KNOWLEDGE REDUCTIONS IN INCONSISTENT INFORMATION SYSTEMS BASED ON DOMINANCE RELATIONS *

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ABSTRACT: Knowledge reduction is one of the most important problems in rough set theory. However, most of information systems are not only inconsistent, but also based on dominance relations because of various factors. To acquire brief decision rules from inconsistent systems based on dominance relations, knowledge reductions are needed. The main aim of this paper is to study the problem. The assignment reduction and approximation reduction are introduced in inconsistent systems based on dominance relations and relationship between them are examined. The judgment theorem and discernibility matrix are obtained, from which we can provide an approach to knowledge reductions in inconsistent systems based on dominance relation.

Keywords: Rough set; Consistent set; Inconsistent system; Knowledge reduction; Discernibility matrix

1 INTRODUCTION

The rough set theory, proposed by Pawlak in the early 1980s[1], is an extension of set theory for the study of intelligent systems characterized by inexact, uncertain or vague information and can serve as a new mathematica tool to soft computing. This theory has been applied successfully in machine learning, patten recognition, decision support systems, expert systems, data analysis, data mining, and so on. Since its introduction, the theory has generated a great deal of interest among more and more researchers.

Knowledge reduction is one of the hot research topics of rough set theory. Much study on this area had been reported and many useful results were obtained until now[2-8]. However, most work was based on consistent information systems, and the main methodology has been developed under equivalence relations which are often called indiscernibility relations. In practise, most of information systems are not only inconsistent, but also based on dominance relations because of various factors. In order to obtain the succinct decision rules from them by using rough set method, knowledge reductions are needed. In recent years, more and more attention has been paid to research of rough set . Many types of knowledge reductions have been proposed in the area of rough sets[9-14]. But useful results of knowledge reductions are very poor in inconsistent information systems based on dominance relations until now. The main aim of the paper is to discuss the problem.

In present paper, we are concerned with approaches to knowledge reductions in inconsistent information systems based dominance relations.The assignment reduction and approximation reduction are introduced in inconsistent systems based on dominance relations, and relationship between them are examined. The judgment theorem and discernibility matrix are obtained, from which we can provide an approach to knowledge reductions in inconsistent systems based on dominance relations. Finally we conclude the paper with a summary and out look for further research.

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2 PRELIMINARIES

This section recalls necessary concepts of rough sets. Detailed description of the theory can be found in [12].

Definition 1 *An ordered quadruple $S = (U, A \cup D, F, G)$ is referred to as an information system with decisions, where*

$U = \{x_1, x_2, \cdots, x_n\}$ is a non-empty finite set of objects;
$A \cup D$ is a non-empty finite attributes set;
$A = \{a_1, a_2, \cdots, a_p\}$ denotes the set of condition attributes;
$D = \{d_1, d_2, \cdots, d_q\}$ denotes the set of decision attributes, and $A \cap D = \phi$;
$F = \{f_k : U \to V_k, k \le p\}$, $f_k(x)$ is the value of a_k on $x \in U$, V_k is the domain of $a_k, a_k \in A$;
$G = \{g_{k'} : U \to V_{k'}, k' \le q\}$, $g_{k'}(x)$ is the value of $d_{k'}$ on $x \in U$, $V_{k'}$ is the domain of $d_{k'}, d_{k'} \in D$.

Pawlak approximation spaces, for an information system $S = (U, A \cup D, F, G)$ and $B \subseteq A$, derive from an equivalence relation (indiscernibility relation). However,there exist many systems which are not based on equivalence relations, but dominance relations in practise. Thus it is necessary to discuss systems based on dominance relations.

Definition 2 *Let $S = (U, A \cup D, F, G)$ be an information system with decisions, for $B \subseteq A$, denote*

$$R_B^{\le} = \{(x_i, x_j) \in U \times U : f_l(x_i) \le f_l(x_j), \forall a_l \in B\};$$
$$R_D^{\le} = \{(x_i, x_j) \in U \times U : g_m(x_i) \le g_m(x_j), \forall d_m \in D\};$$

$R_B^{\le}$ and $R_D^{\le}$ are called dominance relations of information system S. Moreover, we denote information system with decisions based on dominance relations by $S^{\le}$.

