Proceedings of the 26th Chinese Control Conference
July 26-31, 2007, Zhangjiajie, Hunan, China

Fuzziness in Covering Generalized Rough Sets

Xu Weihua[1], Zhang Xiaoyan[2]
1. Faculty of Science, Institute for Information and System Sciences, Xi'an Jiaotong University, Xi'an 710049, P.R. China
E-mail: datongxuweihua@126.com
2. Department of Mathematics and Information, Guangdong Ocean University, Zhanjiang 524088, P. R. China
E-mail: datongzhangxiaoyan@126.com

Abstract: Rough sets theory has been considered as a useful method to model the uncertainty and has been applied successfully in many fields. And every rough set is associated with some amount of fuzziness. On the other hand, rough sets theory has been generalized with coverings instead of classical partition. So it is necessary to consider the amount of fuzziness in generalized rough sets induced by a covering. In this paper, a measure of fuzziness in generalized rough sets induced by a covering is proposed. Moreover, some characterizations and properties of this measure are shown by examples, which is every useful in future research works of generalized rough sets induced by a covering.

Key Words: Rough set, Covering, Fuzzy set, α-cut, Nearest ordinary set, Index of fuzziness

1 INTRODUCTION

Various theories and methods have been proposed to deal with incomplete and insufficient information in classification, concept formation, and data analysis in data mining. For example, fuzzy set theory[17], rough sets[7], computing with words[14,18,19], linguistic dynamic systems[13,14], and many others, have been developed and applied to real-world problems. The focus of this paper is on the rough set theory, a tool originated by Pawlak[7] for data mining, with the particular intention to generalize it for the possible applications in computing with words and linguistic dynamic systems for modeling and analyzing complex systems and for data mining.

Partition or equivalent relation, as the indiscernibility relation in Pawlak's original rough set theory, is still restrictive for many applications. To address this issue, several interesting and meaningful extensions to equivalent relation have been proposed in the past, such as tolerance relations[4], similarity relations[10], others[12,15,16]. Particularly, Zakowski has used coverings a universe for establishing the covering generalized rough set theory[20] and an extensive body of research works has been developed[1,2,3,9]. The covering generalized rough set theory is model with promising potential for applications to data mining. In order to apply this theory to data mining, we address some basic problems in this theory. On the other hand, Dubios and Prade[21] combined fuzzy sets and roughs sets in a fruitful way by defining rough fuzzy sets and fuzzy rough sets. Banerjee and Pal[22] have characterized a measure of roughness of a fuzzy set making use of the concept of rough fuzzy sets. They also suggested some possible applications of the measure in pattern recognition and image analysis problems. Rough sets and fuzzy sets are also studied by [23-27]. Hence, consideration of the amount of fuzziness in covering generalized rough set is needed. This paper discussed the problem mainly.

In this paper the main objective is to study the problem. A measure of fuzziness in covering generalized rough sets is introduced. Furthermore, some characterization and properties of this measure are got with examples, which is every useful for next research works in covering generalized rough sets.

2 FUNDAMENTALS OF FUZZY SETS AND PAWLAK'S ROUGH SETS

The following recalls necessary concepts and preliminaries required in the sequel of our work. Detail description of them can be found in [17].

For a fuzzy set $\tilde{A}$, let denote $\alpha-$cut and strong $\alpha-$cut by $\tilde{A}_\alpha, \tilde{A}_{\bar{\alpha}}$ respectively, where $0 < \alpha \leqslant 1$.

Definition 2.1 Let $\tilde{A}$ be a fuzzy set. Then the nearest ordinary set to $\tilde{A}$ is denoted by $N(\tilde{A})$ and is given by

$$\mu_{N(\tilde{A})}(x) = \begin{cases} 0, & \text{if } \mu_{\tilde{A}}(x) < 0.5 \\ 1, & \text{if } \mu_{\tilde{A}}(x) > 0.5 \\ 0 \text{ or } 1, & \text{if } \mu_{\tilde{A}}(x) = 0.5 \end{cases}$$

By convention, we take $\mu_{N(\tilde{A})}(x) = 0$ for the last case. Thus, $N(\tilde{A}) = \tilde{A}_{0.5}$, where $\tilde{A}_{0.5}$ is the 0.5-cut of $\tilde{A}$.

