# Knowledge reduction and matrix computation in inconsistent ordered information systems

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**Abstract:** In this article, assignment reduction and approximation reduction are proposed for Inconsistent Ordered Information Systems (IOIS). The properties and relationships between assignment reduction and approximation reduction are discussed. The dominance matrix and decision assignment matrix are also proposed for information systems based on dominance relations. The algorithm of assignment reduction is introduced, from which we can provide an approach to knowledge reductions operated in inconsistent systems based on dominance relations. Finally, an example illustrates the validity of the given method, which shows that the method is effective in complicated information systems.

**Keywords:** rough set; ordered information systems; OIS; knowledge reduction; matrix computation.

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## 1 Introduction

The rough set theory, proposed by Pawlak (1982), is an extension of the classical set theory for modelling uncertainty or imprecise information. The research has recently roused great interest in theoretical and application fronts such as machine learning, pattern recognition, data analysis and so on.

Knowledge reduction is one of the hot research topics of the rough set theory. Much study on this area had been reported and many useful results have been obtained even until now (Leuang *et al.*, 2006; Xu and Zhang, 2006; 2007a; Zhang *et al.*, 2001; Wu *et al.*, 2005; Zhang *et al.*, 2003). However, most of the work was based on consistent information systems and the main methodology has been developed under equivalence relations (indiscernibility relations). In practise, most information systems are not only inconsistent, but also based on dominance relations because of various factors. The ordering of the properties of attributes plays a crucial role in those systems. For this reason, Greco *et al.* (1998; 1999; 2001; 2002) and Dembczynski *et al.* (2003a; 2003b) proposed an extension rough sets theory, called the Dominance-Based Rough Sets Approach (DRSA), to take into account the ordering properties of attributes. This innovation is mainly based on the substitution of the indiscernibility relation by a dominance relation. In DRSA, condition attributes and classes are preference ordered. Many studies on DRSA have been made (Sai *et al.*, 2001; Shao and Zhang, 2005; Xu and Zhang, 2006; 2007a; 2008). But simpler results of knowledge reductions are very poor in inconsistent ordered information systems until now.

In this paper, the method operated for knowledge reductions is introduced in inconsistent ordered information systems. The dominance matrix and decision assignment matrix are introduced in information systems based on dominance relations. Furthermore, the algorithm of assignment reduction is obtained, from which we can provide a new approach to knowledge reductions in inconsistent systems based on dominance relations. Finally, an example illustrates the validity of this method, which shows that the method, is effective in complicated information systems.

## 2 Rough sets and OIS

The following paragraphs recall the necessary concepts and preliminaries required in the sequel of our work. A detailed description of the theory can be found in Zhang *et al.* (2001).

An information system with decisions is an ordered quadruple $\mathcal{I} = (U, A \cup D, F, G)$, where:

$U = \{x_1, x_2, \cdots, x_n\}$ is a non-empty finite set of objects

$A \cup D$ is a non-empty finite attributes set

$A = \{a_1, a_2, \cdots, a_p\}$ denotes the set of condition attributes

$D = \{d_1, d_2, \cdots, d_q\}$ denotes the set of decision attributes, $A \cap D = \phi$

$F = \{f_k | U \to V_k, k \le p\}$, $f_k(x)$ is the value of $a_k$ on $x \in U$ and $V_k$ is the domain of $a_k$, $a_k \in A$

$G = \{g_{k'} | U \to V_{k'}, k' \le q\}$, $g_{k'}(x)$ is the value of $d_{k'}$ on $x \in U$ and $V_{k'}$ is the domain of $d_{k'}$, $d_{k'} \in D$.

In an information system, if the domain of an attribute is ordered according to a decreasing or increasing preference, then the attribute is a criterion.

*Definition 2.1 (Zhang et al., 2001)*

An information system is called an Ordered Information System (OIS) if all the condition attributes are criterions.

Assuming that the domain of a criterion $a \in A$ is complete and pre-ordered by an outranking relation $\succeq_a$, then $x \succeq_a y$ means that $x$ is at least as good as $y$ with respect to criterion $a$. We can also say that $x$ dominates $y$. Without any loss of generality, we consider the condition and decision criterions having a numerical domain, that is, $V_a \subseteq \mathcal{R}$ ($\mathcal{R}$ denotes the set of real numbers).

We define $x \succeq y$ by $f(x, a) \geq f(y, a)$ according to increasing preference, where $a \in A$ and $x, y \in U$. For a subset of attributes $B \subseteq A$, $x \succeq_B y$ means that $x \succeq_a y$ for any $a \in B$. That is, $x$ dominates $y$ with respect to all the attributes in $B$. Furthermore, we denote $x \succeq_B y$ by $xR_B^{\geq}y$. In general, we indicate an OIS with a decision by $\mathcal{I}^{\succeq} = (U, A \cup D, F, G)$. Thus, the following definition can be obtained.

Let $\mathcal{I}^{\succeq} = (U, A \cup D, F, G)$ be an OIS with decisions; for $B \subseteq A$, denote:

$$R_B^{\succeq} = \{(x_i, x_j) \in U \times U | f_l(x_i) \geq f_l(x_j), \forall a_l \in B\};$$

$$R_D^{\succeq} = \{(x_i, x_j) \in U \times U | g_m(x_i) \geq g_m(x_j), \forall d_m \in D\}.$$

$R_B^{\succeq}$ and $R_D^{\succeq}$ are called the dominance relations of information system $\mathcal{I}^{\succeq}$.

If we denote:

$$\begin{aligned}[x_i]_B^{\succeq} &= \{x_j \in U | (x_j, x_i) \in R_B^{\succeq}\} \\ &= \{x_j \in U | f_l(x_j) \geq f_l(x_i), \forall a_l \in B\}; \\ [x_i]_D^{\succeq} &= \{x_j \in U | (x_j, x_i) \in R_D^{\succeq}\} \\ &= \{x_j \in U | g_m(x_j) \geq g_m(x_i), \forall d_m \in D\},\end{aligned}$$

then the following properties of a dominance relation are trivial.

