So we have that

$$\underline{A_D^{\succeq}}=\frac{0.1}{u_1}+\frac{0.1}{u_2}+\frac{0.1}{u_3}+\frac{0.6}{u_4}+\frac{0.1}{u_5}+\frac{0.6}{u_6};$$

$$\overline{A_D^{\succeq}}=\frac{0.6}{u_1}+\frac{0.6}{u_2}+\frac{0.9}{u_3}+\frac{0.9}{u_4}+\frac{0.1}{u_5}+\frac{0.6}{u_6}.$$

3. Upper approximation reduction of ordered information system with fuzzy decision

Since dominance relation is not the equivalence relation, the approach to attributes reduction in original information systems is not fitted for ordered information system with fuzzy decision. Therefore, the upper approximation consistent set is constructed and upper approximation reduction is defined, from which the judgment theorem is introduced.

Definition 3.1 Let $I_f^{\succeq}$ be an information system and $B\subseteq A$. If $\overline{A_D^{\succeq}}(u_i)=\overline{B_D^{\succeq}}(u_i)$ for any $u_i\in U$, then B is called upper approximation consistent set of this information system. Moreover, if any proper subset of B is not the upper approximation set, then B is one upper approximation reduction of this information system.

Example 3.1 (Continued from Example 2.1) If take $B=\{a_2,a_3\}$, we will have that $[u_i]_A^{\succeq}=[u_i]_B^{\succeq}$ holds for $u_i\in U$. Hence $\overline{A_D^{\succeq}}(u_i)=\overline{B_D^{\succeq}}(u_i)$, that is to say $\{a_2,a_3\}$ is one upper approximation consistent set. Moreover, if take $C=\{a_1,a_2\}$, then we have

$$[u_1]_C^{\succeq}=\{u_1,u_2,u_5,u_6\}$$
$$[u_2]_C^{\succeq}=\{u_2,u_5,u_6\}$$
$$[u_3]_C^{\succeq}=\{u_1,u_2,u_3,u_4,u_5,u_6\}$$
$$[u_4]_C^{\succeq}=\{u_2,u_4,u_5,u_6\}$$
$$[u_5]_C^{\succeq}=\{u_5\}$$
$$[u_6]_C^{\succeq}=\{u_2,u_5,u_6\}$$

$$\overline{C_D^{\succeq}}=\frac{0.6}{u_1}+\frac{0.6}{u_2}+\frac{0.9}{u_3}+\frac{0.9}{u_4}+\frac{0.1}{u_5}+\frac{0.6}{u_6}=\overline{A_D^{\succeq}}.$$

Hence, $\{a_1,a_2\}$ is also one upper approximation consistent set.

Moreover, we have that $\{a_1\}$ and $\{a_3\}$ are not the upper approximation consistent sets by computing and $\{a_2\}$ is the upper approximation consistent sets. Hence we conclude that this information system has only one upper approximation reduction $\{a_2\}$.

In the next, the judgment theorem is proposed.

Theorem 3.1 Let $I_f^{\succeq}$ be an information system and $B \subseteq A$. Attribute set B is an upper approximation consistent set if and only if for every $u_i, u_j \in U$, if $\overline{A_D^{\succeq}}(u_i) < \overline{A_D^{\succeq}}(u_j)$, then there exist $a_k \in B$ such that $f_k(u_i) > f_k(u_j)$.

Proof. $" \Rightarrow "$: Suppose that $f_k(u_i) \le f_k(u_j)$ for every $a_k \in B$ when $\overline{A_D^{\succeq}}(u_i) < \overline{A_D^{\succeq}}(u_j)$. So we can obtain $u_j \in [u_i]_B^{\succeq}$, that is $[u_j]_B^{\succeq} \subseteq [u_i]_B^{\succeq}$. By

$$\overline{B_D^{\succeq}}(u_i) = \max\{d(u) | \; u \in [u_i]_B^{\succeq}\} \text{ and}$$

$$\overline{B_D^{\succeq}}(u_j) = \max\{d(u) | \; u \in [u_j]_B^{\succeq}\},$$

we can have $\overline{B_D^{\succeq}}(u_i) \ge \overline{B_D^{\succeq}}(u_j)$.

Since B is upper approximation consistent set, we have that $\overline{A_D^{\succeq}}(u_j) = \overline{B_D^{\succeq}}(u_j)$ and $\overline{A_D^{\succeq}}(u_i) = \overline{B_D^{\succeq}}(u_i)$. So we can obtain $\overline{A_D^{\succeq}}(u_i) \ge \overline{A_D^{\succeq}}(u_j)$. This is a contradiction.

$" \Leftarrow "$: Suppose B is not upper approximation consistent set, then there must exist one $u_{i_0} \in U$ such that $\overline{A_D^{\succeq}}(u_{i_0}) \ne \overline{B_D^{\succeq}}(u_{i_0})$. So we have that $\overline{A_D^{\succeq}}(u_{i_0}) < \overline{B_D^{\succeq}}(u_{i_0})$ according to Proposition 2.1.

Since $\overline{B_D^{\succeq}}(u_{i_0}) = \max\{d(u) | \; u \in [u_{i_0}]_B^{\succeq}\}$, we take $u_{j_0} \in [u_{i_0}]_B^{\succeq}$ such that $d(u_{j_0}) = \overline{B_D^{\succeq}}(u_{i_0}) = \max\{d(u) \; | \; u \in [u_{i_0}]_B^{\succeq}\}$. While we have known $u_{j_0} \in [u_{j_0}]_A^{\succeq}$, then we can have $\max\{d(u) | \; u \in [u_{j_0}]_A^{\succeq}\} \ge d(u_{j_0})$. That is to say $\overline{A_D^{\succeq}}(u_{j_0}) \ge d(u_{j_0})$.