Some properties concerning the nearest ordinary sets $N(\tilde{A})$ and $N(\tilde{B})$ are summarized blew:

① $N(\tilde{A} \cap \tilde{B}) = N(\tilde{A}) \cap N(\tilde{B})$;

② $N(\tilde{A} \cup \tilde{B}) = N(\tilde{A}) \cup N(\tilde{B})$;

③ $|\mu_{\tilde{A}}(x) - \mu_{N(\tilde{A})}(x)| = \mu_{\tilde{A}-\tilde{A}^C}(x) = |\mu_{\tilde{A}}(x) - \mu_{\tilde{A}_{0.5}}(x)|$.

where for any set X, $|X|$ denotes the cardinality and X^C the complement of X respectively.

Definition 2.2 The index of fuzziness of a fuzzy set $\tilde{A}$ having n supporting points is defined as

$$v_p(\tilde{A}) = (2/n^p) \cdot d(\tilde{A}, N(\tilde{A}))$$

where $d(\tilde{A}, N(\tilde{A}))$ denotes the distance between the fuzzy set $\tilde{A}$ and its nearest ordinary set $N(\tilde{A})$. The value of p depends on the type of distance function used. e.g. $p = 1$ for a generalized Hamming distance whereas $p = 0.5$ for an Euclidean distance. When $p = 1$, $v_1(\tilde{A})$ is called the linear index of fuzziness of $\tilde{A}$, denoted by $v_l(\tilde{A})$. When $p = 0.5$, $v_{0.5}(\tilde{A})$ is called the quadratic index of fuzziness of $\tilde{A}$, denoted by $v_q(\tilde{A})$.

Next, we will review some basic concepts of Pawlak's rough set in brief which are used in next work.

Let U be a nonempty set and R be an indiscernibility relation or equivalence relation on U. Then (U, R) is called a Pawlak approximation space.

Definition 2.3 For any non-empty subset X of U, the sets

$$\underline{R}(X) = \{x \in U | [x]_R \subseteq X\}$$

and

$$\overline{R}(X) = \{x \in U | [x]_R \cap X \neq \phi\}$$

are, respectively, called the lower and upper approximation of X in (U, R), where $[x]_R$ denotes the equivalence class of the relation R containing the element x.

Using lower and upper approximation, an equivalence relation $\approx_R$ can be defined on the power set of U:

$X \approx_R Y \Leftrightarrow \underline{R}(X) = \underline{R}(Y)$ and $\overline{R}(X) = \overline{R}(Y)$

where $X, Y \in 2^U$, and R is a covering of U.

In addition, this equivalence relation induces a partition on the power set 2^U. Pawlak regards the group of subsets of U with the same upper and lower approximations in (U, R). Moreover specially, the concept is:

Definition 2.4 Given the Pawlak approximation space (U, R) and two sets $A_1, A_2 \in U$, with $A_1 \subseteq A_2$, a Pawlak rough set is the family of subset of U described as follows:

$$R(X) = (A_1, A_2) = \{X \in 2^U | \underline{R}(X) = A_1, \overline{R}(X) = A_2\}$$

Equivalently, a Pawlak rough set containing $X \in 2^U$ can be defined as:

$$[X]_{\approx_R} = \{Y \in 2^U | \underline{R}(X) = \underline{R}(Y), \overline{R}(X) = \overline{R}(Y)\}$$

In other words,

$$[X]_{\approx_R} = (\underline{R}(X), \overline{R}(X))$$

If $\underline{R}(X) = \overline{R}(X)$, X is said to be exact.

For a fixed non-empty subset X of U, the rough set of X i.e. $R(X)$ is unique. Moreover, there are many properties about rough sets, we omitted them for simplification here, and which can be found in [5-7].

3 CONCEPTS AND PROPERTIES OF COVERING GENERALIZED ROUGH SETS

In this section we will list some definitions and results about covering rough sets used in this paper [1,3,9].