*Proposition 2.1 (Zhang et al., 2001)*

Let $R_A^{\succeq}$ be a dominance relation. The following hold:

- $R_A^{\succeq}$ is reflexive, transitive, but not symmetric, so it is not a equivalence relation
- If $B \subseteq A$, then $R_A^{\succeq} \subseteq R_B^{\succeq}$
- If $B \subseteq A$, then $[x_i]_A^{\succeq} \subseteq [x_i]_B^{\succeq}$
- If $x_j \in [x_i]_A^{\succeq}$, then $[x_j]_A^{\succeq} \subseteq [x_i]_A^{\succeq}$ and $[x_i]_A^{\succeq} = \cup\{[x_j]_A^{\succeq} | x_j \in [x_i]_A^{\succeq}\}$
- $[x_j]_A^{\succeq} = [x_i]_A^{\succeq}$ iff $f(x_i, a) = f(x_j, a)$ $(\forall a \in A)$
- $\mathcal{J} = \cup\{[x]_A^{\succeq} | x \in U\}$ constitute a covering of $U$.

For any subset $X$ of $U$ and $A$ of $\mathcal{I}^{\succeq}$, define:

$$\underline{R_A^{\succeq}}(X) = \{x \in U | [x]_A^{\succeq} \subseteq X\};$$

$$\overline{R_A^{\succeq}}(X) = \{x \in U | [x]_A^{\succeq} \cap X \neq \phi\}.$$

$\underline{R_A^{\succeq}}(X)$ and $\overline{R_A^{\succeq}}(x)$ are said to be the lower and upper approximations of $X$ with respect to dominance relation $R_A^{\succeq}$. The approximations also have some properties which are similar to those of the Pawlak approximation spaces.

*Definition 2.2 (Zhang et al., 2001)*

For an OIS with decisions $\mathcal{I}^{\succeq} = (U, A \cup D, F, G)$, if $R_A^{\succeq} \subseteq R_D^{\succeq}$, then this information system is consistent, otherwise, this information system is inconsistent (Inconsistent Ordered Information System or IOIS).

*Example 2.1*

Let us consider an OIS in Table 1.

**Table 1** An ordered information system

| $U$ | $a_1$ | $a_2$ | $a_3$ | $d$ |
|---|---|---|---|---|
| $x_1$ | 1 | 2 | 1 | 3 |
| $x_2$ | 3 | 2 | 2 | 2 |
| $x_3$ | 1 | 1 | 2 | 1 |
| $x_4$ | 2 | 1 | 3 | 2 |
| $x_5$ | 3 | 3 | 2 | 3 |
| $x_6$ | 3 | 2 | 3 | 1 |

From the table, we have:

$$[x_1]_A^{\succeq} = \{x_1, x_2, x_5, x_6\}; \quad [x_2]_A^{\succeq} = \{x_2, x_5, x_6\};$$
$$[x_3]_A^{\succeq} = \{x_2, x_3, x_4, x_5, x_6\}; \quad [x_4]_A^{\succeq} = \{x_4, x_6\};$$
$$[x_5]_A^{\succeq} = \{x_5\}; \quad [x_6]_A^{\succeq} = \{x_6\};$$

and

$$[x_1]_d^{\succeq} = [x_5]_d^{\succeq} = \{x_1, x_5\};$$
$$[x_2]_d^{\succeq} = [x_4]_d^{\succeq} = \{x_1, x_2, x_4, x_5\};$$
$$[x_3]_d^{\succeq} = [x_6]_d^{\succeq} = \{x_1, x_2, x_3, x_4, x_5, x_6\}.$$

Obviously, by the above, we have $R_A^{\succeq} \not\subseteq R_d^{\succeq}$, so the system in Table 1 is inconsistent.

Let $\mathcal{I}^{\succeq} = (U, A \cup D, F, G)$ be an IOIS and denote:

$$\sigma_B^{\succeq}(x) = \{D_j | D_j \cap [x]_B^{\succeq} \neq \phi, x \in U\};$$

$$\eta_B^{\succeq} = \frac{1}{U} \sum_{j=1}^{r} |\overline{R_B^{\succeq}}(D_j)|.$$

For an OIS, the following definitions will be proposed.

*Definition 2.3 (Zhang et al., 2001)*

- If $\sigma_B^{\succeq}(x) = \sigma_A^{\succeq}(x)$, for all $x \in U$, we say that $B$ is an assignment consistent set of $\mathcal{I}^{\succeq}$. If $B$ is an assignment consistent set and no proper subset of $B$ is an assignment consistent set, then $B$ is called an assignment consistent reduction of $\mathcal{I}^{\succeq}$.
- If $\eta_B^{\succeq}(x) = \eta_A^{\succeq}(x)$, for all $x \in U$, we say that B is an approximation consistent set of $\mathcal{I}^{\succeq}$. If $B$ is an approximation consistent set and no proper subset of $B$ is an approximation consistent set, then $B$ is called an approximation consistent reduction of $\mathcal{I}^{\succeq}$.

An assignment consistent set is a subset of attributes set that preserves the possible decisions of every object. An approximation consistent set preserves the upper approximation of every decision class.

*Proposition 2.2 (Zhang et al., 2001)*

Let $\mathcal{I}^{\succeq} = (U, A \cup D, F, G)$ be an OIS, then $B \subseteq A$ is an assignment reduction of $\mathcal{I}^{\succeq}$ if and only if $B$ is an approximation reduction of $\mathcal{I}^{\succeq}$.

For a simple description, the following information system with decisions are based on dominance relations, *i.e*., OIS.