If we denote

$$[x_i]_B^{\le} = \{x_j \in U : (x_i, x_j) \in R_B^{\le}\} = \{x_j \in U : f_l(x_i) \le f_l(x_j), \forall a_l \in B\};$$

$[x_i]_D^{\leqq} = \{x_j \in U : (x_i,x_j) \in R_D^{\leqq}\} = \{x_j \in U : g_m(x_i) \leq g_m(x_j), \forall d_m \in D\}$;

then the following properties of a dominance relation are trivial.

Proposition 1 *Let $R_B^{\leqq}$ be a dominance relation.*
(1) *$R_B^{\leqq}$ is reflexive and transitive, but not symmetric, so it isn't an equivalence relation generally.*
(2) *If $B_1 \subseteq B_2 \subseteq A$, then $R_A^{\leqq} \subseteq R_{B_2}^{\leqq} \subseteq R_{B_1}^{\leqq}$.*
(3) *If $B_1 \subseteq B_2 \subseteq A$, then $[x_i]_A^{\leqq} \subseteq [x_i]_{B_2}^{\leqq} \subseteq [x_i]_{B_1}^{\leqq}$*
(4) *If $x_j \in [x_i]_B^{\leqq}$, then $[x_j]_B^{\leqq} \subseteq [x_i]_B^{\leqq}$.*

For any subset X of U, define

$$\underline{R_B^{\leqq}}(X) = \{x_i \in U : [x_i]_B^{\leqq} \subseteq X\},$$

$$\overline{R_B^{\leqq}}(X) = \{x_i \in U : [x_i]_B^{\leqq} \cap X \neq \phi\},$$

$\underline{R_B^{\leqq}}(X)$ and $\overline{R_B^{\leqq}}(x)$ are said to be the lower and upper approximation of X with respect to a dominance relation $R_B^{\leqq}$. And the approximations have also some properties which are similar to those of Pawlak approximation spaces. Detail description can be found in [12].

Definition 3 *For an information system with decisions $S^{\leq} = (U, A \cup D, F, G)$, if $R_A^{\leqq} \subseteq R_D^{\leqq}$, then this information system is consistent, otherwise, this information system is inconsistent.*

Example 1 Given an information system with decisions based on dominance relations in Table 1.

$U \times (A \cup D)$	a_1	a_2	a_3	d
x_1	1	2	1	3
x_2	3	2	2	2
x_3	1	1	2	1
x_4	2	1	3	2
x_5	3	3	2	3
x_6	3	2	3	1

Table 1

From Table 1, we can see

$$[x_1]_A^{\leqq} = \{x_1,x_2,x_5,x_6\}; [x_2]_A^{\leqq} = \{x_2,x_5,x_6\};$$
$$[x_3]_A^{\leqq} = \{x_2,x_3,x_4,x_5,x_6\}; [x_4]_A^{\leqq} = \{x_4,x_6\};$$
$$[x_5]_A^{\leqq} = \{x_5\}; [x_6]_A^{\leqq} = \{x_6\};$$
$$[x_1]_d^{\leqq} = [x_5]_d^{\leqq} = \{x_1,x_5\}; [x_2]_d^{\leqq} = [x_4]_d^{\leqq} = \{x_1,x_2,x_4,x_5\};$$
$$[x_3]_d^{\leqq} = [x_6]_d^{\leqq} = \{x_1,x_2,x_3,x_4,x_5,x_6\}$$

Obviously, we have $R_A^{\leqq} \nsubseteq R_d^{\leqq}$, so the information system in Table 1 is inconsistent.

By the way, information systems with decisions in following text are based on dominance relations for simple description .