Definition 3.1 Let U be a universe of discourse, $\mathcal{C}$ a family of subsets of U. If none subsets in $\mathcal{C}$ is empty, and $\cup\mathcal{C} = U$, $\mathcal{C}$ is called a covering of U.

It is clear that a partition of U is certainly a covering of U, so the concept of a covering is an extension of the concept of a partition.

In the following discussion, the universe of discourse U is considered to be finite.

Definition 3.2 Let U be the universe, and $\mathcal{C}$ a covering of U. We call the ordered pair $S = (U, \mathcal{C})$ a covering approximation space.

Definition 3.3 Let $S = (U, \mathcal{C})$ be a covering approximation space. For $u \in U$, set family

$$Md(u) = \{K \in \mathcal{C} | u \in K \wedge (\forall H \in \mathcal{C} \wedge u \in H \wedge H \subseteq K \Rightarrow K = H)\}$$

is called the minimal description of u.

Consider a covering approximation space $S = (U, \mathcal{C})$. Similarly to Pawlak approximation space, we interpret the covering as a collection of known concepts. Every set $X \subseteq U$ is a set of examples of some concept (know or un known). Hence to calculate the lower approximation of X, we must find the family of known concepts included in X. To calculate the upper approximation of X, we must find known concepts having at least one example in X that is not an example of another known concept included in X.

Definition 3.4 For a set $X \subseteq U$, set family $\underline{S\mathcal{C}}(X) = \{K \in \mathcal{C} | K \subseteq X\}$ is called the covering lower approximation set family of X.

Set $\underline{\mathcal{C}}(X) = \cup\underline{S\mathcal{C}}(X)$ is called the covering lower approximation of X.

Set family $Bn(X) = \{Md(x) | x \in X - \underline{\mathcal{C}}(X)\}$ is called the covering boundary approximation set family of X.

Set family $\overline{S\mathcal{C}}(X) = \underline{S\mathcal{C}}(X) \cup Bn(X)$ is called the covering upper approximation set family of X.

Set $\overline{\mathcal{C}}(X) = \cup\overline{S\mathcal{C}}(X)$ is called the covering upper approximation of X.

Using covering lower and upper approximation, an equivalence relation $\approx_{\mathcal{C}}$ can be defined on the power set of U:

$X \approx_{\mathcal{C}} Y \Leftrightarrow \underline{\mathcal{C}}(X) = \underline{\mathcal{C}}(Y)$ and $\overline{\mathcal{C}}(X) = \overline{\mathcal{C}}(Y)$

where $X, Y \in 2^U$, and $\mathcal{C}$ is a covering of U.

In addition, this equivalence relation induces a partition on the power set 2^U. An equivalence class of such partition is called a covering generalized rough set. Moreover specially, the concept can be defined as:

Definition 3.5 Given the covering approximation space $S = (U, \mathcal{C})$ and two sets $A_1, A_2 \in U$, with $A_1 \subseteq A_2$, a covering generalized rough set is the family of subset of U described as follows:

$$\mathcal{C}(X) = (A_1, A_2) = \{X \in 2^U | \underline{\mathcal{C}}(X) = A_1, \overline{\mathcal{C}}(X) = A_2\}$$

Equivalently, a covering generalized rough set containing $X \in 2^U$ can be defined as:

$$[X]_{\approx_{\mathcal{C}}} = \{Y \in 2^U | \underline{\mathcal{C}}(X) = \underline{\mathcal{C}}(Y), \overline{\mathcal{C}}(X) = \overline{\mathcal{C}}(Y)\}$$

In other words,

$$[X]_{\approx_{\mathcal{C}}} = (\underline{\mathcal{C}}(X), \overline{\mathcal{C}}(X))$$

If $\underline{\mathcal{C}}(X) = \overline{\mathcal{C}}(X)$, X is said to be exact.

From the flowing Proposition, we can find that a covering generalized rough set will become a classical rough set when the covering is restricted a partition.

Proposition 3.1 *If $\mathcal{C}$ is a partition, $\underline{\mathcal{C}}(X)$ and $\overline{\mathcal{C}}(X)$ are the Pawlak's lower and upper approximations of X.*

Corresponding the properties of Powlak's rough set, covering generalized rough sets have the following results.