## 3 Knowledge reduction in IOIS

### 3.1 Theories of knowledge reduction in IOIS

Let $\mathcal{I}^{\succeq} = (U, A \cup \{d\}, F, G)$ be an OIS and $R_A^{\succeq}, R_d^{\succeq}$ be the dominance relations derived from condition attributes set $A$ and decision attributes set $\{d\}$, respectively. For $B \subseteq A$, denote:

$$U/R_B^{\succeq} = \{[x_i]_B^{\succeq} | x_i \in U\};$$

$$U/R_d^{\succeq} = \{D_1, D_2, \ldots, D_r\};$$

$$\sigma_B^{\succeq}(x) = \{D_j | D_j \cap [x]_B^{\succeq} \neq \phi, x \in U\};$$

$$\eta_B^{\succeq} = \frac{1}{U} \sum_{j=1}^{r} |\overline{R_B^{\succeq}(D_j)}|,$$

where $[x]_B^{\succeq} = \{y \in U : (x, y) \in R_B^{\succeq}\}$.

From the above, we can have the following propositions immediately.

*Proposition 3.1*

Let $\mathcal{I}^{\succeq} = (U, A \cup \{d\}, F, G)$ be an IOIS:

- $\overline{R_B^{\succeq}}(D_j) = \cup\{[x]_B^{\succeq} | D_j \in \sigma_B^{\succeq}(x)\}$
- If $B \subseteq A$, then $\sigma_A^{\succeq}(x) \subseteq \sigma_B^{\succeq}(x), \forall x \in U$
- If $[x]_B^{\succeq} \supseteq [y]_B^{\succeq}$, then $\sigma_B^{\succeq}(x) \supseteq \sigma_B^{\succeq}(y), \forall x, y \in U$.

*Definition 3.1*

Let $\mathcal{I}^{\succeq} = (U, A \cup \{d\}, F, G)$ be an IOIS:

- If $\sigma_B^{\succeq}(x) = \sigma_A^{\succeq}(x)$, for all $x \in U$, we say that $B$ is an assignment consistent set of $\mathcal{I}^{\succeq}$. If $B$ is an assignment consistent set and no proper subset of $B$ is an assignment consistent set, then $B$ is called an assignment consistent reduction of $\mathcal{I}^{\succeq}$.
- If $\eta_B^{\succeq}(x) = \eta_A^{\succeq}(x)$, for all $x \in U$, we say that $B$ is an approximation consistent set of $\mathcal{I}^{\succeq}$. If $B$ is an approximation consistent set and no proper subset of $B$ is an approximation consistent set, then $B$ is called an approximation consistent reduction of $\mathcal{I}^{\succeq}$.

An assignment consistent set is a subset of an attributes set that preserves the possible decisions of every object. An approximation consistent set preserves the upper approximation of every decision class.

*Example 3.1*

Consider the IOIS in Example 2.1.

For the IOIS in Example 2.1 (Table 1), we denote:

$$D_1 = [x_1]_d^{\succeq} = [x_5]_d^{\succeq};$$
$$D_2 = [x_2]_d^{\succeq} = [x_4]_d^{\succeq};$$
$$D_3 = [x_3]_d^{\succeq} = [x_6]_d^{\succeq}.$$

Thus, we can acquire that:

$$\sigma_A^{\succeq}(x_1) = \sigma_A^{\succeq}(x_2) = \sigma_A^{\succeq}(x_3) = \sigma_A^{\succeq}(x_5) = \{D_1, D_2, D_3\};$$
$$\sigma_A^{\succeq}(x_4) = \{D_2, D_3\};$$
$$\sigma_A^{\succeq}(x_6) = \{D_3\}.$$

When $B = \{a_2, a_3\}$, it can be easily checked that $[x]_A^{\succeq} = [x]_B^{\succeq}$ for all $x \in U$. So $\sigma_B^{\succeq}(x) = \sigma_A^{\succeq}(x)$ is true. Thus, $B = \{a_2, a_3\}$ is an assignment consistent set of $\mathcal{I}^{\succeq}$. Furthermore, we can examine that $\{a_2\}$ and $\{a_3\}$ are not a consistent set of $\mathcal{I}^{\succeq}$. That is, $B = \{a_2, a_3\}$ is an assignment consistent reduction of $\mathcal{I}^{\succeq}$.

When $B' = \{a_1, a_3\}$, we have:

$$[x_1]_{B'}^{\succeq} = \{x_1, x_2, x_3, x_4, x_5, x_6\}; \quad [x_2]_{B'}^{\succeq} = \{x_2, x_5, x_6\};$$
$$[x_3]_{B'}^{\succeq} = \{x_2, x_3, x_4, x_5, x_6\}; \quad [x_4]_{B'}^{\succeq} = \{x_4, x_6\};$$
$$[x_5]_{B'}^{\succeq} = \{x_2, x_5, x_6\}; \quad [x_6]_{B'}^{\succeq} = \{x_6\};$$

and

$$\sigma_{B'}^{\succeq}(x_1) = \sigma_{B'}^{\succeq}(x_2) = \sigma_{B'}^{\succeq}(x_3) = \sigma_{B'}^{\succeq}(x_4) = \{D_1, D_2, D_3\};$$
$$\sigma_{B'}^{\succeq}(x_4) = \{D_2, D_3\};$$
$$\sigma_{B'}^{\succeq}(x_6) = \{D_3\}.$$

For all $x \in U$, we have $\sigma_{B'}^{\succeq}(x) = \sigma_B^{\succeq}(x)$ and $B' = \{a_1, a_3\}$ is another assignment consistent set of $\mathcal{I}^{\succeq}$. Moreover, it can be easily calculated that $a_1$ is not an assignment consistent set of $\mathcal{I}^{\succeq}$. Hence, $B' = \{a_1, a_3\}$ is another assignment reduction of $\mathcal{I}^{\succeq}$. Furthermore, it can be easily verified that $\{a_1, a_2\}$ is not an assignment consistent set of $\mathcal{I}^{\succeq}$.

Thus, there exist two assignment reductions of $\mathcal{I}^{\succeq}$ in the system of Table 1, which are $\{a_1, a_3\}$ and $\{a_2, a_3\}$.