We have $\overline{A_D^{\succeq}}(u_{j_0}) \ge d(u_{j_0}) = \overline{B_D^{\succeq}}(u_{i_0}) > \overline{A_D^{\succeq}}(u_{i_0})$. So there exist $a_k \in B$ such that $f_k(u_{i_0}) > f_k(u_{j_0})$. It is a contradiction with $u_{j_0} \notin [u_{i_0}]_B^{\succeq}$.

The theorem is proved. □

4. Approach to upper approximation reduction

In theorem 3.1, we proposed an equivalent description of upper approximation consistent set, in which case it can be used to estimate the attribute set, which is an upper approximation consistent set or not. In next, the concept of discernability matrix is introduced first and reduction approach is constructed immediately.

Definition 4.1 Let $I_f^{\succeq}$ be an information system and $D_f^{\succeq} = \{(u_i, u_j) | \overline{A_D^{\succeq}}(u_i) < \overline{A_D^{\succeq}}(u_j)\}$. If denote

$$D_f = \begin{cases} \{a_k \in A \mid f_k(u_i) > f_k(u_j)\} & (u_i, u_j) \in D_f^{\succeq} \\ \varnothing & (u_i, u_j) \notin D_f^{\succeq} \end{cases},$$

then D_f is called to the upper approximation discernability attribute set between u_i and u_j .

From which $M_f = \left(D_f(u_i, u_j)\right)_{u_i, u_j \in U}$ is the discernability matrix of this information system.

Theorem 4.1 Let $I_f^{\succeq}$ be an information system and $B \subseteq A$, B is upper approximation consistent set if and only if $B \cap D_f(u_i, u_j) \ne \varnothing$ for all $(u_i, u_j) \in D_f^{\succeq}$.

Proof. $" \Rightarrow "$: By theorem 3.1 we have that there exist $a_k \in B$ such that $f_k(u_i) > f_k(u_j)$ for $(u_i, u_j) \in D_f^{\succeq}$, then $a_k \in D_f(u_i, u_j)$, hence $B \cap D_f(u_i, u_j) \ne \varnothing$.

$" \Leftarrow "$: For $(u_i, u_j) \in D_f^{\succeq}$, if $B \cap D_f(u_i, u_j) \ne \varnothing$, then there exist $a_k \in B$ s.t. $a_k \in D_f(u_i, u_j)$, that is $f_k(u_i) > f_k(u_j)$. By Theorem 3.1 we have that B is an upper approximation consistent set of A . □

Definition 4.2 For an information system $I_f^{\succeq}$ and $B \subseteq A$, the upper approximation discernability equation is denoted as

$$\begin{aligned} F_f &= \wedge\{\vee\{a_k | \; a_k \in D_f(u_i, u_j)\}, u_i, u_j \in U\} \\ &= \wedge\{\vee\{a_k | \; a_k \in D_f(u_i, u_j)\}, u_i, u_j \in D_f^{\succeq}\} \end{aligned}.$$

Theorem 4.2 Let $I_f^{\succeq}$ be an information system and the minimum alternative normal form is defined as $E_f = \vee_{k=1}^{p}(B_f^k)$. If take $B_f^k = \{a_s, s = 1, 2, \dots, q_k\}$ then $\{B_f^k, k = 1, 2, \dots, p\}$ is the set with all its elements have the upper approximation reduction form.

Proof. For any $(u_i, u_j) \in D_f^{\succeq}$, by the definition of minimum alternative normal form, we have that B_f^k is upper approximation consistent set. If one element of B_f^k in $F_f = \vee_{k=1}^{p}(B_f^k)$ is deleted, in which case B_f^k becomes $B_f^k(*)$, there must exist $(u_{i_0}, u_{j_0}) \in D_f^{\succeq}$ s.t. $B_f^k(*) \cap D_f(u_{i_0}, u_{j_0}) = \varnothing$. So $B_f^k(*)$ is not an upper approximation consistent set. Hence B_f^k is an upper approximation reduction. Because M_f includes all $D_f(u_i, u_j)$, there is not upper approximation reduction with other forms.

Example 4.1 (Continued from Example 3.1-3.2) By computing we have that

Table 2.

U	u_1	u_2	u_3	u_4	u_5	u_6
u_1	$\emptyset$	$\emptyset$	a_2	a_2	$\emptyset$	$\emptyset$
u_2	$\emptyset$	$\emptyset$	a_1, a_2	a_2	$\emptyset$	$\emptyset$
u_3	$\emptyset$	$\emptyset$	$\emptyset$	$\emptyset$	$\emptyset$	$\emptyset$
u_4	$\emptyset$	$\emptyset$	$\emptyset$	$\emptyset$	$\emptyset$	$\emptyset$
u_5	a_1, a_2, a_3	a_2	a_1, a_2	a_1, a_2	$\emptyset$	a_2
u_6	$\emptyset$	$\emptyset$	a_1, a_2	a_1, a_2	$\emptyset$	$\emptyset$

Hence, $E_f = (a_1 \vee a_2 \vee a_3) \wedge \{a_2\} \wedge \{a_1 \vee a_2\} = a_2$, that is, $\{a_2\}$ is the one and only upper approximation reduction, from which we have that the conclusion of this Example is consistent with Example 3.2.

5. Conclusion

Attributes reduction, as one research problem, has played an important role in rough set theory. However, most of information systems are based on dominance relations because of various factors. To acquire brief decision rules from inconsistent ordered information systems with fuzzy decision, attributes reductions are needed. The main aim of this paper is to study the problem. In this paper, the upper approximation consistent set is constructed and approach to attribute reduction in ordered information system with fuzzy decision is proposed. Moreover, the judgment theorem of upper approximation consistent set is obtained and some useful works are done in detail.

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