Proposition 3.2 *For a covering $\mathcal{C}$, the covering lower and upper approximations have the following properties:*

① $\underline{\mathcal{C}}(U) = U$; $\overline{\mathcal{C}} = U$

② $\underline{\mathcal{C}}(\phi)$; $\overline{\mathcal{C}} = \phi$

③ $\underline{\mathcal{C}}(X) \subseteq X$; $X \subseteq \overline{\mathcal{C}}(X)$

④ $\underline{\mathcal{C}}(\underline{\mathcal{C}}(X)) = \underline{\mathcal{C}}(X)$; $\overline{\mathcal{C}}(\overline{\mathcal{C}}(X)) = \overline{\mathcal{C}}(X)$

⑤ $X \subseteq Y \Rightarrow \underline{\mathcal{C}}(X) \subseteq \underline{\mathcal{C}}(Y)$

⑥ $\forall K \in \mathcal{C}, \mathcal{C}(K) = K, \overline{\mathcal{C}}(K) = K$

4 FUZZINESS IN COVERING GENERALIZED ROUGH SETS

Let $S = (U, \mathcal{C})$ be a covering approximation space and suppose $X \subseteq U$. In the covering $\mathcal{C}$, the rough set of X is say $\mathcal{C}(X) = (\underline{\mathcal{C}}(X), \overline{\mathcal{C}}(X))$. Thus in the space $S = (U, \mathcal{C})$, the set X is approximated by two approximations, one from the inner side called the lower approximation of X, and another from the outer side called the upper approximation of X.

Definition 4.1 For an element $u \in U$ and the rough set of X, $\mathcal{C}(X) = (\underline{\mathcal{C}}(X), \overline{\mathcal{C}}(X))$, degree of rough belongingness of u in X, denoted by $D(u, X)$, is defined by

$$D(u, X) = \frac{|(\cup Md(u)) \cap X|}{|\cup Md(u)|}$$

Clearly, $\forall u \in U, D(u, X) \in [0, 1]$. Hence, this immediately induces a fuzzy set $\widetilde{F_X^{\mathcal{C}}}$ of U and membership function of $\widetilde{F_X^{\mathcal{C}}}$ is given by

$$\mu_{\widetilde{F_X^{\mathcal{C}}}}(u) = \frac{|(\cup Md(u)) \cap X|}{|\cup Md(u)|}$$

Definition 4.2 The fuzziness in the rough set $\mathcal{C}(X)$ of X is denoted by $f_X^{\mathcal{C}}$ and is defined by the amount of fuzziness present in the fuzzy set F_X^{C}. The amount of fuzziness can be measured by a suitable index of fuzziness (linear or quadratic). The linear and the quadratic indices of fuzziness of the rough set $\widetilde{F_X^{\mathcal{C}}}$ are respectively called the linear fuzziness and the quadratic fuzziness of rough set $\mathcal{C}(X)$. They are denoted by $(f_X^{\mathcal{C}})_l$ and $(f_X^{\mathcal{C}})_q$, respectively.

Example 4.1 Given an information table T based on an dominance relation in Tab.1.

Tab. 1

$U \times A$	a_1	a_2	a_3
x_1	1	2	1
x_2	3	2	2
x_3	1	1	2
x_4	2	1	3
x_5	3	3	2
x_6	3	2	3

Here, $U = \{x_i; i = 1, 2, \cdots, 6\}$ is a non-empty finite set of objects and $A = \{a_1, a_2, a_3\}$ denotes the set of attributes. And dominance relation $R_A^{\leqslant}$ of information table T is defined by

$$R_A^{\leqslant} = \{(x_i, x_j) \in U \times U : f_l(x_i) \leqslant f_l(x_j), \forall a_l \in A\}$$

where $f_l(x)$ is the value of a_l on $x \in U$. Moreover if we denote

$$\begin{aligned} [x_i]_A^{\leqslant} &= \{x_j \in U : (x_i, x_j) \in R_A^{\leqslant}\} \\ &= \{x_j \in U : f_l(x_i) \leqslant f_l(x_j), \forall a_l \in A\} \end{aligned}$$