*Theorem 3.1*

Let $\mathcal{I}^{\succeq} = (U, A \cup \{d\}, F, G)$ be an OIS, then $B \subseteq A$ is an assignment consistent set of $\mathcal{I}^{\succeq}$ if and only if $B$ is an approximation consistent set of $\mathcal{I}^{\succeq}$.

*Proof*

Assume that $B \subseteq A$ is an assignment consistent set of $\mathcal{I}^{\succeq}$, that is, $\sigma_B^{\succeq}(x) = \sigma_A^{\succeq}(x)$ for all $x \in U$. By the definition, for $\forall j \geq r$, we have:

$$\begin{aligned} x \in \overline{R_B^{\succeq}}(D_j) &\Leftrightarrow [x]_B^{\succeq} \cap D_j \neq \phi \Leftrightarrow D_j \in \sigma_B^{\succeq}(x) \\ &\Leftrightarrow D_j \in \sigma_A^{\succeq}(x) \Leftrightarrow [x]_A^{\succeq} \cap D_j \neq \phi \\ &\Leftrightarrow x \in \overline{R_A^{\succeq}}(D_j). \end{aligned}$$

$\overline{R_B^{\succeq}}(D_j) = \overline{R_A^{\succeq}}(D_j)$, that is, $\eta_B^{\succeq} = \eta_A^{\succeq}$. Hence, $B$ is an approximation consistent set of $\mathcal{I}^{\succeq}$.

Conversely, if $B$ is an assignment consistent set of $\mathcal{I}^{\succeq}$, then $\eta_B^{\succeq} = \eta_A^{\succeq}$, which indicates:

$$\sum_{j=1}^{r} |\overline{R_B^{\succeq}}(D_j)| = \sum_{j=1}^{r} |\overline{R_A^{\succeq}}(D_j)|.$$

On the other hand, since $\overline{R_B^{\succeq}}(D_j) \supseteq \overline{R_A^{\succeq}}(D_j)$ for $\forall j \geq r$, $\overline{R_B^{\succeq}}(D_j) = \overline{R_A^{\succeq}}(D_j)$ holds. Thus, for all $x \in U$, we have:

$$\begin{aligned} D_j \in \sigma_B^{\succeq}(x) &\Leftrightarrow [x]_B^{\succeq} \cap D_j \neq \phi \Leftrightarrow x \in \overline{R_B^{\succeq}}(D_j) \\ &\Leftrightarrow x \in \overline{R_A^{\succeq}}(D_j) \Leftrightarrow [x]_A^{\succeq} \cap D_j \neq \phi \\ &\Leftrightarrow D_j \in \sigma_A^{\succeq}(x). \end{aligned}$$

Hence, $\sigma_B^{\succeq}(x) = \sigma_A^{\succeq}(x)$ is true for all $x \in U$, which show that $B$ is an assignment consistent set of $\mathcal{I}^{\succeq}$.

*Corollary 3.1*

Let $\mathcal{I}^{\succeq} = (U, A \cup \{d\}, F, G)$ be an OIS, then $B \subseteq A$ is an assignment reduction of $\mathcal{I}^{\succeq}$ if and only if $B$ is an approximation reduction of $\mathcal{I}^{\succeq}$.

*Theorem 3.2*

Let $\mathcal{I}^{\succeq} = (U, A \cup \{d\}, F, G)$ be an OIS, then $B \subseteq A$ is an assignment consistent set of $\mathcal{I}^{\succeq}$ if and only if when $\sigma_A^{\succeq}(x) \cap \sigma_A^{\succeq}(y) \neq \sigma_A^{\succeq}(y)$; $[x]_B^{\succeq} \cap [y]_B^{\succeq} \neq [y]_B^{\succeq}$ holds for $x, y \in U$.

*Proof*

Assume that when $\sigma_A^{\succeq}(x) \cap \sigma_A^{\succeq}(y) \neq \sigma_A^{\succeq}(y)$, $[x]_B^{\succeq} \cap [y]_B^{\succeq} \neq [y]_B^{\succeq}$ does not hold, implying that $[x]_B^{\succeq} \cap [y]_B^{\succeq} = [y]_B^{\succeq}$. We have $[x]_B^{\succeq} \supseteq [y]_B^{\succeq}$ and $\sigma_B^{\succeq}(x) \supseteq \sigma_B^{\succeq}(y)$ can be obtained by Proposition 3.1(3). On the other hand, since $B$ is an assignment consistent set of $\mathcal{I}^{\succeq}$, we have $\sigma_A^{\succeq}(x) \supseteq \sigma_A^{\succeq}(y)$, which is in contradiction with $\sigma_A^{\succeq}(x) \cap \sigma_A^{\succeq}(y) \neq \sigma_A^{\succeq}(y)$.

Conversely, we only prove $\sigma_B^{\succeq}(x) \subseteq \sigma_A^{\succeq}(x)$ by Proposition 3.1(2).

For all $x$, $y \in U$, if $\sigma_A^{\succeq}(x) \cap \sigma_A^{\succeq}(y) \neq \sigma_A^{\succeq}(y)$ implies $[x]_B^{\succeq} \cap [y]_B^{\succeq} \neq [y]_B^{\succeq}$, which means that $[x]_B^{\succeq} \cap [y]_B^{\succeq} = [y]_B^{\succeq}$ implies $\sigma_A^{\succeq}(x) \cap \sigma_A^{\succeq}(y) = \sigma_A^{\succeq}(y)$, that is, $[x]_B^{\succeq} \supseteq [y]_B^{\succeq}$ implies $\sigma_A^{\succeq}(x) \supseteq \sigma_A^{\succeq}(y)$.