3 THEORIES OF KNOWLEDGE REDUCTIONS IN INCONSISTENT INFORMATION SYSTEMS

Let $S^{\leq} = (U, A \cup D, F, G)$ be an information system with decisions , and $R_B^{\leqq}, R_D^{\leqq}$ be dominance relations derived from condition attributes set A and decision attributes set D respectively, For $B \subseteq A$, denote

$$U/R_B^{\leqq} = \{[x_i]_B^{\leqq} : x_i \in U\},$$

$$U/R_D^{\leqq} = \{D_1, D_2, \cdots, D_r\},$$

$$\sigma_B^{\leqq}(x) = \{D_j : D_j \cap [x]_B^{\leqq} \neq \phi, x \in U\},$$

$$\eta_B^{\leqq} = \frac{1}{U} \sum_{j=1}^{r} |\overline{R_B^{\leqq}}(D_j)|,$$

where $[x]_B^{\leqq} = \{y \in U : (x,y) \in R_B^{\leqq}\}$.

From the above, we can have the following propositions immediately.

Proposition 2 (1) $\overline{R_B^{\leqq}}(D_j) = \cup\{[x]_B^{\leqq} : D_j \in \sigma_B^{\leqq}(x)\}$.
(2) If $B \subseteq A$, then $\sigma_A^{\leqq}(x) \subseteq \sigma_B^{\leqq}(x), \forall x \in U$.
(3) If $[x]_B^{\leqq} \supseteq [y]_B^{\leqq}$, then $\sigma_B^{\leqq}(x) \supseteq \sigma_B^{\leqq}(y), \forall x,y \in U$.

Definition 4 *Let $S^{\leq} = (U, A \cup D, F, G)$ be an information system with decisions.*

(1) *If $\sigma_B^{\leqq}(x) = \sigma_A^{\leqq}(x)$, for all $x \in U$, we say that B is an assignment consistent set of S. If B is an assignment consistent set, and no proper subset of B is assignment consistent set, then B is called an assignment consistent reduct of $S^{\leq}$.*

(2) *If $\eta_B^{\leqq}(x) = \eta_A^{\leqq}(x)$, for all $x \in U$, we say that B is an approximation consistent set of S. If B is an approximation consistent set, and no proper subset of B is approximation consistent set, then B is called an approximation consistent reduct of $S^{\leq}$.*

An assignment consistent set is a subset of attributes set that preserves the possible decisions of every object. And an approximation consistent set preserves the upper approximation of every decision class.

Example 2 Consider the inconsistent information system with decisions in Table 1.

For the information system with decisions in Table 1, we denote $D_1 = [x_1]_d^{\leqq} = [x_5]_d^{\leqq}, D_2 = [x_2]_d^{\leqq} = [x_4]_d^{\leqq}, D_3 = [x_3]_d^{\leqq} = [x_6]_d^{\leqq}$, we can observe that

$$\sigma_A^{\leqq}(x_1) = \sigma_A^{\leqq}(x_2) = \sigma_A^{\leqq}(x_3) = \sigma_A^{\leqq}(x_5) = \{D_1, D_2, D_3\}$$
$$\sigma_A^{\leqq}(x_4) = \{D_2, D_3\}, \sigma_A^{\leqq}(x_6) = \{D_3\}$$

When $B = \{a_2, a_3\}$, it can be easily checked that $[x]_A^{\leqq} = [x]_B^{\leqq}$, for all $x \in U$. So that $\sigma_B^{\leqq}(x) = \sigma_A^{\leqq}(x)$ is true, and $B = \{a_2, a_3\}$ is an assignment consistent set of $S^{\leq}$. Furthermore, we can examine that $\{a_2\}$ and $\{a_3\}$ are not consistent set of $S^{\leq}$. That is to say $B = \{a_2, a_3\}$ is an assignment consistent reduction of $S^{\leq}$.