then from Tab.1 we can see

$$\begin{aligned} \mathcal{C}_1 &= [x_1]_A^{\leqslant} = \{x_1, x_2, x_5, x_6\} \\ \mathcal{C}_2 &= [x_2]_A^{\leqslant} = \{x_2, x_5, x_6\} \\ \mathcal{C}_3 &= [x_3]_A^{\leqslant} = \{x_2, x_3, x_4, x_5, x_6\} \\ \mathcal{C}_4 &= [x_4]_A^{\leqslant} = \{x_4, x_6\} \\ \mathcal{C}_5 &= [x_5]_A^{\leqslant} = \{x_5\} \\ \mathcal{C}_6 &= [x_6]_A^{\leqslant} = \{x_6\} \end{aligned}$$

and $\mathcal{C}_i \neq \phi; \cup \mathcal{C}_i = U, i = 1, 2, \cdots, 6$. So $\mathcal{C}$ is a covering of U and $S = (U, \mathcal{C})$ is a covering approximation space. In addition, the figure of the covering of S in Table 1 is following. Moreover, we can know: $Md(x_i) = \mathcal{C}_i, (i = 1, 2, \cdots, 6)$ by term of the following figure(Fig.1).

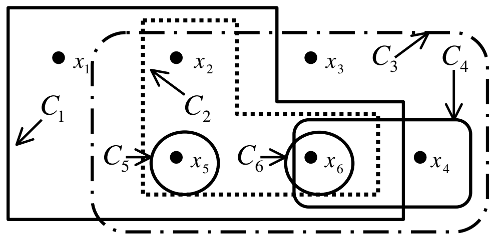

Fig.1 Covering of S in Tab.1

Now let's consider a subset $X = \{x_1, x_3, x_4, x_5\}$ of U. It is easy to obtain $\underline{SC}(X) = \mathcal{C}_5$. So we have $\underline{\mathcal{C}} = \mathcal{C}_5$. On the other hand, we can calculate $Bn(X) = \{Md(x_1), Md(x_3), Md(x_4)\} = \{x_1, x_2, x_3, x_4, x_5, x_6\} = U$. That is to say $\overline{\mathcal{C}} = U$. Therefore the rough set of X is $\mathcal{C}(X) = (\mathcal{C}_5, U)$.

By above definitions we can obtain

$$\mu_{\widetilde{F_X^{\mathcal{C}}}}(x_1) = 1/2, \quad \mu_{\widetilde{F_X^{\mathcal{C}}}}(x_2) = 1/3, \quad \mu_{\widetilde{F_X^{\mathcal{C}}}}(x_3) = 3/5$$
$$\mu_{\widetilde{F_X^{\mathcal{C}}}}(x_4) = 1/2, \quad \mu_{\widetilde{F_X^{\mathcal{C}}}}(x_5) = 1, \quad \mu_{\widetilde{F_X^{\mathcal{C}}}}(x_6) = 0$$

Hence,

$$\widetilde{F_X^{\mathcal{C}}} = \left\{\frac{1}{2}/x_1, \frac{1}{3}/x_2, \frac{3}{5}/x_3, \frac{1}{2}/x_4, 1/x_5, 0/x_6\right\}$$

In another way we know

$$N(\widetilde{F_X^{\mathcal{C}}}) = \{0/x_1, 0/x_2, 1/x_3, 0/x_4, 1/x_5, 0/x_6\}$$

Thus the linear fuzziness in the rough set $\mathcal{C}(X)$ is

$$\begin{aligned} (f_X^{\mathcal{C}})_l &= (2/6) \cdot d(\widetilde{F_X^{\mathcal{C}}}, N(\widetilde{F_X^{\mathcal{C}}})) \\ &= \frac{1}{3} \cdot (\frac{1}{2} + \frac{1}{3} + \frac{2}{5} + \frac{1}{2}) \\ &= 0.5778 \end{aligned}$$

where $d(\widetilde{F_X^{\mathcal{C}}}, N(\widetilde{F_X^{\mathcal{C}}}))$ denotes the Hamming distance between them.