On the other hand, suppose $D_k \in \sigma_B^{\succeq}(x)$, that is, $[x]_B^{\succeq} \cap D_k \neq \phi$. Assume that $y \in [x]_B^{\succeq} \cap D_k$, then $y \in [x]_B^{\succeq}$ and $y \in D_k$. By Proposition 1(4), we obtain that $[x]_B^{\succeq} \supseteq [y]_B^{\succeq}$ is true, which implies $\sigma_A^{\succeq}(x) \supseteq \sigma_A^{\succeq}(y)$. Since $y \in [y]_A^{\succeq}$, we have $y \in [y]_A^{\succeq} \cap D_k$, which means $[y]_A^{\succeq} \cap D_k = \phi$. We observe $D_k \in \sigma_A^{\succeq}(y) \subseteq \sigma_A^{\succeq}(x)$, that is, $D_k \in \sigma_A^{\succeq}(x)$. Thus, we conclude that $\sigma_B^{\succeq}(x) \subseteq \sigma_A^{\succeq}(x)$, *i.e.*, $B$ is an assignment consistent set of $\mathcal{I}^{\succeq}$.

*Corollary 3.2*

Let $\mathcal{I}^{\succeq} = (U, A \cup \{d\}, F, G)$ be an OIS, then $B \subseteq A$ is an approximation consistent set of $\mathcal{I}^{\succeq}$ if and only if $\sigma_A^{\succeq}(x) \cap \sigma_A^{\succeq}(y) \neq \sigma_A^{\succeq}(y)$; $[x]_B^{\succeq} \cap [y]_B^{\succeq} \neq [y]_B^{\succeq}$ holds for $x, y \in U$.

### *3.2 Approach to knowledge reduction in IOIS*

This section provides an approach to assignment reduction in IOIS. Let us first give the following notes.

*Definition 3.2*

Let $\mathcal{I}^{\succeq} = (U, A \cup \{d\}, F, G)$ be an IOIS. We denote:

$$D^* = \{(x_i, x_j) | \sigma_A^{\succeq}(x_i) \subset \sigma_A^{\succeq}(x_j)\}.$$

Let denote the value of $a_k$ by $f_{a_k}$. So we get:

$$D(x_i, x_j) = \begin{cases} \{a_k \in A | f_{a_k}(x_i) > f_{a_k}(x_j)\}, & (x_i, x_j) \in D^* \\ A, & (x_i, x_j) \notin D^* \end{cases}.$$

Then, $D(x_i, x_j)$ is said to be an assignment discernibility attributes set. $M = (D(x_i, x_j), x_i, x_j \in U)$ is referred to as the assignment discernibility matrix of $\mathcal{I}^{\succeq}$.

*Theorem 3.3*

Let $\mathcal{I}^{\succeq} = (U, A \cup \{d\}, F, G)$ be an IOIS and $B \subseteq A$, then $B$ is an assignment consistent set if and only if $B \cap D(x, y) \neq \phi$ for all $(x, y) \in D^*$.

*Proof*

Assume that $B$ is an assignment consistent set of $\mathcal{I}^{\succeq}$. For any $(x, y) \in D^*$, we can obtain $\sigma_A^{\succeq}(x) \subset \sigma_A^{\succeq}(y)$, that is, $\sigma_A^{\succeq}(x) \cap \sigma_A^{\succeq}(y) \neq \sigma_A^{\succeq}(y)$. From Theorem 3.2, we have $[x]_B^{\succeq} \cap [y]_B^{\succeq} \neq [y]_B^{\succeq}$. Thus, there exist the following three cases between $[x]_B^{\succeq}$ and $[y]_B^{\succeq}$, which are:

1 $[x]_B^{\succeq} \subset [y]_B^{\succeq}$

2 $[x]_B^{\succeq} \cap [y]_B^{\succeq} = \phi$

3 both $[x]_B^{\succeq} \cap [y]_B^{\succeq} \subset [x]_B^{\succeq}$ and $[x]_B^{\succeq} \cap [y]_B^{\succeq} \subset [y]_B^{\succeq}$.

We will prove that $B \cap D(x, y) \neq \phi$ always holds in every case:

Case 1 If $[x]_B^{\succeq} \subset [y]_B^{\succeq}$, then there necessarily exists an element $z \in [y]_B^{\succeq}$, but $z \notin [x]_B^{\succeq}$. From $z \notin [x]_B^{\succeq}$, we can certainly find an element $a_k \in B$, such that $f_{a_k}(x) > f_{a_k}(z)$. On the other hand, the fact $f_{a_k}(y) \geq f_{a_k}(z)$ is true according to $z \in [y]_B^{\succeq}$. From the above, we can obtain $f_{a_k}(x) > f_{a_k}(y)$. Hence, we have $a_k \in D(x, y)$, *i.e.*, $B \cap D(x, y) \neq \phi$.

Case 2 If $[x]_B^{\succeq} \cap [y]_B^{\succeq} = \phi$, then there necessarily exists an element $a_k \in B$, such that $f_{a_k}(x) > f_{a_k}(y)$, *i.e.*, $B \cap D(x, y) \neq \phi$. Otherwise, if for all $a_l \in B$, $f_{a_l}(x) \geq f_{a_l}(y)$ always holds, then we observe $y \in [x]_B^{\succeq}$. This is a contradiction.

Case 3 The proof is similar to Case 1, because we can also certainly find an element $z \in [y]_B^{\succeq}$, but $z \notin [x]_B^{\succeq}$ in the case.

Thus, we can conclude that $B \cap D(x, y) \neq \phi$ for all $(x, y) \in D^*$.