When $B' = \{a_1, a_3\}$, we have

$$[x_1]_{B'}^{\leqq} = \{x_1,x_2,x_3,x_4,x_5,x_6\}; [x_2]_{B'}^{\leqq} = \{x_2,x_5,x_6\};$$
$$[x_3]_{B'}^{\leqq} = \{x_2,x_3,x_4,x_5,x_6\}; [x_4]_{B'}^{\leqq} = \{x_4,x_6\};$$
$$[x_5]_{B'}^{\leqq} = \{x_2,x_5,x_6\}; [x_6]_{B'}^{\leqq} = \{x_6\};$$

and

$$\sigma_{B'}^{\leqq}(x_1) = \sigma_{B'}^{\leqq}(x_2) = \sigma_{B'}^{\leqq}(x_3) = \sigma_{B'}^{\leqq}(x_4) = \{D_1, D_2, D_3\};$$
$$\sigma_{B'}^{\leqq}(x_4) = \{D_2, D_3\}; \sigma_{B'}^{\leqq}(x_6) = \{D_3\}$$

So for all $x \in U$ we have $\sigma_{B'}^{\leqq}(x) = \sigma_A^{\leqq}(x)$, and $B' = \{a_1, a_3\}$ is another assignment consistent set of $S^{\leq}$. Moreover, it can be easily calculated that a_1 is not an assignment consistent set of $S^{\leq}$. Hence $B' = \{a_1, a_3\}$ is another assignment reduct of

$S^{\leq}$. Furthermore, it can be easily verified that $\{a_1, a_2\}$ isn't an assignment consistent set of $S^{\leq}$.

Thus there exist only two assignment reduct of $S^{\leq}$ in the system of Table 1, which are $\{a_1, a_3\}$ and $\{a_2, a_3\}$.

Theorem 1 *Let $S^{\leq} = (U, A \cup D, F, G)$ be an information system with decisions, then $B \subseteq A$ is an assignment consistent set of $S^{\leq}$ if and only if B is an approximation consistent set of $S^{\leq}$.*

Proof. Assume that $B \subseteq A$ is an assignment consistent set of $S^{\leq}$, that is $\sigma_B^{\leqq}(x) = \sigma_A^{\leqq}(x)$ for all $x \in U$. By the definition, for $\forall j \leq r$ we have $x \in \overline{R_B^{\leqq}}(D_j) \Leftrightarrow [x]_B^{\leqq} \cap D_j \neq \phi \Leftrightarrow D_j \in \sigma_B^{\leqq}(x) \Leftrightarrow D_j \in \sigma_A^{\leqq}(x) \Leftrightarrow [x]_A^{\leqq} \cap D_j \neq \phi \Leftrightarrow x \in \overline{R_A^{\leqq}}(D_j)$. So $\overline{R_B^{\leqq}}(D_j) = \overline{R_A^{\leqq}}(D_j)$, that is $\eta_B^{\leqq} = \eta_A^{\leqq}$. Hence B is an approximation consistent set of $S^{\leq}$.

Conversely, if B is an assignment consistent set of $S^{\leq}$, then $\eta_B^{\leqq} = \eta_A^{\leqq}$, which means

$$\sum_{j=1}^{r} |\overline{R_B^{\leqq}}(D_j)| = \sum_{j=1}^{r} |\overline{R_A^{\leqq}}(D_j)|.$$

On the other hand, since $\overline{R_B^{\leqq}}(D_j) \supseteq \overline{R_A^{\leqq}}(D_j)$ for $\forall j \leq r$, $\overline{R_B^{\leqq}}(D_j) = \overline{R_A^{\leqq}}(D_j)$ holds, and for all $x \in U$ we have $D_j \in \sigma_B^{\leqq}(x) \Leftrightarrow [x]_B^{\leqq} \cap D_j \neq \phi \Leftrightarrow x \in \overline{R_B^{\leqq}}(D_j) \Leftrightarrow x \in \overline{R_A^{\leqq}}(D_j) \Leftrightarrow [x]_A^{\leqq} \cap D_j \neq \phi \Leftrightarrow D_j \in \sigma_A^{\leqq}(x)$. Hence, $\sigma_B^{\leqq}(x) = \sigma_A^{\leqq}(x)$ is true for all $x \in U$, which means that B is an assignment consistent set of $S^{\leq}$.

