Knowledge Reduction Based on Evidence Reasoning Theory in Ordered Information Systems*

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Abstract. Rough set theory has been considered as a useful tool to model the vagueness, imprecision, and uncertainty, and has been applied successfully in many fields. Knowledge reduction is one of the most important problems in rough set theory. However, in real-world most of information systems are based on dominance relations in stead of the classical rough set because of various factors. To acquire brief decision rules from systems based on dominance relations, knowledge reductions are needed. The main aim of this paper is to study the problem. The basic concepts and properties of knowledge reduction based on evidence reasoning theory are discussed. Furthermore, the characterization and knowledge reduction approaches based on evidence reasoning theory are obtained with examples in several kinds of ordered information system, which is every useful in future research works of the ordered information systems.

1 Introduction

The rough set theory, proposed by Pawlak in the early 1980s[1], is an extension of set theory for the study of intelligent systems characterized by inexact, uncertain or vague information and can serve as a new mathematica tool to soft computing. This theory has been applied successfully in machine learning, patten recognition, decision support systems, expert systems, data analysis, data mining, and so on. Since its introduction, the theory has generated a great deal of interest among more and more researchers.

Knowledge reduction is one of the hot research topics of rough set theory. Much study on this area had been reported and many useful results were

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obtained until now[2-8]. However, most work was based on consistent information systems, and the main methodology has been developed under equivalence relations which are often called indiscernibility relations. In practise, most of information systems are not only inconsistent, but also based on dominance relations because of various factors. In order to obtain the succinct decision rules from them by using rough set method, knowledge reductions are needed. In recent years, more and more attention has been paid to research of rough set. Many types of knowledge reductions have been proposed in the area of rough sets[9-15].

However, the original rough sets theory approach does not consider attributes with preference-ordered domains, that is, criteria. In many real situations, we are often face with the problems in which the ordering of properties of the considered attributes plays a crucial role. One such type of problem is the ordering of objects. For this reason, Greco, Matarazzo, and Slowinski[16-20]proposed an extension rough sets theory, called the dominance-based rough sets approach(DRSA) to take into account the ordering properties of criteria. This innovation is mainly based on substitution of the indiscernibility relation by a dominance relation. In DRSA, where condition attributes are criteria and classes are preference ordered, and many studies have been made in DRSA[21-25]. But useful results of knowledge reductions are very poor in ordered information systems until now.

In this paper the main objective is to study the problem. The basic concepts and properties of knowledge reduction based on evidence reasoning theory are discussed. Furthermore, the characterization and knowledge reduction approaches based on evidence reasoning theory are obtained with examples in several kinds of ordered information system, which is every useful in future research works of the ordered information systems.

2 Rough Sets and Ordered Information Systems

This section recalls necessary concepts of rough sets and ordered information systems. Detailed description of the theory can be found in [12, 24].

In rough set theory, an information system(IS) is an quadruple $\mathcal{I} = (U, AT, V, f)$, where U is a finite nonempty set of objects and AT is a finite nonempty set of attributes, $V = \bigcup_{a\in AT} V_a$ and V_a is a domain of attribute a, $f : U \times AT \to V$ is a total function such that $f(x, a) \in V_a$ for every $a \in AT, x \in U$ called information function.

A decision table is a special case of an information system in which, among the attributes, we distinguish one called a decision attribute. The other attributes are called condition attributes. Therefore, $\mathcal{I} = (U, AT \cup \{d\}, V, f)$ and $AT \cap \{d\} = \phi$,where set AT contains so-called condition attributes and d, the decision attribute.

For an information system (U, AT, V, f), $A \subseteq AT$,

$$R_A = \{(x_i, x_j) | f(x_i, a) = f(x_j, a), a \in A\}$$

is an equivalence relation(indiscernibility relation,Pawlak). So U can be classified in terms of R_A. The set which includes x can be expressed as $[x]_A$ and has the following properties:

$$[x]_{AT} \subseteq [x]_A, \quad R_{AT} \subseteq R_A.$$

The total of the classifications of U in terms of R_A can be represented as following:

$$U/R_A = \{[x]_A | x \in U\}.$$

It describes the meta-knowledge that can be represented by attribute A. In addition, the object set involved with the meta-knowledge of U/R_A can be represented by attribute A. It is denoted as $\sigma(U/R_A)$.