Conversely, if every $(x, y) \in D^*$ satisfies $B \cap D(x, y) \neq \phi$, then we can select an $a_k \in B$, such that $a_k \in D(x, y)$. That is, $f_{a_k}(x) > f_{a_k}(y)$ so $y \notin [x]_B^{\succeq}$. Since $y \in [y]_B^{\succeq}$ is true, we can obtain $[x]_B^{\succeq} \cap [y]_B^{\succeq} \neq [y]_B^{\succeq}$. On the other hand, since $(x, y) \in D^*$, we have $\sigma_A^{\succeq}(x) \subset \sigma_A^{\succeq}(y)$, which implies $\sigma_A^{\succeq}(x) \cap \sigma_A^{\succeq}(y) \neq \sigma_A^{\succeq}(y)$. Hence, we find that when $\sigma_A^{\succeq}(x) \cap \sigma_A^{\succeq}(y) \neq \sigma_A^{\succeq}(y)$, $[x]_B^{\succeq} \cap [y]_B^{\succeq} \neq [y]_B^{\succeq}$ holds. Thus, we know that $B$ is an assignment consistent set of $\mathcal{I}^{\succeq}$ in terms of Theorem 3.2.

*Definition 3.3*

Let $\mathcal{I}^{\succeq} = (U, A \cup \{d\}, F, G)$ be an IOIS and let $M = (D(x_i, x_j), x_i, x_j \in U)$ be the assignment discernibility matrix of $\mathcal{I}^{\succeq}$. Denote:

$$F = \wedge\{\vee\{a_k | a_k \in D(x_i, x_j)\}, x_i, x_j \in U\}$$

$$= \wedge\{\vee\{a_k | a_k \in D(x_i, x_j)\}, x_i, x_j \in D^*\},$$

where $F$ is called the assignment discernibility function.

*Theorem 3.4*

Let $\mathcal{I}^{\succeq} = (U, A \cup \{d\}, F, G)$ be an IOIS. The minimal disjunctive normal form of the assignment discernibility function $F$ is:

$$F = \bigvee_{k=1}^{p} \left( \bigwedge_{s=1}^{q} \right) a_s .$$

Denote $B_k = \{a_s | s = 1, 2, \cdots, q_k\}$, then $\{B_k | k = 1,2,\cdots, p\}$ is just a set of all the assignment reductions of $\mathcal{I}^{\succeq}$.

*Proof*

It follows directly from Theorem 4.1.

Theorem 3.4 provides a practical approach to the assignment reduction of information systems with decisions based on dominance relations. Next, we will consider the IOIS in Table 1 by using this approach.

*Example 3.2*

Table 2 is the assignment discernibility matrix of the IOIS in Example 2.1.

**Table 2** The assignment discernibility matrix in Example 2.1

| $x_i, x_j$ | $x_1$ | $x_2$ | $x_3$ | $x_4$ | $x_5$ | $x_6$ |
|---|---|---|---|---|---|---|
| $x_1$ | $A$ | $A$ | $A$ | $A$ | $A$ | $A$ |
| $x_2$ | $A$ | $A$ | $A$ | $A$ | $A$ | $A$ |
| $x_3$ | $A$ | $A$ | $A$ | $A$ | $A$ | $A$ |
| $x_4$ | $a_1, a_3$ | $a_3$ | $a_1, a_3$ | $A$ | $a_3$ | $A$ |
| $x_5$ | $A$ | $A$ | $A$ | $A$ | $A$ | $A$ |
| $x_6$ | $a_1, a_3$ | $a_3$ | $A$ | $a_1, a_2$ | $a_3$ | $A$ |

Consequently, we have:

$$F = (a_1 \vee a_2 \vee a_3) \wedge (a_1 \vee a_3) \wedge (a_1 \vee a_2) \wedge a_3$$

$$= (a_1 \wedge a_3) \vee (a_2 \wedge a_3).$$

Therefore, from Theorem 3.4, we get that $\{a_1, a_3\}$ and $\{a_2, a_3\}$ are all assignment reductions of the IOIS in Table 1, which accords with the result of Example 3.1.

# 4 Algorithm of matrix computation

## *4.1 Dominance matrices and assignment decision matrices*

In this section, the dominance matrices and assignment decision matrices are proposed and some properties are obtained.

*Definition 4.1*

Let $\mathcal{I}^{\succeq} = (U, A \cup D, F, G)$ be an OIS, and denote:

$$M_B = (m_{ij})_{n\times n} = \begin{cases} 1, & x_j \in [x_i]_B^{\succeq}, \\ 0, & \text{otherwise.} \end{cases}$$

The matrix $M_B$ is called the dominance matrix of attributes set $B \subseteq A$. If $|B| = l$, we say that the order of $M_B$ is $l$.

*Definition 4.2*

Let $\mathcal{I}^{\succeq} = (U, A \cup D, F, G)$ be an OIS and $M_B$, $M_C$ be dominance matrices of attributes sets $B$, $C \subseteq A$. The intersection of $M_B$ and $M_C$ is defined by:

$$M_B \cap M_C = (m_{ij})_{n\times n} \cap (m'_{ij})_{n\times n} = (\min\{m_{ij}, m'_{ij}\})_{n\times n}.$$

From above definition, we can have the following properties.

*Proposition 4.1*

Let $M_B$, $M_C$ be the dominance matrices of attributes sets $B$, $C \subseteq A$; the following results always hold:

- $m_{ii} = 1$
- if $M_B$, $M_C$, then $M_{B\cup C} = M_B \cap M_C$.

*Definition 4.3*

Let $\mathcal{I}^{\succeq} = (U, A \cup D, F, G)$ be an OIS and denote:

$$M_D = (r_{ij})_{n\times n} = \begin{cases} 1, & [x_j]_D^{\succeq} \in \sigma_A^{\succeq}(x_i), \\ 0, & \text{otherwise.} \end{cases}$$

The matrix $M_D$ is called the decision assignment matrix of $\mathcal{I}^{\succeq}$.

From the above, we can see that the dominance relation of objects is decided by dominance matrices and the different decisions of objects is decided by the decision assignment matrix.

*Definition 4.4*

Let $\alpha = (a_1, a_2, \ldots, a_n)$ and $\beta = (b_1, b_2, \ldots, b_n)$ be two $n$ dimension vectors. If $a_i \le b_i$, $(i = 1, 2, \ldots, n)$; we say vector $\alpha$ is less than vector $\beta$, denoted by $\alpha \le \beta$.