And the quadratic fuzziness in the rough set $\mathcal{C}(X)$ is

$$\begin{aligned} (f_X^{\mathcal{C}})_q &= (2/\sqrt{6}) \cdot d(\widetilde{F_X^{\mathcal{C}}}, N(\widetilde{F_X^{\mathcal{C}}})) \\ &= (2/\sqrt{6}) \cdot \sqrt{\frac{1}{4} + \frac{1}{9} + \frac{4}{25} + \frac{1}{4}} \\ &= 0.62960 \end{aligned}$$

where $d(\widetilde{F^{\mathcal{C}}_X}, N(\widetilde{F^{\mathcal{C}}_X}))$ denotes the Euclidean distance between them.

Next, we will discuss some properties of the measure of fuzziness in covering generalized rough set.

Proposition 4.1 *Let $S = (U, \mathcal{C})$ be a covering approximation space. The following always holds:*

① $(f^{\mathcal{C}}_U)_l = 0$;
② $(f^{\mathcal{C}}_\phi)_l = 0$;
③ $(f^{\mathcal{C}}_U)_q = 0$;
④ $(f^{\mathcal{C}}_\phi)_q = 0$.

Proof Here we prove only (1). Proofs of the rest are similar and we omitted them.

For any $u \in U$, we have

$$\mu_{\widetilde{F^{\mathcal{C}}_U}}(u) = \frac{|(\cup Md(u)) \cap U|}{|\cup Md(u)|} = \frac{|\cup Md(u)|}{|\cup Md(u)|} = 1$$

Therefore, $\mu_{\widetilde{F^{\mathcal{C}}_U} \cap (\widetilde{F^{\mathcal{C}}_U})^C}(u) = 0$, where $(\widetilde{F^{\mathcal{C}}_U})^C$ denotes the complement of the fuzzy set $\widetilde{F^{\mathcal{C}}_U}$. Hence, $(f^{\mathcal{C}}_U)_l = (2/n) \cdot d(\widetilde{F^{\mathcal{C}}_U}, N(\widetilde{F^{\mathcal{C}}_U})) = (2/n) \cdot \sum |\mu_{\widetilde{F^{\mathcal{C}}_U}}(u) - \mu_{N(\widetilde{F^{\mathcal{C}}_U})}(u)| = (2/n) \cdot \sum \mu_{\widetilde{F^{\mathcal{C}}_U} \cap (\widetilde{F^{\mathcal{C}}_U})^C}(u) = 0$.

Thus the proof is completed.

Proposition 4.2 *The fuzziness in an exact set of a covering approximation space is 0.*

Proof Let $\mathcal{C}(X) = (X, X)$ be an exact set of a covering approximation $S = (U, \mathcal{C})$. Then, for any $u \in U$ we have

$$\mu_{\widetilde{F^{\mathcal{C}}_X}}(u) = \frac{|(\cup Md(u)) \cap X|}{|\cup Md(u)|} = \frac{|\cup Md(u)|}{|\cup Md(u)|} = 1$$

And for each $u \in U - X$, $Md(u) \cap X = \phi$. Therefore for any $u \in U - X$,$\mu_{\widetilde{F^{\mathcal{C}}_X}}(u) = 0$. Hence we have $f^{\mathcal{C}}_X = (2/n^p) \cdot d(\widetilde{F^{\mathcal{C}}_X}, N(\widetilde{F^{\mathcal{C}}_X})) = (2/n^p) \cdot \sum \mu_{\widetilde{F^{\mathcal{C}}_X} \cap (\widetilde{F^{\mathcal{C}}_X})^C}(u) = 0$.

The proof is completed.

Proposition 4.3 *For any two sets X and Y in a covering approximation space $S = (U, \mathcal{C})$, if $X \subseteq Y$, then $\widetilde{F^{\mathcal{C}}_X} \subseteq \widetilde{F^{\mathcal{C}}_Y}$.*

Proof Obviously, for any $u \in U$, $X \subseteq Y$ implies $|(\cup Md(u)) \cap X| \leqslant |(\cup Md(u)) \cap Y|$. So we have

$$\frac{|(\cup Md(u)) \cap X|}{|\cup Md(u)|} \leqslant \frac{|(\cup Md(u)) \cap Y|}{|\cup Md(u)|}$$

That is to say $\mu_{\widetilde{F^{\mathcal{C}}_X}}(u) \leqslant \mu_{\widetilde{F^{\mathcal{C}}_Y}}(u)$. Thus $\widetilde{F^{\mathcal{C}}_X} \subseteq \widetilde{F^{\mathcal{C}}_Y}$.