For any $X \subseteq U$, the upper and lower approximations can be represented as

$$\overline{R_A}(X) = \{x | [x]_A \cap X \neq \phi\}$$

$$\underline{R_A}(X) = \{x | [x]_A \subseteq X\}.$$

If $\overline{R_A}(X) = \underline{R_A}(X) = X$, X is the knowledge which can be represented by A and X is called a definable set. Otherwise, X is the knowledge which cannot be represented by A, and is called a rough set.

In an information systems, if the domain(scale) of a condition attributes is ordered according to a decreasing or increasing preference, then the attributes is a criterion.

Definition 2.1. An information system is called an ordered information system(OIS) if all condition attributes are criterions.

It is assumed that the domain of a criterion $a \in AT$ is complete pre-ordered by an outranking relation $\succeq_a$, and $x \succeq_a y$ means that x is at least as good as y with respect to criterion a. In the following, without any loss of generality, we consider a condition criterion having a numerical domain, that is, $V_a \subseteq \mathcal{R}$($\mathcal{R}$ denotes the set of real numbers) and being of type gain , that is, $x \succeq y \Leftrightarrow f(x,a) \geq f(y,a)$(according to increasing preference) of $x \succeq y \Leftrightarrow f(x,a) \leq f(y,a)$(according to decreasing preference), where $a \in AT, x, y \in U$. For a subset of attributes $A \subseteq AT$, we define $x \succeq_A y \Leftrightarrow x \succeq_a y, \forall a \in A$. That is to say x is at least as good as y with respect to all attributes in A. In general, the domain of the condition criterion may be also discrete, but the preference order between its values has to be provided.

The dominance relation that identifies granules of knowledge is defined as follows.

For a given OIS, we say that x dominates y with respect to $A \subseteq AT$, if $x \succeq_A y$, and denoted by $xR_A^{\geq}y$. Namely,

$$R_A^{\geq} = \{(y,x) \in U \times U | y \succeq_A x\}.$$

If $(y,x) \in R_A^{\geq}$, then y dominates x with respect to A.

Given $A \subseteq AT$ and $A = A_1 \cup A_2$,where attributes set A_1 according to increasing preference, A_2 according to decreasing preference. The granules of knowledge induced by the dominance relation $R_A^{\geq}$ are the set of objects dominating x,

$$\begin{aligned}[x]_A^{\geq} &= \{y \in U | f(y, a_1) \geq f(x, a_1)\ (\forall a_1 \in A_1) \\ &\quad \textit{and}\ \ f(y, a_2) \leq f(x, a_2)\ (\forall a_2 \in A_2)\} \\ &= \{y \in U | (y, x) \in R_A^{\geq}\}\end{aligned}$$

and the set of objects dominated by x,

$$\begin{aligned}[x]_A^{\leq} &= \{y \in U | f(y, a_1) \leq f(x, a_1)\ (\forall a_1 \in A_1) \\ &\quad \textit{and}\ \ f(y, a_2) \geq f(x, a_2)\ (\forall a_2 \in A_2)\} \\ &= \{y \in U | (x, y) \in R_A^{\geq}\}\end{aligned}$$

Which are called the $A - \textit{dominating}$ set and $A - \textit{dominated}$ set with respect to $x \in U$, respectively.

Let $U/R_A^{\geq}$ denote classification, which is the family set $\{[x]_A^{\geq} | x \in U\}$. Any element from $U/R_A^{\geq}$ will be called a dominance class. Dominance classes in $U/R_A^{\geq}$ do not constitute a partition of U in general. They may be overlap.

In the following, for simplicity, without any loss of generality, we only consider condition attributes with increasing preference.

Proposition 2.1. Let $R_A^{\geq}$ be a dominance relation. The following hold.

(1) $R_A^{\geq}$ is reflexive,transitive, but not symmetric, so it is not a equivalence relation.
(2) If $B \subseteq A \subseteq AT$, then $R_{AT}^{\geq} \subseteq R_A^{\geq} \subseteq R_B^{\geq}$.
(3) If $B \subseteq A \subseteq AT$, then $[x_i]_{AT}^{\geq} \subseteq [x_i]_A^{\geq} \subseteq [x_i]_B^{\geq}$
(4) If $x_j \in [x_i]_A^{\geq}$, then $[x_j]_A^{\geq} \subseteq [x_i]_A^{\geq}$ and $[x_i]_A^{\geq} = \cup\{[x_j]_A^{\geq} | x_j \in [x_i]_A^{\geq}\}$.
(5) $[x_j]_A^{\geq} = [x_i]_A^{\geq}$ iff $f(x_i, a) = f(x_j, a)$ $(\forall a \in A)$.
(6) $\mathcal{J} = \cup\{[x]_A^{\geq} | x \in U\}$ constitute a covering of U.