Corollary 1 *Let $S^{\leq} = (U, A \cup D, F, G)$ be an information system with decisions, then $B \subseteq A$ is an assignment reduction of $S^{\leq}$ if and only if B is an approximation reduction of $S^{\leq}$.*

Theorem 2 *Let $S^{\leq} = (U, A \cup D, F, G)$ be an information system with decisions, then $B \subseteq A$ is an assignment consistent set of $S^{\leq}$ if and only if when $\sigma_A^{\leqq}(x) \cap \sigma_A^{\leqq}(y) \neq \sigma_A^{\leqq}(y)$, $[x]_B^{\leqq} \cap [y]_B^{\leqq} \neq [y]_B^{\leqq}$ holds for $x, y \in U$.*

Proof. Assume that when $\sigma_A^{\leqq}(x) \cap \sigma_A^{\leqq}(y) \neq \sigma_A^{\leqq}(y)$, $[x]_B^{\leqq} \cap [y]_B^{\leqq} \neq [y]_B^{\leqq}$ doesn't hold, that implies $[x]_B^{\leqq} \cap [y]_B^{\leqq} = [y]_B^{\leqq}$. So we have $[x]_B^{\leqq} \supseteq [y]_B^{\leqq}$, and $\sigma_B^{\leqq}(x) \supseteq \sigma_B^{\leqq}(y)$ can be obtained by Proposition 2(3). On the other hand, since B is an assignment consistent set of S, we have $\sigma_A^{\leqq}(x) \supseteq \sigma_A^{\leqq}(y)$, which is in contradiction with $\sigma_A^{\leqq}(x) \cap \sigma_A^{\leqq}(y) \neq \sigma_A^{\leqq}(y)$.

Conversely, we only prove $\sigma_B^{\leqq}(x) \subseteq \sigma_A^{\leqq}(x)$ by Proposition 2(2).

For all $x, y \in U$, if $\sigma_A^{\leqq}(x) \cap \sigma_A^{\leqq}(y) \neq \sigma_A^{\leqq}(y)$ implies $[x]_B^{\leqq} \cap [y]_B^{\leqq} \neq [y]_B^{\leqq}$, which means that $[x]_B^{\leqq} \cap [y]_B^{\leqq} = [y]_B^{\leqq}$ implies $\sigma_A^{\leqq}(x) \cap \sigma_A^{\leqq}(y) = \sigma_A^{\leqq}(y)$, and that is to say $[x]_B^{\leqq} \supseteq [y]_B^{\leqq}$ implies $\sigma_A^{\leqq}(x) \supseteq \sigma_A^{\leqq}(y)$.

On the other hand, suppose $D_k \in \sigma_B^{\leqq}(x)$, that is $[x]_B^{\leqq} \cap D_k \neq \phi$. Assume that $y \in [x]_B^{\leqq} \cap D_k$, then $y \in [x]_B^{\leqq}$ and $y \in D_k$. By Proposition 1(4), we obtain that $[x]_B^{\leqq} \supseteq [y]_B^{\leqq}$ is true, which implies $\sigma_A^{\leqq}(x) \supseteq \sigma_A^{\leqq}(y)$. Since $y \in [y]_A^{\leqq}$, we have $y \in [y]_A^{\leqq} \cap D_k$, which means $[y]_A^{\leqq} \cap D_k \neq \phi$. So we observe $D_k \in \sigma_A^{\leqq}(y) \subseteq \sigma_A^{\leqq}(x)$, that is $D_k \in \sigma_A^{\leqq}(x)$. Thus we conclude that $\sigma_B^{\leqq}(x) \subseteq \sigma_A^{\leqq}(x)$, i.e., B is an assignment consistent set of $S^{\leq}$.