*Definition 4.5*

Let $M_A = (\alpha_1, \alpha_2, \ldots, \alpha_n)^T$ and $M_B = (\beta_1, \beta_2, \ldots, \beta_n)^T$ be two matrices, where $\alpha_i$ and $\beta_i$ be vectors respectively. If $\alpha_i \le \beta_i$, we say $M_A$ is less than $M_B$, denoted by $M_A \le M_B$.

By the definitions, dominance matrices have the following properties.

*Proposition 4.2*

Let $\mathcal{I}^{\succeq} = (U, A \cup D, F, G)$ be an OIS and $B \subseteq A$. If $M_A$ and $M_B$ are the dominance matrices, then $M_A \le M_B$.

## *4.2 Algorithm of matrix computation for knowledge reductions*

In the section, we will give the method of matrix computation for knowledge reduction in OIS.

*Theorem 4.1*

Let $\mathcal{I}^{\succeq} = (U, A \cup D, F, G)$ be an OIS and $B \subseteq A$. If $M_B \leq M_D$ does not hold, an element $x$ in $U$ exists such that $\sigma_A^{\succeq}(x) \neq \sigma_B^{\succeq}(x)$.

*Proof*

Since $M_B \leq M_D$ does not hold, elements $m_{ij} \in M_B$ and $r_{ij} \in M_D$ exist such that $m_{ij} > r_{ij}$. But $M_B$ and $M_D$ is a 0-1 matrix. Hence, $m_{ij} = 1$ and $r_{ij = 0}$. We have $x_j \in [x_i]_B^{\succeq}$ by $m_{ij} = 1$. If we denote $D_0 = [x_j]_D^{\succeq}$, then $D_0 \cap [x_i]_B^{\succeq} \neq \phi$. Thus, $D_0 \in \sigma_B^{\succeq}(x_i)$. On the other hand, we can obtain $D_0 = [x_j]_D^{\succeq} \notin \sigma_A^{\succeq}(x_i)$ by $r_{ij} = 0$.

The theorem is proved.

*Corollary 4.1*

Let $\mathcal{I}^{\succeq} = (U, A \cup D, F, G)$ be an OIS and $B \subseteq A$. If it satisfies $\sigma_A^{\succeq}(x) \neq \sigma_B^{\succeq}(x)$ for all $x \in U$, then $M_B \leq M_D$ holds.

*Theorem 4.2*

Let $\mathcal{I}^{\succeq} = (U, A \cup D, F, G)$ be an OIS and $B \subseteq A$. $M_B \leq M_D$ if and only if $\sigma_A^{\succeq}(x) = \sigma_B^{\succeq}(x)$ always holds for all $x \in U$.

*Proof*

"$\Leftarrow$" It can be obtained directly by Corollary 4.1.

"$\Rightarrow$" Here, we only prove $\sigma_B^{\leq}(x) \subseteq \sigma_A^{\leq}(x)$, because of $\sigma_A^{\succeq}(x) \subseteq \sigma_B^{\succeq}(x)$, for all $x \in U$ and $B \subseteq A$.

For an arbitrary $D_0 = [x_j]_D^{\succeq} \in \sigma_B^{\succeq}(x)$, we have $D_0 \cap [x]_B^{\succeq} \neq \phi$. Assume $x_k \in D_0 \cap [x]_B^{\succeq}$, $m_{ik} = 1$ can be obtained by $x_k \in [x]_B^{\succeq}$. Another, because $M_B \leq M_D$, we get $r_{ik} = 1$. Thus, we can obtain $[x_k]_D^{\succeq} \in \sigma_A^{\succeq}(x)$, *i.e.*, $[x_k]_D^{\succeq} \cap [x]_A^{\succeq} \neq \phi$ by Definition 4.3. On the other hand, $x_k \in D_0 = [x_j]_D^{\succeq} \in \sigma_A^{\succeq}(x)$, so $[x_k]_D^{\succeq} \subseteq [x_j]_D^{\succeq}$. Thus, $[x_j]_D^{\succeq} \cap [x]_A^{\succeq} \neq \phi$, *i.e.*, $D_0 = [x_j]_D^{\succeq} \in \sigma_A^{\succeq}(x)$. Hence, $\sigma_B^{\succeq}(x) \subseteq \sigma_A^{\succeq}(x)$.

The theorem is proved.

*Corollary 4.2*

Let $\mathcal{I}^{\succeq} = (U, A \cup D, F, G)$ be an OIS and $B \subseteq A$. $B$ is an assignment reduction of $I$ if and only if $M_B \leq M_D$ and $M_{B'} \leq M_D$ does not hold for every proper subset $B'$ of $B$.

We can obtain the following algorithm by Theorems 4.1 and 4.2.

*Algorithm*

The algorithm of matrix computation for knowledge reduction in IOIS is described as follows:

- Input – an IOIS $\mathcal{I}^{\succeq} = (U, A \cup D, F, G)$, where $U = \{x_1, x_2, \ldots, x_n\}$ and $A = \{a_1, a_2, \ldots, a_p\}$.
- Output – assignment reductions of $\mathcal{I}^{\succeq} = (U, A \cup D, F, G)$.

Step 1 Simplify the system by combining the objects with the same values of every attribute.

Step 2 Calculate the decision assignment matrix of $\mathcal{I}^{\succeq}$ : $M_D = (\gamma_1, \gamma_2, \ldots, \gamma_n)^T$.

Step 3 For all $a_l \in A$, $(1 \le l \le p)$, calculate the first-order dominance matrices:

$$M_{\{a_l\}} = M_{\{a_l\}}^{(1)} = (\tau_1^{(1)}, \tau_2^{(1)}, \ldots, \tau_n^{(1)})^T.$$

Let $i = 1$ to $n$.
If $0 \ne \tau_i^{(1)} \le \gamma_i$, then let $\tau_i^{(1)} = 0$.
Denote the new matrix by $FM_{\{a_l\}}^{(1)}$ and go to the next step.