For any subset X of U, and A of AT define

$$\underline{R_A^{\geq}}(X) = \{x \in U | [x]_A^{\geq} \subseteq X\},$$

$$\overline{R_A^{\geq}}(X) = \{x \in U | [x]_A^{\geq} \cap X \neq \phi\},$$

$\underline{R_A^{\geq}}(X)$ and $\overline{R_A^{\geq}}(x)$ are said to be the lower and upper approximation of X with respect to a dominance relation $R_A^{\geq}$. And the approximations have also some properties which are similar to those of Pawlak approximation spaces.

Proposition 2.2. Let (U, AT, V, f) be an OIS and $X, Y \subseteq U$, then its lower and upper approximations satisfy the following properties.

(1) $\underline{R_A^{\geq}}(X) \subseteq X \subseteq \overline{R_A^{\geq}}(X)$.
(2) $\overline{R_A^{\geq}}(X \cup Y) = \overline{R_A^{\geq}}(X) \cup \overline{R_A^{\geq}}(Y)$;
$\underline{R_A^{\geq}}(X \cap Y) = \underline{R_A^{\geq}}(X) \cap \underline{R_A^{\geq}}(Y)$.
(3) $\underline{R_A^{\geq}}(X) \cup \underline{R^{\geq}}(Y) \subseteq \underline{R_A^{\geq}}(X \cup Y)$;
$\overline{R_A^{\geq}}(X \cap Y) \subseteq \overline{R_A^{\geq}}(X) \cap \overline{R^{\geq}}(Y)$.

(4) $\underline{R_A^{\geq}}(\sim X) = \sim \overline{R_A^{\geq}}(X)$; $\overline{R_A^{\geq}}(\sim X) = \sim \underline{R_A^{\geq}}(X)$.

(5) $\underline{R_A^{\geq}}(U) = U$; $\overline{R_A^{\geq}}(\phi) = \phi$.

(6) $\underline{R_A^{\geq}}(X) \subseteq \underline{R_A^{\geq}}(\underline{R^{\geq_A}}(X))$; $\overline{R_A^{\geq}}(\overline{R_A^{\geq}}(X)) \subseteq \overline{R_A^{\geq}}(X)$.

(7) If $X \subseteq Y$, then $\underline{R_A^{\geq}}(X) \subseteq \underline{R_A^{\geq}}(Y)$ and $\overline{R_A^{\geq}}(X) \subseteq \overline{R_A^{\geq}}(Y)$.

where $\sim X$ is the complement of X.

Example 2.1. Given an OIS in Table 1.

Table 1

$U \times AT$	a_1	a_2	a_3
x_1	1	2	1
x_2	3	2	2
x_3	1	1	2
x_4	2	1	3
x_5	3	3	2
x_6	3	2	3

From Table 1, we can see that the dominance classes determined by AT are

$$[x_1]_{AT}^{\geq} = \{x_1, x_2, x_5, x_6\}; [x_2]_{AT}^{\geq} = \{x_2, x_5, x_6\};$$

$$[x_3]_{AT}^{\geq} = \{x_2, x_3, x_4, x_5, x_6\}; [x_4]_{AT}^{\geq} = \{x_4, x_6\};$$

$$[x_5]_{AT}^{\geq} = \{x_5\}; [x_6]_{AT}^{\geq} = \{x_6\};$$

If $X = \{x_2, x_3, x_5\}$, then

$$\underline{R_{AT}^{\geq}}(X) = \{x_5\} \subseteq X; \overline{R_{AT}^{\geq}}(X) = \{x_1, x_2, x_3, x_5\} \supseteq X$$

Definition 2.2. An ordered decision table(ODT) is an ordered information system $\mathcal{I} = (U, AT \cup \{d\}, V, f)$, where $d(d \notin AT)$ is an overall preference called the decision, and all the elements of AT are criterions.