Corollary 2 *Let $S^{\leq} = (U, A \cup D, F, G)$ be an information system with decisions, then $B \subseteq A$ is an approximation consistent set of $S^{\leq}$ if and only if when $\sigma_A^{\leqq}(x) \cap \sigma_A^{\leqq}(y) \neq \sigma_A^{\leqq}(y)$, $[x]_B^{\leqq} \cap [y]_B^{\leqq} \neq [y]_B^{\leqq}$ holds for $x, y \in U$.*

4 APPROACHES TO KNOWLEDGE REDUCTIONS IN INCONSISTENT INFORMATION SYSTEMS

This section provides approaches to assignment reduction based on dominance relation rough set model. Let first give the following notions.

Definition 5 *Let $S^{\leq} = (U, A \cup D, F, G)$ be an information system with decisions. we denote*

$$D^* = \{(x_i, x_j) : \sigma_A^{\leqq}(x_i) \subset \sigma_A^{\leqq}(x_j)\}$$

Denoted by f_{a_k} the value of a_k w.r.t. the object x. Define

$$D(x_i, x_j) = \begin{cases} \{a_k \in A : f_{a_k}(x_i) > f_{a_k}(x_j)\}, & (x_i, x_j) \in D^* \\ A, & (x_i, x_j) \notin D^* \end{cases}$$

Then $D(x_i, x_j)$ is said to be assignment discernibility attributes set. And $M = (D(x_i, x_j), x_i, x_j \in U)$ is referred as to assignment discernibility matrix of $S^{\leq}$.

Theorem 3 *Let $S^{\leq} = (U, A \cup D, F, G)$ be an information system with decisions, $B \subseteq A$, then B is an assignment consistent set if and only if $B \cap D(x, y) \neq \phi$, for all $(x, y) \in D^*$.*

Proof. Assume that B is an assignment consistent set of $S^{\leq}$. For any $(x, y) \in D^*$, we can obtain $\sigma_A^{\leqq}(x) \subset \sigma_A^{\leqq}(y)$, that is $\sigma_A^{\leqq}(x) \cap \sigma_A^{\leqq}(y) \neq \sigma_A^{\leqq}(y)$. From the Theorem 2, we have $[x]_B^{\leqq} \cap [y]_B^{\leqq} \neq [y]_B^{\leqq}$. Thus means there exist the following three cases between $[x]_B^{\leqq}$ and $[y]_B^{\leqq}$, which are (1) $[x]_B^{\leqq} \subset [y]_B^{\leqq}$, (2) $[x]_B^{\leqq} \cap [y]_B^{\leqq} = \phi$, (3) both $[x]_B^{\leqq} \cap [y]_B^{\leqq} \subset [x]_B^{\leqq}$ and $[x]_B^{\leqq} \cap [y]_B^{\leqq} \subset [y]_B^{\leqq}$. We will prove that $B \cap D(x, y) \neq \phi$ always holds in every case.

Case 1. If $[x]_B^{\leqq} \subset [y]_B^{\leqq}$, then there necessarily exist an element $z \in [y]_B^{\leqq}$, but $z \notin [x]_B^{\leqq}$. From $z \notin [x]_B^{\leqq}$, we can certainly find an element $a_k \in B$, such that $f_{a_k}(x) > f_{a_k}(z)$. On the other hand, the fact $f_{a_k}(y) \leq f_{a_k}(z)$ is true according to $z \in [y]_B^{\leqq}$. From the above, we can obtain $f_{a_k}(x) > f_{a_k}(y)$. Hence, we have $a_k \in D(x, y)$, i.e., $B \cap D(x, y) \neq \phi$.

Case 2. If $[x]_B^{\leqq} \cap [y]_B^{\leqq} = \phi$, then there exists necessarily an element $a_k \in B$, such that $f_{a_k}(x) > f_{a_k}(y)$, i.e. $B \cap D(x, y) \neq \phi$. Otherwise, if for all $a_l \in B$, $f_{a_l}(x) \leq f_{a_l}(y)$ always holds, then we observe $y \in [x]_B^{\leqq}$. This is contradiction.

Case 3. The proof is similar to Case 1, because we can also find certainly an element $z \in [y]_B^{\leqq}$, but $z \notin [x]_B^{\leqq}$ in the case.