The proposition is proved.

Proposition 4.4 *For any set X in a covering approximation space $S = (U, \mathcal{C})$, $(\widetilde{F^{\mathcal{C}}_X})^C = \widetilde{F^{\mathcal{C}}_{X^C}}$ holds.*

Proof For any $u \in U$, we have

$$\begin{aligned} & \mu_{\widetilde{F^{\mathcal{C}}_X}}(u) + \mu_{\widetilde{F^{\mathcal{C}}_{X^C}}}(u) \\ = & \frac{|(\cup Md(u)) \cap X| + |(\cup Md(u)) \cap X^C|}{|\cup Md(u)|} \\ = & \frac{|\cup Md(u)|}{|\cup Md(u)|} = 1 \end{aligned}$$

Hence, it is proved.

Proposition 4.5 *For any two sets X and Y in a covering approximation space $S = (U, \mathcal{C})$, the following holds:*

① $\widetilde{F^{\mathcal{C}}_{X \cup Y}} \supseteq \widetilde{F^{\mathcal{C}}_X} \cup \widetilde{F^{\mathcal{C}}_Y}$;
② $\widetilde{F^{\mathcal{C}}_{X \cup Y}} = \widetilde{F^{\mathcal{C}}_X} \cup \widetilde{F^{\mathcal{C}}_Y}$, *if either $X \subseteq Y$ or $Y \subseteq X$.*

Proof ① For any $u \in U$, we have

$$\begin{aligned} \mu_{\widetilde{F^{\mathcal{C}}_{X \cup Y}}}(u) &= \frac{|(\cup Md(u)) \cap (X \cup Y)|}{|\cup Md(u)|} \\ &= \frac{|((\cup Md(u)) \cap X) \cup ((\cup Md(u)) \cap Y)|}{|\cup Md(u)|} \\ &\geqslant \frac{\max\{|((\cup Md(u)) \cap X)|, |((\cup Md(u)) \cap Y)|\}}{|\cup Md(u)|} \\ &= \max\{\frac{|((\cup Md(u)) \cap X)|}{|\cup Md(u)|}, \frac{|((\cup Md(u)) \cap Y)|}{|\cup Md(u)|}\} \\ &= \max\{\mu_{\widetilde{F^{\mathcal{C}}_X}}(u), \mu_{\widetilde{F^{\mathcal{C}}_Y}}(u)\} \\ &= \mu_{\widetilde{F^{\mathcal{C}}_X} \cup \widetilde{F^{\mathcal{C}}_Y}}(u). \end{aligned}$$

Thus completed.

② Straightforward.

In a similar way, the following proposition can be obtained.

Proposition 4.6 *For any two sets X and Y in a covering approximation space $S = (U, \mathcal{C})$, the following holds:*

① $\widetilde{F^{\mathcal{C}}_{X \cap Y}} \subseteq \widetilde{F^{\mathcal{C}}_X} \cap \widetilde{F^{\mathcal{C}}_Y}$;
② $\widetilde{F^{\mathcal{C}}_{X \cap Y}} = \widetilde{F^{\mathcal{C}}_X} \cap \widetilde{F^{\mathcal{C}}_Y}$, *if either $X \subseteq Y$ or $Y \subseteq X$.*

5 CONCLUSIONS

It is well-known that rough set theory has been regarded as a generalization of the classical set theory in one way. Furthermore, this is an important mathematical tool to deal with vagueness. As a natural need, it is a fruitful way to combine fuzzy sets and rough sets by defining rough fuzzy sets and fuzzy rough sets. In addition, it is necessary to combine fuzzy sets and covering generalized rough sets. In the paper, a measure of fuzziness in covering generalized rough set is introduced, and some basic properties is considered. Moreover, some characterization of this measure are made with examples.

REFERENCES

[1] Bonikowski Z, Bryniarski E, Wybraniex U. Extensions and intentions in the rough set theory[J]. Inform. Sci.,1998, 107:149-167.