Definition 2.3. For an ODT $\mathcal{I} = (U, AT \cup \{d\}, V, f)$, if $R_{AT}^{\geq} \subseteq R_d^{\geq}$, then this ODT is consistent, denoted by CODT, otherwise, this ODT is inconsistent(IODT).

Example 2.2. Given an CODT based on Table 1 in Table 2.

From the table, we have

$$[x_1]_d^{\geq} = [x_3]_d^{\geq} = \{x_1, x_2, x_3, x_4, x_5, x_6\};$$

$$[x_2]_d^{\geq} = [x_5]_d^{\geq} = [x_6]_d^{\geq} = \{x_2, x_5, x_6\};$$

$$[x_4]_d^{\geq} = \{x_2, x_4, x_5, x_6\}$$

Table 2

$U \times (AT \cup d)$	a_1	a_2	a_3	d
x_1	1	2	1	1
x_2	3	2	2	3
x_3	1	1	2	1
x_4	2	1	3	2
x_5	3	3	2	3
x_6	3	2	3	3

Obviously, by the above and Example 2.1, we have $R_{AT}^{\geq} \subseteq R_d^{\geq}$, so the DOT in Table 2 is CODT.

Example 2.3. We can obtain a IODT(Table 3) in stead of the value domain of d by {3,2,1,2,3,1}, respectively in Example 2.2.

Table 3

$U \times (AT \cup d)$	a_1	a_2	a_3	d
x_1	1	2	1	3
x_2	3	2	2	2
x_3	1	1	2	1
x_4	2	1	3	2
x_5	3	3	2	3
x_6	3	2	3	1

From the table, we have

$$[x_1]_d^{\geq} = [x_5]_d^{\geq} = \{x_1, x_5\}; \quad [x_2]_d^{\geq} = [x_4]_d^{\geq} = \{x_1, x_2, x_4, x_5\};$$

$$[x_3]_d^{\geq} = [x_6]_d^{\geq} = \{x_1, x_2, x_3, x_4, x_5, x_6\}.$$

Obviously, by the above and Example 2.1, we have $R_{AT}^{\geq} \not\subseteq R_d^{\geq}$, so the ODT in Table 3 is IODT.

3 Knowledge Reduction Approach Based on Evidence Reasoning in OIS and ODT

For an information system (U, AT, V, f) in Pawlak rough set theory, if $R_A = R_{AT}$ when $A \subset AT$, for any $a \in A$, $R_{A-\{a\}} \neq R_{AT}$, then A is a reduction of the information system. Moreover, reduction exists and is not unique[11].The set of attributes that is included in all reductions is called the core. Similarly, the following can be found in [16].

Definition 3.1. For an ordered information system OIS (U, AT, V, f), if $R_A^{\geq} = R_{AT}^{\geq}$ when $A \subset AT$, for any $a \in A$, $R_{A-\{a\}}^{\geq} \neq R_{AT}^{\geq}$, then A is a reduction of the information system. The set of attributes that is included in all reductions is called the core.

Definition 3.2. For an consistent ordered decision table CODT $\mathcal{I} = (U, AT \cup \{d\}, V, f)$, if $R_A^{\geq} \subseteq R_d^{\geq}$ when $A \subset AT$, for any $a \in A$, $R_{A-\{a\}}^{\geq} \nsubseteq R_d^{\geq}$, then A is a reduction of the CODT.

Let $\mathcal{I} = (U, AT \cup \{d\}, V, f)$ be an IODT , and for any set $A \subseteq AT$, $R_A^{\geq}, R_d^{\geq}$ be dominance relations derived from condition attributes set AT and decision attributes set $\{d\}$ respectively,denote

$$U/R_A^{\geq} = \{[x_i]_A^{\geq} | x_i \in U\},$$

$$U/R_d^{\geq} = \{d_1, d_2, \cdots, d_r\},$$

$$\sigma_A^{\geq}(x) = \{d_j | d_j \cap [x]_A^{\geq} \neq \phi, x \in U\},$$

where $[x]_A^{\geq} = \{y \in U | (y, x) \in R_A^{\geq}\}$.

From the above, we can have the following propositions immediately.

Proposition 3.1. The following always hold.

(1) $\overline{R_A^{\geq}}(d_j) = \cup\{[x]_A^{\geq} : d_j \in \sigma_A^{\geq}(x)\}$.

(2) If $B \subseteq A$, then $\sigma_A^{\geq}(x) \subseteq \sigma_B^{\geq}(x), \forall x \in U$.