Thus we can conclude that $B \cap D(x, y) \neq \phi$ for all $(x, y) \in D^*$.

Conversely, if every $(x, y) \in D^*$ satisfies $B \cap D(x, y) \neq \phi$, then we can select an $a_k \in B$, such that $a_k \in D(x, y)$. That is $f_{a_k}(x) > f_{a_k}(y)$, so $y \notin [x]_B^{\leqq}$. Since $y \in [y]_B^{\leqq}$ is true, we can obtain $[x]_B^{\leqq} \cap [y]_B^{\leqq} \neq [y]_B^{\leqq}$. On the other hand, since $(x, y) \in D^*$, we have $\sigma_A^{\leqq}(x) \subset \sigma_A^{\leqq}(y)$, which implies $\sigma_A^{\leqq}(x) \cap \sigma_A^{\leqq}(y) \neq \sigma_A^{\leqq}(y)$. Hence, we find that when $\sigma_A^{\leqq}(x) \cap \sigma_A^{\leqq}(y) \neq \sigma_A^{\leqq}(y)$, $[x]_B^{\leqq} \cap [y]_B^{\leqq} \neq [y]_B^{\leqq}$ holds. Thus we know that B is an assignment consistent set of $S^{\leq}$ in term of Theorem 2.

Definition 6 *Let $S^{\leq} = (U, A \cup D, F, G)$ be an information system with decisions, and $M = (D(x_i, x_j), x_i, x_j \in U)$ be assignment discernibility matrix of $S^{\leq}$. Denote*

$$F = \wedge\{\vee\{a_k : a_k \in D(x_i,x_j)\}, x_i,x_j \in U\}$$
$$= \wedge\{\vee\{a_k : a_k \in D(x_i,x_j)\}, x_i,x_j \in D^*,$$

F is called assignment discernibility function.

Theorem 4 *Let $S^{\leq} = (U, A \cup D, F, G)$ be an information system with decisions. The minimal disjunctive normal form of assignment discernibility function F is*

$$F = \vee_{k=1}^{p}(\wedge_{s=1}^{q})a_s.$$

Denote $B_k = \{a_s : s = 1,2,\cdots,q_k\}$, then $\{B_k : k = 1,2,\cdots,p\}$ is just set of all assignment reducts of $S^{\leq}$.

Proof. It follows directly from Theorem 3 and the definition of minimal disjunctive normal of the discernibility function.

Theorem 4 provides a practical approach to assignment reduction of information systems with decisions based on dominance relation. The following we will consider the information system in Table 1 using this approach.

Example 3 The following table (Table 2) is the assignment discernibility matrix of information system in Table 1.

x_i,x_j	x_1	x_2	x_3	x_4	x_5	x_6
x_1	A	A	A	A	A	A
x_2	A	A	A	A	A	A
x_3	A	A	A	A	A	A
x_4	a_1,a_3	a_3	a_1,a_3	A	a_3	A
x_5	A	A	A	A	A	A
x_6	a_1,a_3	a_3	A	a_1,a_2	a_3	A

Table 2

Consequently, we have

$$F = (a_1 \vee a_2 \vee a_3 \vee) \wedge (a_1 \vee a_3) \wedge (a_1 \vee a_2) \wedge a_3$$
$$= (a_1 \wedge a_3) \vee (a_2 \wedge a_3)$$

Therefore, from Theorem 4 we obtain that $\{a_1,a_3\}$ and $\{a_2,a_3\}$ are all assignment reducts of information system in Table 1, which accords with the result of Example 2.

5 CONCLUSION

It is well known that most of information systems are not only inconsistent, but also based on dominance relations because of various factors in practise. Therefore, it is meaningful to study the knowledge reductions in inconsistent information systems based on dominance relations. In this paper, we are concerned with approaches to the problem. The assignment reduction and approximation reduction are introduced in inconsistent systems based on dominance relations, and relationship between them are examined. The judgment theorem and discernibility matrix are obtained, from which we can provide the approach to knowledge reductions in inconsistent systems based on dominance relations.

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