Step 4 Call matrix $FM_{\{a_l\}}^{(1)} = (\tau_1^{(1)}, \tau_2^{(2)}, \ldots, \tau_n^{(1)})^T$ to be the first-order assignment matrix, where $a_l \in A(1 \le l \le p)$. If $FM_{\{a_l\}}^{(1)} = 0$, then obtain the first-order assignment reduction: $\{a_l\}$. Otherwise, go to the next step.

Step 5 Calculate the intersection of all the first-order nonzero matrices which were obtained in Step 3 and call new matrices to be the second-order dominance matrices, denoted by $M_{\{a_l a_s\}}^{(2)}$, $(M_{\{a_l a_s\}}^{(2)} \ne M_{\{a_l\}}^{(1)}, M_{\{a_l a_s\}}^{(2)} \ne M_{\{a_s\}}^{(1)})$.
Go back to Step 3 and calculate all the second-order assignment reductions.

Step 6 Obtain the higher-order assignment reductions by repeating Step 5. If the new matrices are zero matrices, then output all assignment reductions.

From the algorithm above, we can know that the complication of times is $O(|U^2|2^{|A|})$.

## 5 An example

*Example*

Let us consider an OIS in Table 3.

**Table 3** An ordered information system

| $U$ | $a_1$ | $a_2$ | $a_3$ | $d$ |
|---|---|---|---|---|
| $x_1$ | 1 | 2 | 1 | 3 |
| $x_2$ | 3 | 2 | 2 | 2 |
| $x_3$ | 1 | 1 | 2 | 1 |
| $x_4$ | 2 | 1 | 3 | 2 |
| $x_5$ | 3 | 3 | 2 | 3 |
| $x_6$ | 3 | 2 | 3 | 1 |

From the table, we can compute the dominance matrices and decision assignment matrices, which are:

$$M_{\{a_1\}} = \begin{bmatrix} 1 & 1 & 1 & 1 & 1 & 1 \\ 0 & 1 & 0 & 0 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 \\ 0 & 1 & 0 & 1 & 1 & 1 \\ 0 & 1 & 0 & 0 & 1 & 1 \\ 0 & 1 & 0 & 0 & 1 & 1 \end{bmatrix};$$

$$M_{\{a_2\}} = \begin{bmatrix} 1 & 1 & 0 & 0 & 1 & 1 \\ 1 & 1 & 0 & 0 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 1 & 1 & 0 & 0 & 1 & 1 \end{bmatrix};$$

$$M_{\{a_3\}} = \begin{bmatrix} 1 & 1 & 1 & 1 & 1 & 1 \\ 0 & 1 & 1 & 1 & 1 & 1 \\ 0 & 1 & 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & 1 & 0 & 1 \\ 0 & 1 & 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & 1 & 0 & 1 \end{bmatrix};$$

$$M_D = M_{\{d\}} = \begin{bmatrix} 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 \\ 0 & 1 & 1 & 1 & 0 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 \\ 0 & 0 & 1 & 0 & 0 & 1 \end{bmatrix}.$$

By comparing matrices $M_{\{a_1\}}$, $M_{\{a_2\}}$, $M_{\{a_3\}}$ and $M_{\{d\}}$, we can find that the vectors of the first, second, third and fifth rows in matrices $M_{\{a_1\}}$ and $M_{\{a_2\}}$ are less then those in matrix $M_{\{d\}}$, respectively. The system does not have the first-order assignment reduction. Thus, the first-order assignment matrices are as follows:

$$FM^{(1)}_{\{a_1\}} = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 & 1 \end{bmatrix};$$

$$FM^{(1)}_{\{a_2\}} = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 1 & 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 & 1 & 1 \end{bmatrix};$$

$$FM^{(1)}_{\{a_3\}} = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 1 \end{bmatrix}.$$

Furthermore, the second-order assignment matrices are:

$$M^{(2)}_{\{a_1a_2\}} = FM^{(1)}_{\{a_1\}} \cap FM^{(1)}_{\{a_2\}} = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 & 1 \end{bmatrix};$$

$$M^{(2)}_{\{a_1a_3\}} = FM^{(1)}_{\{a_1\}} \cap FM^{(1)}_{\{a_3\}} = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix};$$

$$M^{(2)}_{\{a_2a_3\}} = FM^{(1)}_{\{a_2\}} \cap FM^{(1)}_{\{a_3\}} = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}.$$

We can see that $M^{(2)}_{\{a_1a_3\}} = M^{(2)}_{\{a_2a_3\}}$ and their sixth row vectors are less then those of $M_{\{d\}}$, respectively, by comparing $M^{(2)}_{\{a_1a_2\}}$, $M^{(2)}_{\{a_1a_3\}}$, $M^{(2)}_{\{a_2a_3\}}$ and $M_{\{d\}}$. Hence, we can obtain all the second-order assignment reductions, which are $\{a_1, a_2\}$, $\{a_2, a_3\}$.

Hence, we have:

$$FM^{(2)}_{\{a_1a_3\}} = FM^{(2)}_{\{a_2a_3\}} = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}.$$

The algorithm is finished.

Thus, all the assignment reductions are $\{a_1, a_3\}$, $\{a_2, a_3\}$ in the system of the example above.

From the example, we can find that the algorithm is valid, and operated simply, for systems with a great deal of objects and attributes.

## 6 Conclusion

It is well known that most information systems are based on dominance relations because of various factors in practise. Therefore, it is meaningful to study the knowledge reductions in IOIS. In this article, assignment reduction and approximation reduction were proposed for IOIS. The properties and relationships between assignment reduction and approximation reduction were discussed. The dominance matrix and decision assignment matrix were also proposed for information systems based on dominance relations. The algorithm of assignment reduction was introduced, from which we can provide an approach to knowledge reductions operated in inconsistent systems based on dominance relations. Finally, an example illustrated the validity of this given method, which shows that the method is effective in a complicated information system.

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