(3) If $[x]_A^{\geq} \supseteq [y]_A^{\geq}$, then $\sigma_A^{\geq}(x) \supseteq \sigma_A^{\geq}(y), \forall x, y \in U$.

Definition 3.3. Let $\mathcal{I} = (U, AT \cup \{d\}, V, f)$ be an IODT. If $\sigma_A^{\geq}(x) = \sigma_{AT}^{\geq}(x)$, for all $x \in U$, we say that A is an assignment consistent set of $\mathcal{I}$. If A is an assignment consistent set, and no proper subset of A is assignment consistent set, then A is called an assignment consistent reduction of IODT.

An assignment consistent set is a subset of attributes set that preserves the possible decisions of every object.

Obviously, the reductions of OIS and ODT also exist and is not unique.

In evidence reasoning, for a universe U a mass function can be defined by a map $m : 2^U \to [0, 1]$, which is called a basic probability assignment and satisfies two axioms:

$$(1) \quad m(\phi) = 0$$

$$(2) \quad \sum_{X \subseteq U} m(X) = 1.$$

A subset $X \subseteq U$ with $m(X) > 0$ is called a focal element. Using the basic probability assignment, belief and plausibility of X are expressed as

$$Bel(X) = \sum_{Y \subseteq X} m(Y),$$

$$Pl(X) = \sum_{Y \cap X \not\phi} m(Y).$$

In [26], the authors discussed the interpretations of belief functions in the theory of Pawlak rough sets. For an information system (U, AT, V, f), $X \subseteq U, A \subseteq AT$, it is represented as follows:

$$Bel(X) = \frac{|\underline{R_A}(X)|}{|U|} = \sum_{Y \subseteq X} m(Y)$$

$$Pl(X) = \frac{|\overline{R_A}(X)|}{|U|} = \sum_{Y \cap X = \phi} m(Y)$$

Then $Bel(X)$is the belief function and $Pl(X)$ is the plausibility function of U.

For an OIS and for any set $A \subseteq AT$, the classification of $U = \{x_1, x_2, \cdots, x_k\}$ by the dominance relation $R^{\geq}_{AT}$ is denoted as

$$U/R^{\geq}_{AT} = \{[x_1]^{\geq}_{AT}, [x_2]^{\geq}_{AT}, \cdots, [x_k]^{\geq}_{AT}\}.$$

Let

$$D = \{(x_i, x_j)|i, j \in \{1, 2, , \cdots, k\}\}$$

then the element number of D is k^2.

And we note that

$$W(x_i, x_j) = \{a|f(x_i, a) < f(x_j, a)\}$$

Specially, when $W(x_i, x_j) = \phi$, we denoted as

$$D' = \{(x_i, x_j)|W(x_i, x_j) = \phi\}$$

$$H(A) = \{(x_i, x_j)|W(x_i, x_j) = A\}.$$

Then

$$m(A) = \frac{|H(A)|}{|D - D'|} \qquad (A \subseteq AT)$$

is the mass function on AT. As a result, we have belief function $Bel(A)$ and plausibility function $Pl(A)$.

Proposition 3.2. For an OIS $\mathcal{I} = (U, AT, V, f)$, if $A \subset AT, Pl(A) = 1$ and if $B \subseteq A$ and $B \neq A$, we have $Pl(B) < 1$, then A is a reduction of the OIS $\mathcal{I}$.

Proof. Since $Pl(A) = 1$ if and only if

$$\sum_{B \cap A \neq \phi} m(B) = 1.$$

This means that, for any $m(B) \neq 0$, B must have the form of $B \cap A \neq \phi$, i.e. for any $H(B) \neq \phi$, we have $B \cap A \neq \phi$. Then $U/R^{\geq}_{AT}$ can be identified by A. For the dame reason, $Pl(B) < 1$ if there exist B' such that $H(B') \neq \phi$ but $B' \cap B = \phi$. Therefore $U/R^{\geq}_{AT}$ cannot be identified by B' completely.