[2] Bonikowski Z. Algebraic structures of rough sets[M]. Ziarko W ed. Rough Sets, Fuzzy Sets and Knowledge Discovery. London: Springer-Verlag, 1994:243-247.

[3] Bryniaski E. Acalulus of rough sets of the first order[J]. Bull. Pol. Acad. Sci.,1998, 16:71-77.

[4] Cattaneo G. Abatract approximation spaces for tough theories[M]. Polkowski, Skoeron eds. Rough Sets in Knowledge Discovery 1: Methodolgy and applications. Heidelberg: Physica-Verlag, 1998:59-98.

[5] Pawlak Z. Rough sets basic notions[R]. ICS PAS Rep., 1981, 436.

[6] Pawlak Z. Rough sets power set hierarchy[R]. ICS PAS Rep., 1982, 470.

[7] Pawlak Z. Rough sets[J]. International Journal of Computer and Information Sciences, 1982, 11:341-356.

[8] Pawlak Z. Rough Sets, Theoretical Aspects of Reasoning About Data[M]. Boston: Kluwer Academic Publishers, 1991.

[9] Pomykala J A. Approximaion opearions in approximation space[J]. Bull. Pol. Acad. Sci., 1987, 9/10:653-662.
[10] Skowron A, Stepaniuk J. Tolerance approxximation space[J]. Fundam. Inform., 1996, 27:245-253.
[11] Slowinski R, Vanderpooten D. A generalized definition of rough approximations based on similarity[J]. IEEE Trans. Data knowledge Eng., 2000, 2:331-336.
[12] Wasilewska A. Topological rough algebras[M]. Lin Cercone eds. Rough Sets & data Mining. Boston: Kuwer Academic Publishers, 1997:441-425.
[13] Wang F. Modeling, analysis and synthesis of linguistic dynamic systems: a computaional theory[C]// Proceedings of IEEE International Workshop on Architecture for Semiotic Modeling and situation Control in Large Complex Systems, Monterery, CA, 27-30 August 1995:173-178.
[14] Wagn F. Outline of a computational theory for lingusitic dynamic systems: toward computing with words[J]. Int. J. Intell. contr. Syst., 1998, 2:211-224.
[15] Yao Y Y. Relational interpretations of neighborhood operators and rough set approximation operators[J]. Inform. Sci., 1998, 101:239-259.
[16] Yao Y Y. Constructive and algebraic methods of theory of rough sets[J]. Inform. Sci., 1998, 109:21-47.
[17] Zadeh L A. Fuzzy sets[J]. Inform. contr., 1965, 89:338-353.
[18] Zadeh L A. The concept of a linguistic variable and its application to approximate reasoning-I, II, III[J]. Inform. Sci., 1975, 8(9):199-249, 301-357, 43-80.
[19] Zadeh L A. Fuzzy logic–computing with words[J]. IEEE Trans. Fuzzy Syst., 1996, 4:103-111.
[20] Zakowski W. Approximations in the soace[J]. Demonstratio Eathematica., 1983, 16:761-769.
[21] Dubios D, Prade H. Rough fuzzy sets and fuzzy rough sets[J]. Int. J. General systems., 1990, 17:191-208.
[22] Banerjee M, Pal K K. Roughness of a fuzzy set[J]. Information science, 1995, 93:235-246.
[23] Biswas R. On rough sets and fuzzy rough sets[J]. Bull. Pol. Acad. Sci. Math., 1992, 42:345-349.
[24] Biswas R. On rough fuzzy sets[J]. Bull. Pil. Acad. Sci. Math., 1994, 42:352-355.
[25] Nakamura A. Fuzzy rough sets[J]. Note on Multiple-valued Logic in Japan, 1998, 9(8):1-8.
[26] Nanda S. Fuzzy rough sets[J]. Fuzzy sets and Systems, 1992, 45:157-160.
[27] Xu Wei-hua, Zhang Wen-xiu. Measuring roughness of generalized rough sets induced by a covering[J]. Fuzzy Sets and Systems, 2007, doi:10.1016/j.fss.2007.03.018.