Example 3.1. Let we consider the OIS $\mathcal{I} = (U, AT, V, f)$ in Example 2.1 here. Note that

$$A_1 = \{a_1, a_3\} \quad A_2 = \{a_3\} \quad A_3 = \{a_2\} \quad A_4 = \{a_1, a_2\}$$
$$A_5 = AT = \{a_1, a_2, a_3\}$$

The classification of $U/R^{\geq}_{AT}$ is as follows:

$$[x_1]^{\geq}_{AT} = \{x_1, x_2, x_5, x_6\}; [x_2]^{\geq}_{AT} = \{x_2, x_5, x_6\};$$
$$[x_3]^{\geq}_{AT} = \{x_2, x_3, x_4, x_5, x_6\}; [x_4]^{\geq}_{AT} = \{x_4, x_6\};$$
$$[x_5]^{\geq}_{AT} = \{x_5\}; [x_6]^{\geq}_{AT} = \{x_6\};$$

Then the matrix of $W(x_i, x_j)$ is as in Table 4.

Table 4

	x_1	x_2	x_3	x_4	x_5	x_6
x_1	ϕ	A_1	A_2	A_1	A_5	A_1
x_2	ϕ	ϕ	ϕ	A_2	A_3	A_2
x_3	A_3	A_4	ϕ	A_1	A_4	A_5
x_4	A_3	A_4	ϕ	ϕ	A_4	A_4
x_5	ϕ	ϕ	ϕ	A_2	ϕ	A_2
x_6	ϕ	ϕ	ϕ	ϕ	A_3	ϕ

From the above, we have $|D - D'| = 20$, and $m(A_1) = 4/20$, $m(A_2) = 5/20$, $m(A_3) = 4/20$, $m(A_4) = 5/20$, $m(A_5) = 2/20$.

Therefore, for $A = \{a_2, a_3\}$, we can find $A \cap A_i \neq \phi (i = 1, 2, \cdots, 5)$, and $Pl(A) = 1$. Since $Pl(\{a_2\}) = Pl(A_3) = 4/20$ and $Pl(\{a_3\}) = Pl(A_2) = 5/20$. Hence, $A = \{a_2, a_3\}$ is a reduction of the OIS.

Next, we will mainly consider the method of the reduction in ODT.

Firstly, the CODT is considered.

For the consistent information system $\mathcal{I} = (U, AT \cup \{d\}, V, f)$ with target d, i.e. CODT.

For any set $A \subseteq AT$ we note that

$$W(x_i, x_j) = \begin{cases} \{a | f(x_i, a) < f(x_j, a)\}, & f(x_i, d) < f(x_j, d). \\ \phi, & f(x_i, d) \geq f(x_j, d). \end{cases}$$

And

$$H(A) = \{(x_i, x_j) | W(x_i, x_j) = A\}.$$
$$D = \{(x_i, x_j) | i, j \in \{1, 2, \cdots, k\}\}.$$

Another, when $W(x_i, x_j) = \phi$, we denoted as

$$D' = \{(x_i, x_j) | W(x_i, x_j) = \phi\}$$

Then

$$m(A) = \frac{|H(A)|}{|D - D'|} \qquad (A \subseteq AT)$$

is the mass function on AT. As a result, we can calculate the belief function $Bel(A)$ and plausibility function$Pl(A)$.

Proposition 3.3. For an CODT $\mathcal{I} = (U, AT \cup \{d\}, V, f)$, if $A \subset AT, Pl(A) = 1$ and if $B \subseteq A$ and $B \neq A$, we have $Pl(B) < 1$, then A is a reduction of the CODT $\mathcal{I}$.

Example 3.2. Here the CODT $\mathcal{I} = (U, AT \cup \{d\}, V, f)$ in Example 2.2 be considered. Note that

$$A_1 = \{a_1, a_3\} \quad A_2 = \{a_1, a_2\} \qquad A_3 = AT = \{a_1, a_2, a_3\}$$

Then the matrix of $W(x_i, x_j), i, j \in \{1, 2, \cdots, 6\}$ is as in Table 5.

Table 5

	x_1	x_2	x_3	x_4	x_5	x_6
x_1	ϕ	A_1	ϕ	A_1	A_3	A_1
x_2	ϕ	ϕ	ϕ	ϕ	ϕ	ϕ
x_3	ϕ	A_2	ϕ	A_1	A_2	A_3
x_4	ϕ	A_2	ϕ	ϕ	A_2	A_2
x_5	ϕ	ϕ	ϕ	ϕ	ϕ	ϕ
x_6	ϕ	ϕ	ϕ	ϕ	ϕ	ϕ

We have $|D - D'| = 11$, and $m(A_1) = 4/11$, $m(A_2) = 5/11$, $m(A_3) = 2/11$.

Therefore, for $A = \{a_2, a_3\}$ and $A' = \{a_1\}$, we can find $A \cap A_i \neq \phi$, and$A' \cap A_i \neq \phi (i = 1, 2, 3)$, moreover $Pl(A) = Pl(A') = 1$. Since $Pl(\{a_2\}) = 7/11$ and $Pl(\{a_3\}) = 6/11$. Hence, $A = \{a_2, a_3\}$ and $\{a_1\}$is a reduction of the CODT.

Finally, we will give the approach to reduction of IODT.

For any set $A \subseteq AT$ we note that

$$W(x_i, x_j) = \begin{cases} \{a | f(x_i, a) < f(x_j, a)\}, & \sigma^{\geq}_{AT}(x_i) \subset \sigma^{\geq}_{AT}(x_j). \\ \phi, & \sigma^{\geq}_{AT}(x_i) \not\subset \sigma^{\geq}_{AT}(x_j). \end{cases}$$

And

$$H(A) = \{(x_i, x_j) | W(x_i, x_j) = A\}.$$

$$D = \{(x_i, x_j) | i, j \in \{1, 2, \cdots, k\}\}.$$

Another, when $W(x_i, x_j) = \phi$, we denoted as

$$D' = \{(x_i, x_j) | W(x_i, x_j) = \phi\}$$

Then

$$m(A) = \frac{|H(A)|}{|D - D'|} \qquad (A \subseteq AT)$$

is the mass function on AT.

Hence, we can obtain the following.

Proposition 3.4. For an IODT $\mathcal{I} = (U, AT \cup \{d\}, V, f)$, if $A \subset AT, Pl(A) = 1$ and if $B \subseteq A$ and $B \neq A$, we have $Pl(B) < 1$, then A is an assignment consistent reduction of the IODT $\mathcal{I}$.

Example 3.3. IODT $\mathcal{I} = (U, AT \cup \{d\}, V, f)$ in Example 2.3 be considered. Note that

$$A_1 = \{a_1, a_3\} \quad A_2 = \{a_1, a_2\} \quad A_3 = \{a_3\} \quad A_4 = AT = \{a_1, a_2, a_3\}$$

Then the matrix of $W(x_i, x_j), i, j \in \{1, 2, \cdots, 6\}$ is as in Table 6.

Table 6

	x_1	x_2	x_3	x_4	x_5	x_6
x_1	ϕ	ϕ	ϕ	A_1	ϕ	A_1
x_2	ϕ	ϕ	ϕ	A_3	ϕ	A_3
x_3	ϕ	ϕ	ϕ	A_1	ϕ	A_4
x_4	ϕ	ϕ	ϕ	ϕ	ϕ	A_2
x_5	ϕ	ϕ	ϕ	A_3	ϕ	A_3
x_6	ϕ	ϕ	ϕ	ϕ	ϕ	ϕ

We have $|D - D'| = 9$, and $m(A_1) = 3/9$, $m(A_2) = 1/9$, $m(A_3) = 4/9$, $m(A_4) = 1/9$.

Therefore, for $A = \{a_2, a_3\}$ and $A' = \{a_1, a_3\}$, we can find $A \cap A_i \neq \phi$, and$A' \cap A_i \neq \phi (i = 1, 2, 3, 4)$, moreover $Pl(A) = Pl(A') = 1$. Since $Pl(\{a_1\}) = 5/9$, $Pl(\{a_2\}) = 2/9$ and $Pl(\{a_3\}) = 8/9$. Hence, $A = \{a_2, a_3\}$ and $\{a_1, a_3\}$is an assignment consistent reduction of the IODT.

4 Conclusion

It is well-known that rough set theory has been regarded as a generalization of the classical set theory in one way. Furthermore, this is an important mathematical tool to deal with vagueness. We proposed a new technique of knowledge reduction using rough sets with evidence reasoning theory. The basic concepts and properties of knowledge reduction based on evidence reasoning theory are discussed. Furthermore, the characterization and knowledge reduction approaches based on evidence reasoning theory are obtained with examples in several kinds of ordered information system, which is every useful in future research works of the ordered information systems. The successful applications of rough set theory in a variety of intelligent systems will amply demonstrate heir usefulness and versatility.

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